Factor Analysis of the Electrocardiogram
Test of Electrocardiographic Theory: Normal Hearts

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With the assistance of O. F. Brown

Earlier electrocardiographic recording procedures utilized the limb leads described by Waller and standardized by Einthoven. In 1913, Einthoven, Fahr, and de Waart made the first vectorial approach to the study of electrocardiographic potentials, using the standard limb leads to estimate the mean direction and magnitude of the electromotive forces produced by the heart in the plane defined by these leads. Their assumptions, and the inherent limitations, are well known. In 1932, the recording of the unipolar leads was developed by Wilson and his co-workers, and in 1938, Wilson and Johnston described a technique for recording a planar vectorcardiogram on a cathode ray oscilloscope. In this procedure, 2 voltages recorded from "perpendicular co-planar" leads are fed into the vertical and horizontal deflection plates of an oscilloscope and a continuous vector loop is described. Procedures for stereo-vectorcardiography (3-dimensional vectorcardiography) have been described by Schmitt, but generally vector loops are recorded sequentially in 2 perpendicular planes.

Modern electrocardiographic practice relies generally on scalar "unipolar" recording techniques. Since Wilson's time, however, much research has concerned the vectorial approach to electrocardiography. In general, the conflict between these systems concerns ventricular depolarization. To reduce these 2 approaches to their simplest forms (and in so doing we may be reducing to absurdity), we can say that the recording of chest leads rests partially on the assumption that during ventricular depolarization some current sources within the heart are sufficiently close to the body surface to cause potentials which are, in part, local or "unique" at particular precordial points. An increase in the number of recording points thus should increase the amount of information. The vectorcardiographic approach, on the other hand, is based on the assumption that the potentials on the body surface during ventricular depolarization can be considered to arise largely from a single fixed-center current dipole lying somewhere within the chest and varying in magnitude and direction.

During ventricular depolarization, the boundaries between active and resting myocardium follow a complex pathway through the ventricles. Since at any instant such a boundary is composed of an infinite number of dipoles, it may seem unlikely that a single dipole can represent all of ventricular depolarization. However, if a current source is sufficiently removed from recording points, all but dipolar components will cancel. We may thus ask if the heart is so far from the body surface that only "dipolar" voltages are left.

A dipole can be resolved into 3 mutually perpendicular current sources which share a center. The instantaneous value of each of these sources equals the instantaneous projection of the dipole on the X, Y, and Z axes. The instantaneous voltages $E_p$ recorded at a body surface point $P$ is proportional to the magnitude of each of these projections:

$$E_p = k'X + k''Y + k'''Z.$$  

The constants $k'$, $k''$, and $k'''$ are determined by the geometry and resistivity of the body.
and are constant throughout QRS. A dipolar system is thus a 3-function system and is a special case of the general situation in which each of 3 current generators, $P_1$, $P_2$, and $P_3$, is connected to electrodes fixed on or within the body. In the more general case, the electrodes connected to each generator can be of any number, size, shape, or position. In the dipole, each pair of input "electrodes" is perpendicular to the other 2 and they share a center. In either case, the potential at a given body surface point at a particular instant is a linear function of the current in each of the 3 generators. For a nondipolar 3-function system, $E_p = k_1P_1 + k_2P_2 + k_3P_3$. Again, the constants are geometrically fixed. Either 3-function system is obviously a system about which we can learn no more than the instantaneous magnitude of each of the 3 primary generators. If there are more than 3 current sources within the chest, each source will contribute to the potential at a point on the body surface:

$$E_p = k_1P_1 + k_2P_2 + k_3P_3 + k_4P_4 \ldots k_nP_n.$$  

The aim of electrocardiography is to describe, as well as possible from body surface recordings, the current sources within the heart, or at least to distinguish normal from abnormal hearts. The number of recordings necessary to describe the equivalent generator is equal to the number of sources contributing to this generator. For an $n$-function system, we need $n$ (nonredundant) recordings. If we record fewer than $n$ electrocardiographic leads, we will have missed information. If we record more than $n$ leads, we will have redundancy.

It was demonstrated by Helmholtz\textsuperscript{7} and is implicit in Green's theorem\textsuperscript{8} that if one knows only the potential distribution on a surface, one cannot determine a unique internal generator. This limiting theorem is very important in electrocardiography where we can know only the voltages on the body surface. Despite this limitation, it is possible to know how many basic functions or voltage generators are necessary to account for the voltages recorded on the body surface. The procedure known as factor analysis has been widely used in psychologic testing and elsewhere in the social sciences.\textsuperscript{9} As the name indicates, factor analysis is a way of examining mathematically a large number of pieces of information to determine the smallest number of "factors" or independent variables which (in linear combination) can account for all the information. The method is quite ideal for electrocardiographic study, since any number of simultaneous and related measurements (i.e., electrocardiographic leads) can be examined, and the number of unique pieces of information necessary to account for all of the information\textsuperscript{*} or squared deviation from zero in these leads can be determined. The calculations also tell us how much of the squared deviation is accounted for by a first factor, by a second factor, by a third factor, and so on. In this study, an IBM 650 computer was used for the calculations. The technical details of the procedure employed for factor analysis will be presented as an appendix to this paper, but for the nontechnical reader, a further discussion follows.

The previous equation, $E_p = k_1P_1 + k_2P_2 + k_3P_3$, can be geometrically or arithmetically tested. In the geometrical test, 4 electrocardiographic leads, $A$, $B$, $C$, $D$ are recorded and a 3-dimensional plot is made in space with the coordinates $A/D$, $B/D$, and $C/D$. If the system is a 1-function system, all values of $A/D$, $B/D$, and $C/D$ will fall at a single point. In a 2-function system, they will define a line and in a 3-function system a plane. This test is applicable only for 4 or fewer leads and for 3 or fewer factors. We have used this procedure extensively and find the results entirely in accord with those to be re-

\textsuperscript{*}Information is used here rather than "variance." For practical purposes, however, the term variance might be used. Variance of a curve is the sum of all squared deviations from the mean divided by the number of observations where the means have previously been set to zero. In our procedure, the variances of the curves are set equal but the deviations from the baseline are used instead of the deviations from the mean in the above formula. This preserves the "electrical axis."
ported in this paper. The arithmetic test has been described by Burger and used by others. In it, 3 leads are fed into amplifiers and a variable fraction of each lead is extracted, using potentiometers. These fractions are linearly combined to see if they can make a fourth lead. We have also used this system extensively and obtained results which, although less quantitative, do not differ from those reported here. The factor analysis can be considered a geometrical test in which the number of initial leads is not limited, or as an arithmetical test in which the initial number of inputs is not limited.

Methods

Between 8 and 36 "simultaneous" electrocardiographic leads were recorded for 17 normal persons. The subjects were mostly young men, 20 to 40 years old. In practice, 8 to 12 leads were recorded at 1 time; all but 1 or 2 of these leads were then changed and a second record was taken, and so on. The leads which were repeated served as time references and made it possible to line up the other leads for time correspondence. In general, all of the conventional bipolar limb leads were recorded, plus the unipolar chest (V) leads. Where more than 8 leads were recorded, high and low V leads and back leads were used. In some experiments, a back or leg electrode was used as an indifferent point, and the potential difference was recorded between this and the extremity and chest electrodes. In addition, in a number of later experiments, the potential difference was recorded between adjacent chest lead positions, that is, V1-V2, V5-V6, etc.

The recordings were made with an amplifier-oscilloscope system which has a frequency response flat to well over 20,000 cycles. The preamplifiers used had an input impedance of 1,000 megohms and are push-pull and direct-coupled. At times, a 3-second time constant was used. Simultaneous time marks at 2-msec. intervals were fed into all channels from a single time-pip generator to make possible the reading of all records at identical instants. A 16-channel oscilloscope, or a Ryecom oscilloscope, was used with moving film. Film speed was sufficient to resolve clearly 2 msec. in time. After the electrocardiograms had been recorded and developed, they were photographically enlarged and the value of each lead carefully read at each time pip. In all experiments, the ratio of trace width to maximum amplitude was such that it was possible to read each deflection to 1.0 or, at worst, 1.5 per cent of maximal deflection.

A data matrix was thus obtained consisting of 8 to 36 electrocardiographic leads with the value (i.e., magnitude) for each at about 40 instants in time. The data matrix was punched onto cards for the IBM computer. The factor analysis produced the following information: 1. A set of "factors" or hypothetical electrocardiograms \( P_1, P_2, P_3, \) etc. 2. A set of factor loadings or constants \( (K) \) to be multiplied by each factor to produce each recorded lead; a separate constant relates each lead to each factor:

\[
E_1 = K_1P_1 + K_2P_2 + K_3P_3, \text{ etc.}
\]

3. A computation of the percentage of the squared deviation removed from the entire input data by each factor (plus its loading constants) combined sequentially. 4. A computation of the percentage of the squared deviation removed from each recorded lead by each factor plus its loading constants, also combined sequentially. 5. Data for the replotting of each lead as it would be reconstructed from 1, 2, 3, or more, factors plus loading constants. 6. The correlation \( (r) \) of each lead with each factor. It was possible to have leads and predicted leads plotted by the computer's print-out unit.

Results

In all normal subjects, the first factor accounts for 50 to over 80 per cent of the squared deviation of all leads; that is, the sum of squared discrepancies between predicted and observed curves divided by the sum of squared values of the observed curve is less than .50 and less than .20 in some cases. The second factor, plus the first, usually accounts for over 85 to 99 per cent of the squared deviation, and 3 factors account for more than 95 per cent of the over-all squared deviation. The percentage of squared deviation left in the over-all recordings and the percentage of squared deviation left in the individual leads after 3 factors have been extracted are indicated in table 1. Figure 1 shows original leads (normalized) from 1 experiment, the leads as predicted from 3 factors, the 3 factors extracted by the computer, and the loading constants (in tabular form). As can be seen, the predicted leads are virtually identical with the recorded leads. A control experiment was done in a water bath with 3 input functions and 8 recorded leads. Here the percentage of squared deviation accounted for by 3 factors (99.9 per cent) was greater than in any of the individuals studied.

Discussion

These results indicate that the potentials found on the body surface during QRS in our series of normal adults can almost entirely be accounted for by 3 factors. Even when poten-
Burger\textsuperscript{10} described the procedure of linearly combining 3 leads to produce a fourth. Okada et al.\textsuperscript{11} modified this procedure and attempted to combine 2 leads to produce a third. They felt that they had eliminated the need to combine 3 leads by selecting their recording points. The "synthesis coefficients" which indicate the "nondipole" content of the leads used (i.e., those voltages which did not result from 2 functions) averaged 23 ± 7 per cent in 7 measurements in each of 19 normals. In Burger's study, no figure was given for the accuracy of the 3-function concept.\textsuperscript{10} In Okada's study, the fit from 2 factors was far poorer than in our study. This discrepancy results largely from the methods used to describe the effectiveness of curve fitting. Okada's figures are closer to the maximum deviation, ours to the average deviation squared. The effectiveness of curve fitting is shown in figure 1. Where our results indicate a 95 per cent fit, \( r \), the correlation between predicted and observed curves, will be about .91; where the fit is 90 per cent, \( r \) will exceed .80. The actual discrepancy between predicted and observed curves (non-3-function residual) is close to the noise level and consists largely of 60-cycle pickup, and other artifacts. It is also possible that Okada et al.\textsuperscript{11} are testing the dipole hypothesis in either a general or specific form, while our study tests for 3 functions. We do not consider it profitable to devise systems for recording non-3-function components of body surface potentials.

These studies will be extended to include more normal adults, children, and individuals with electrocardiographic pathology. The conclusions in this group of normals, however, raise certain questions:

A first question concerns the relationship between ventricular depolarization and the body surface QRS complex. The events within the heart are sufficiently complex\textsuperscript{6} and the heart is anatomically so close to the body surface that these results are contrary to the intuitive predictions of the writers when this study was initiated. Further, the experiments have been purposely designed to find non-3-
Figure 1

This figure shows 7 leads recorded in 1 experiment (a total of 17 leads were actually recorded in this experiment). The original leads are indicated by the black lines. The leads, as predicted from 3 factors, are indicated by the dotted lines. On the right are shown the 3 basic factors extracted by the factor analysis procedure in the order of their contribution to the record. The total duration of the leads and of the factors is about 90 msec. The readings were made at each 2 msec. The leads are all normalized to the same mean value. The recording positions are indicated on each lead. MB means mid-back and conventional V lead designations are used; V4H is 1 interspace above V4 and AAL indicates anterior axillary line of the fourth intercostal space. The first 2 leads have reversed polarity. Factor loading constants are presented in tabular form at the lower right. The instantaneous predicted value of a lead is determined by adding the products of the instantaneous values of each factor times its loading constant.

The conductivity of the blood within the cavities acts to shunt current and decreases the “information” at the body surface. Since no answer has been given to this or related questions, any attempt to derive the body surface electrocardiogram from knowledge of intracardiac activity must at present be qualitative.
A second question concerns the nature of the equivalent generator at the body surface demonstrated by this technic. We can state that the equivalent generator is a 3-function generator, but we must ask if it is any particular type of 3-function system such as a dipole. We may further ask whether this study indicates that vectorcardiographic recording is a justifiable procedure. As indicated previously, the dipole is a special case of the more general 3-function generator and, too, a given body surface potential distribution can be explained by an infinite number of internal generators. There thus appears to be no unique generator which can account for these body surface potentials. Many possible internal generators of equal complexity, some of which may or may not be dipolar, can account for the potentials on the body surface. It has been considered that a dipole within the chest would produce a complete set of "mirror" potentials on the body surface. That is to say, for every potential recorded on the body surface, another potential of opposite sign and of magnitude proportional throughout the QRS complex would be recorded at some other site. Schmitt and Simonsouir found many such mirror patterns. The previously discussed study by Frank was considered by him to demonstrate the existence of mirror images. It does not appear that this condition separates the dipolar 3-function system from the more general 3-function systems or from systems involving more than 3 generators, since a number of possible voltage generators which are not dipolar would give rise to a complete set of mirror images on the surface of the body. For instance, if we idealize the body as a cylinder, any arrangement of inputs in which each is symmetrical with respect to the long axis, will produce a complete set of mirror images on the surface. Furthermore, it has not been demonstrated that an eccentric dipole within the chest would produce a complete set of mirror images on the body surface. However, Frank was able to duplicate reasonably many body surface potentials by energizing a fixed dipole in a torso model. No studies to date demonstrate the nature of the equivalent generator as seen at the body surface.

One piece of evidence which bears on the 3-function versus the dipole argument, is found in the work of Nelson who measured the potentials at 1 level around the torso with 26 electrodes arranged in a belt. He found that at an instant during QRS, it was possible to have several intersections of zero potential with the body surface. This finding would not be incompatible with the 3-function theory, but would be incompatible with the dipole hypothesis.

Summary
The factor analysis of electrocardiograms from 17 normal individuals indicates that over 95 per cent of all the electrocardiographic "information" is accounted for by 3 factors. In the entire series, all individual leads were more than 83 per cent accounted for by 3 factors. These results indicate that there are no significant voltages at the body surface in normal individuals which can be ascribed to more than 3 internal generators. While these results indicate a 3-function system, they do not indicate a dipolar system since a dipole is a special case of a more general 3-function system.

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Appendix*
The analysis of electrocardiographic data may be regarded as a problem in curve fitting. The fundamental equation of the system is:

\[ f_i(t) = a_{1i}g_1(t) + a_{2i}g_2(t) + \ldots + a_{pi}g_p(t) + e_i \]

where \( f_i(t) \) is the voltage curve at point \( i \), \( g_j(t) \) is one of \( p \) generating curves, \( a_{ji} \) is a weighting constant, and \( e_i \) is an error component. Since we

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are ordinarily unable to specify the exact forms of the curves \( f_i(t_k) \) and \( g_j(t_k) \), we would take for a given set of \( n \) instants in time:

\[
\begin{align*}
\hat{f}_i(h) & = a_i g_i(t_k) + a_i g_i(t_k) + \cdots + a_i g_i(t_k) + \epsilon_i, \\
& \qquad \qquad \qquad \qquad \quad \text{for } i = 1, 2, \ldots, n, \\
\hat{g}_j(h) & = b_j g_j(t_k) + b_j g_j(t_k) + \cdots + b_j g_j(t_k) + \epsilon_j, \\
& \qquad \qquad \qquad \qquad \quad \text{for } j = 1, 2, \ldots, m.
\end{align*}
\]  

Alternately we may write:

\[
\begin{align*}
\hat{x}'_k & = [f_1(t_k), f_2(t_k), \ldots, f_n(t_k)] \\
\hat{z}'_k & = [g_1(t_k), g_2(t_k), \ldots, g_m(t_k)]
\end{align*}
\]  

and

\[
X = Z A' + E,
\]

where

\[
A' = \begin{bmatrix} a_{11} & \cdots & a_{1m} \\ a_{21} & \cdots & a_{2m} \\ \cdots & \cdots & \cdots \\ a_{n1} & \cdots & a_{nm} \end{bmatrix}
\]

and

\[
E = \begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \cdots & \epsilon_{1m} \\ \epsilon_{21} & \epsilon_{22} & \cdots & \epsilon_{2m} \\ \cdots & \cdots & \cdots \\ \epsilon_{n1} & \epsilon_{n2} & \cdots & \epsilon_{nm} \end{bmatrix}.
\]

In fitting such a linear system as \( Z A' \) to \( X \) we might require that for our choice of \( Z \) and \( A \) (the number of generating functions) the sum of squared discrepancies between \( X \) and \( Z A' \) be minimized, that is, that the sum of squares of all elements of \( E \) be minimized.\(^{17,18}\) This may easily be accomplished if we let \( A \) be the latent vectors or characteristic vectors of the matrix \( X'X \) associated with the \( P \) largest roots of \( X'X \) and \( Z \) be equal to \( X A \). However, this solution is not unique for once \( Z \) and \( A \) have been obtained, we may transform \( Z \) and \( A \) with any square nonsingular matrix \( H \). We let:

\[
ZH = Y
\]

and

\[
H^{-1}A' = B',
\]

then

\[
YB' = ZA'.
\]

However if we require that the sum of squared elements in the matrices:

\[
X - Z_1 A'_1, \qquad \text{and} \qquad X - Z_m A'_m - Z_A A',
\]

be successively minimized, that is, that the sum of squared elements in \( E_1, E_2, \ldots, E_p \) be sequentially minimized, it can be shown that the solution indicated is unique up to a diagonal transformation matrix. This procedure also has the merit of enabling one to examine the fit provided by different values of \( p \).

The solution of finding the latent roots and vectors of \( X'X \) is identical with the method of "factor analysis," usually called principal components,\(^{17,18}\) except that in factor analysis either a correlation or covariance matrix is factored rather than a moment matrix.

In general, the more parameters required by a model the more closely can a given set of observations be fitted to that model. In this particular case, no location parameters are required by the model, hence a better test is provided if the moment matrix is factored rather than a correlation matrix.

The procedure was modified in this case by scaling the columns of \( X \), so that the sums of squares of elements in each column (or lead) were equal. This has the effect of putting all observations into the same unit or of making all columns equally important in determining the number of factors necessary to account for the data. The effect seems a desirable one, for if the magnitudes of the observations made on 2 variables are extremely disparate, a factor that is largely determined by the variable with observations of small magnitude may be overlooked as unimportant as the reduction it effects in the total sum of squared discrepancies will be very small.

The procedure permits a rough test of the fit of the model to the data provided by dividing the total sum of squared elements of \( E \) by the sum of the diagonal elements of \( X'X \). The result is the proportion of "information" unaccounted for by the model or factors, and 1 minus this figure is the proportion of the "information" accounted for by the factors.

**Summario in Interlingua**

Le analyse del factors de electrocardiogrammas ab 17 subjectos normal indica que plus que 95 pro cento de omne la "information" electrocardiographic es coperite per 3 factors. In le serie complete, omne le derivations individual essera coperite a 95 pro cento per 3 factors. Iste resultatos indica que il non existe voltages significative al superficie del corpore de subjectos normal que pote esser ascribite a plus que 3 generatores interne. Durante que iste resultatos indica un systema a 3 functiones, illos non indica un systema dipolar, proque un dipolo es un caso special de un systema plus general a 3 functiones.
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