A Model Study of Volume Conductor Effects on Endocardial and Intracavitary Potentials

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An idealized mathematical model was developed to study the effects of variations in conductive and geometric parameters on measured endocardial and intracavitary potentials. The model consists of a spherical multielectrode probe located eccentrically within a system of concentric spheres that represent a blood cavity, myocardium, lung region, and surface muscle layer. Solutions were found for endocardial and intracavitary probe potentials produced by two different configurations of equivalent myocardial sources: 1) multiple activation wave fronts oriented radially, representing global fronts in the myocardium; and 2) pairs of equal and opposite dipoles on a line oriented tangentially to the endocardial surface, representing cardiac sources during early ectopic activation. It was found that the complexities of the cardiac source configurations are reflected in the endocardial potential but not in the associated probe potential, which exhibits a smoothed-out, low-amplitude distribution. In addition, probe potential depends on probe size and location within the cavity. Furthermore, endocardial and probe potentials are influenced by variations in the conductivity of different regions; an increase in blood conductivity results in a decrease in both endocardial and probe potential magnitudes produced by either type of cardiac sources, and an increase in myocardial conductivity results in an increase in both potential magnitudes, whereas an increase in lung conductivity results in an increase in the magnitude of the potential produced by radial sources but a small decrease in the magnitude of the potential produced by tangential sources. The effects of variations in skeletal muscle conductivity are negligible. The volume conductor effects of myocardial anisotropy (9:1 anisotropy ratio) are to attenuate both endocardial and probe potentials by as much as 60% and 71%, respectively, for radial sources and by 96% and 85%, respectively, for tangential sources. In conclusion, volume conductor influences should be considered in the interpretation of measured cavity potentials. Multiple myocardial events are resolved in endocardial potentials but not in potentials measured by an intracavitary multielectrode probe. This observation indicates that for the purpose of resolving cardiac activity, efforts should be directed at reconstruction of endocardial potentials measured with an intracavitary probe. (Circulation Research 1992;71:511–525)

KEY WORDS • endocardial potential • intracavitary potential • volume conductor

T he current trend toward surgical and catheter-based nonsurgical ablation techniques for the management of ventricular arrhythmias requires accurate determination of the arrhythmogenic site within the myocardium. Since most ventricular arrhythmias are subendocardial in origin,1 endocardial potential mapping has become an important tool for the study of arrhythmias and a method for localizing the arrhythmogenic site before treatment.2,3 Recently, an intracavitary electrode catheter with 41 evenly distributed electrodes on its surface was developed by Tacardi and colleagues.4 The multielectrode probe can be percutaneously introduced into the ventricular cavity without occluding it and, hence, permits the simultaneous recording of intracavitary potentials from multiple sites during electrophysiological testing. It was demonstrated that intracavitary potential mapping using this probe can provide information on the site of origin of ectopic paced beats in a normal canine heart.4–6

The electrical activity of cardiac muscle cells results in electrical potentials distributed throughout the heart–torso volume conductor. The determination of electrical potentials from known sources, together with knowledge of the electrical properties and geometry of the volume conductor, is known as the “forward problem.” Potential fields in different regions of the heart–torso volume conductor reflect both the heart electrical generators and the nature of inhomogeneities (i.e., regions of different conductivities).

The objective of the present study was to analyze the effects of variations in conductive and geometric parameters on the magnitudes and distributions of endocardial and intracavitary potentials. The study presented here is part of our general effort to relate potential fields generated by the heart to the process of cardiac excitation. Our previous work dealt mostly with potentials in the volume conductor external to the heart and on the epicardial surface.2,7–11 This has also been the focus of most other forward problem studies in electrocardiology (for a comprehensive review, see Reference

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Supported by National Institutes of Health grant HL-33343, American Heart Association National Center Grant-in-Aid 91006370, and American Heart Association, Northeast Ohio Affiliate, Grant-in-Aid 4710.

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Received May 23, 1991; accepted April 17, 1992.
In the present study, we focus on potential fields generated by cardiac excitation in the blood cavity and on the endocardial surface. An attempt to calculate intracavitary potentials from experimentally measured endocardial potentials was recently presented. As mentioned above, cavity potentials are routinely measured during cardiac electrophysiological studies. Endocardial potentials can also be measured directly, either with an electrode catheter that is brought in direct contact with the endocardial surface, or by using an endocardial multielectrode balloon during surgery. Since the goal of these measurements is to determine the state of electrical activity in the myocardium, a theoretical basis for the interpretation of these measurements in terms of electric sources within the myocardium and for understanding volume conductor influences that affect the measured potentials is highly desirable.

In the study presented here, an idealized mathematical model was developed to study the effects of variations in conductive and geometric parameters of the heart–torso volume conductor on measured endocardial and intracavitary potentials for given cardiac electrical sources. The model consists of a spherical multielectrode probe located within a system of concentric spheres that contain a blood cavity, myocardium, lung region, and surface muscle layer. The spherical intracavitary probe represents the multielectrode probe introduced by Taccardi et al. The model incorporates the important torso components and yet is simple enough to permit analytical solutions for endocardial and intracavitary potentials. A major advantage of the analytic approach is that model parameters can be varied easily and continually. In this article, the forward problem is solved to gain insight into the effects of volume conductor inhomogeneities and geometry on endocardial and intracavitary potentials. The focus of this study is on the isolated effect of the volume conductor for a given source configuration. Moreover, we address here a question that is global in nature, namely the volume conductor effects on the field generated by macroscopic myocardial sources. Therefore, we represent the cardiac generators by two different configurations of equivalent sources: 1) double-layer surfaces oriented radially, representing global activation fronts in the myocardium, and 2) two equal and opposite dipoles on a line tangential to the endocardial surface (and, therefore, oriented along the hypothetical local fiber direction), representing cardiac sources during early ectopic activation.

The solutions (i.e., endocardial and probe potentials) are evaluated for their ability to distinguish local cardiac events—multiple maxima and minima reflecting multiple activation fronts and initiation sites within the myocardium.

**Materials and Methods**

**Model**

Study of the effects of the volume conductor properties on endocardial and intracavitary potentials is based on the analytic mathematical approach presented by Rudy and Plonsey and applied previously to the study of body surface and epicardial potentials. In the model (Figure 1), the multielectrode-probe is represented as a sphere located eccentrically within a blood volume that in turn is bounded by a system of concentric spheres representing the ventricular wall, the lung region, and the surface muscle layer. The model allows for the adjustment of the location of the probe to different eccentric positions within the blood cavity.

Two different representations of cardiac electrical sources are used: 1) radial double layers, and 2) two equal and opposite dipoles oriented tangentially to the blood cavity. The double-layer spherical caps (activation wave fronts) are placed concentrically within the myocardium and are an equivalent-source representation of macroscopic global myocardial activation fronts propagating from endocardium to epicardium (Figures 2A and 2B). We preserve the spherical symmetry of the sources so that a closed-form analytic solution can be constructed. We successfully used a similar representation previously to study volume conductor effects on epicardial and torso potentials. The second source distribution consists of two equal and opposite dipoles on a line tangential to the endocardial surface (and therefore oriented along the hypothetical local fiber direction) (Figures 2C and 2D). This equivalent source configuration is an approximate source obtained from the oblique dipole-layer model of excitation wave fronts in the anisotropic myocardium. It represents the cardiac sources during early ectopic activation, with the ectopic focus located at the midpoint of the line con-
the upper term is for a radial dipole and the lower for a tangential dipole; $\Phi$ is the potential calculated at an arbitrary point of spherical coordinates $r$, $\theta$, and $\phi$ (the potential is independent of $\phi$ for a radial dipole); $A_n$ and $B_n$ are expansion coefficients; and $P_n^r$ and $P_n^\theta$ are associated Legendre polynomials. To solve for the coefficients, appropriate boundary conditions are applied at each interface within the model. Boundary conditions used are provided in Table 1. Boundary condition 4 applies only for a radial double-layer source. The subcutaneous fat layer is not included in the model because its effects were shown to be negligible.7-9

Following the standard techniques of solving boundary value problems,10 substitution of the potentials into the boundary conditions results in a set of algebraic equations for the coefficients, which are solved using the Gaussian elimination method.18 Endocardial potentials are computed at the boundary between the blood region and the myocardium, whereas intracavitary probe potentials are computed on the surface of the spherical probe. The expression for endocardial potential produced by a radial double-layer source is

$$\Phi_2(r_2, \theta) = \frac{2\pi a^2}{r_0^2} \left[ \sum_{n=1}^{s} \left[ A_n \frac{1}{r_2^n} \right. \left. + B_n \frac{1}{r_0^{n+1}} \right] \right] P_n^\theta (\cos \theta) \cdot \Phi_2 (r_2, \theta)$$

where $r_2$ is the radius of the blood cavity, $P_n^\theta$ is the derivative of Legendre polynomial of degree $n$, $r_0$ is the radius of the double-layer spherical cap, $2\theta_0$ is the central angle of the double layer, and $a = r_0 (\sin \theta_0)$ (Figure 2B). The origin of $\theta$ is at the center of the blood cavity. The position of the probe inside the cavity is introduced as an eccentricity parameter $d$ (distance between the probe center and the cavity center). A translation operation from the cavity coordinate system to the probe coordinate system permits the incorporation of the eccentricity parameter $d$.19 The expression for the probe potential, after applying boundary condition 10 in Table 1, produced by a radial double-layer source is

$$\Phi_1 (r_1, \theta) = \frac{2\pi a^2}{r_0^2} \left[ \sum_{n=1}^{s} \left[ \sum_{s=1}^{n} \frac{A_n}{d^{s+1}} \right] P_n^\theta (\cos \theta_0) \right]$$

$$\cdot \left[ 1 + \frac{s}{s+1} \right] \left[ \sum_{n=1}^{s} \frac{A_n}{d^{s+1}} \right] P_n^\theta (\cos \theta_0)$$

$$- 2\pi (1 - \cos \theta_0) \left[ \sum_{n=1}^{s} \frac{A_n}{d^{s+1}} \right] P_n^\theta (\cos \theta_0)$$

where $r_1$ is the probe radius (Figure 1). The origin of $\theta$ is now at the probe center. The eccentricity parameter $d$ introduces the dependence of the probe potential on the location of the probe within the blood cavity. The mathematical formulation for the potentials produced by a tangential dipole oriented in the $Y$ direction (Figure 2D) is based on that presented by Cuffin.20 Probe eccentricity within the blood cavity is introduced with the same transformation used above. Potentials

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Figure 2. The two different configurations of equivalent myocardial sources used in the simulations. A radial activation front within the myocardium is depicted in panel A as a double-layer cap (with radius $r_0$) traveling from endocardium to epicardium and appears as a dashed arc in the cross section in panel B. The double layer is marked by $+$ and $-$ on its positive and negative surfaces, respectively. A cardiac source during early ectopic activation, consisting of two equal and opposite dipoles on a line tangential to the endocardial surface, is depicted in panel C. The cross section in panel D shows a separation between the two dipoles of 5 mm that is used in all the simulations. The dashed ellipses in panels C and D represent cross sections of a hypothetical ellipsoidal wave front generated by point stimulation. Region 3, (3a) is part of the myocardium interior (exposed) to the sources.

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**Method of Solution**

The model is formulated as a quasi-static problem15; steady-state conditions apply at any instant of time. The analytic solution to the model in Figures 1 and 2 is constructed by writing the general solution to Laplace’s equation, for each region of different conductivity, as a series expansion in Legendre polynomials16,17:

$$\Phi(r, \theta, \phi) = \sum_{n=0}^{s} [A_n r^n + B_n r^{-n+1}] \cdot P_n^r (\cos \theta)$$

where $\Phi$ is the potential calculated at an arbitrary point of spherical coordinates $r$, $\theta$, and $\phi$. The coefficients $A_n$ and $B_n$ are solved using the Gaussian elimination method.18 Endocardial potentials are computed at the boundary between the blood region and the myocardium, whereas intracavitary probe potentials are computed on the surface of the spherical probe. The expression for endocardial potential produced by a radial double-layer source is
produced by multiple radial or tangential dipole sources are computed by superposition.

**Geometric Parameters**

Typical values for geometric parameters used in the computations are as follows (Figure 1): $r_1$ (radius of probe) = 0.25, 0.75, or 1.25 cm; $r_2$ (radius of blood cavity; endocardium) = 2 cm; $r_3$ (external radius of myocardium; epicardium) = 2.5 cm; $r_4$ (external radius of lung region) = 5.5 cm; and $r_5$ (external radius of skeletal muscle layer) = 6 cm. Parameter $p$ is introduced and is equal to the distance between the probe surface ($d+r_1$) and the endocardium ($r_2$) at $\theta=0^\circ$; i.e., $p=r_2-(d+r_1)$, where $d$ is the eccentricity as defined previously.

Torsos dimensions are chosen to correspond to the physical human-torso tank system which was the source of our experimental data for the torso-to-endocardium inverse computations. The same experimental set-up will be used to obtain data for our future probe-endocardium computations in a realistic geometric model. The human-torso tank was molded around a 9-year-old child. This was done to preserve the torso/heart size ratio that is typical for an adult human, since a canine heart was used in the experiments. The size of the spherical ventricular cavity chosen in the model is typical of the canine heart.

We previously demonstrated that, unlike torso potentials, cardiac potentials are not significantly affected by the eccentricity of the heart within the torso volume conductor. The epicardial potentials are almost completely independent of the location of the heart within the torso. For an increase of heart eccentricity from 1 to 5 cm (where the heart is touching the anterior chest wall), the epicardial potential increases by only 4%. Since the present study focuses on potentials within the heart volume conductor, concentric heart position can be used in the model without significant loss of accuracy. The ability to use this approximation simplifies the mathematical expression (only one eccentricity transformation for probe position is needed instead of two transformations). In addition, with this simplification, the simulations run faster, numerical error is minimized, and convergence is assured. Simulations for different heart/torso ratios performed with the model presented here also demonstrated that endocardial and probe potentials are not sensitive to geometric variations in the torso volume conductor.

The spherical geometry of the probe is dictated by symmetry considerations that are necessary to obtain an analytical solution for the potential field. Probe potentials are computed for three different probe sizes to obtain a qualitative measure of the dependence of potential magnitude and resolution on probe size. The
spherical approximation of the geometry is evaluated by comparing potentials on spherical probes to potentials on cylindrical and olive-shaped probes that are used experimentally.\textsuperscript{4} Potentials on the surfaces of the cylindrical and olive-shaped probes are computed by discretizing the boundary integral equation relating probe potential to endocardial potential. To evaluate the spherical approximation to the probe geometry, potentials computed on the midcircumference of spherical, cylindrical, and olive-shaped probes are compared in Figure 3. Potentials are computed for two radial activation wave fronts and for a probe position at the center of the blood cavity. Qualitatively, the potential on a spherical probe 0.25 cm in radius is similar to the potential on the cylindrical probe used experimentally (4×0.3 cm). The peak-to-peak potential amplitude for the spherical probe is only 0.2% greater than that for the cylindrical probe. Similarly, the potential on a spherical probe 0.75 cm in radius approximates closely the potential on the olive-shaped probe (2.5×1.2 cm), with a peak-to-peak potential amplitude difference of only 3%. This relation shows the feasibility of using a spherical probe and the analytic solution obtained from spherical symmetry considerations for simulating the volume conductor effects on intracavitary probe potentials. Note that, for all probe geometries, the probe potentials exhibit only one broad minimum, not reflecting the two discrete double-layer sources in the myocardium. This "smoothing effect" is an important property that is well simulated by the spherical probe and will be discussed in detail later.

\textbf{Conductive Parameters}

The following values are used for typical conductivities under normal conditions (Figure 1): $\sigma_2$ (blood)=0.006 mho/cm; $\sigma_3$ (myocardium)=0.002 mho/cm; $\sigma_4$ (lunv)=0.0005 mho/cm; and $\sigma_5$ (skeletal muscle)=0.00125 mho/cm.

We introduce the anisotropic property of the surface muscle layer by replacing it with an equivalent layer of isotropic conductivity ($\sigma_a$), using the boundary extension transformation technique.\textsuperscript{21} Under this transformation, the thickness of the muscle layer is modified, being multiplied by a factor of 3, so that the effective thickness used in the computations is 1.5 cm. The validity of the boundary extension technique was demonstrated by Schmidt and Pilkington\textsuperscript{22}; this equivalent representation of the muscle yielded an error of only 2% in peak body surface potential as compared with a rigorous computation using explicit representation of the anisotropy.

The boundary extension technique is also applied to the study of the volume conductor effect of myocardial anisotropy on endocardial and intracavitary probe potentials. We evaluated the applicability of this approach by comparing the estimated effect of myocardial anisotropy on peak body surface potential to that obtained with an explicit representation of myocardial anisotropy by Schmidt and Pilkington.\textsuperscript{22} Our computations using the boundary extension technique showed that for myocardial anisotropy conductivity ratios of 3:1, 2:1, and 1.5:1, peak body surface potentials were attenuated by 14.9%, 8.1%, and 4.2%, respectively, providing an excellent estimate of the corresponding attenuation of 14.4%, 8%, and 4.4% computed with the explicit representation.\textsuperscript{22} In the study presented here, the variations in myocardial anisotropy are set up such that the mean of the longitudinal (along fiber axis) and transverse (perpendicular to fiber axis) conductivities is equal to 0.002 mho/cm.\textsuperscript{7} The anisotropy ratios considered are 9:1, 3:1, 2:1, and 1.5:1. Therefore, the longitudinal/transverse conductivities considered are 0.0036/0.0004, 0.003/0.001, 0.00267/0.00133, and 0.0024/0.0016, and the corresponding geometric extension factors are 3, 1.732, 1.414, and 1.225, respectively. The thickness of the myocardium is multiplied by the geometric extension factor; the thickness of all other compartments is kept the same. Specifically, the distance from the endocardium to the surface of the probe was kept constant.

Unless otherwise stated, potentials produced by radial sources were computed for two activation wave fronts with central angles (extent) of 10° each and a separation of 40° (i.e., situated from −20° to +30° and from +20° to +30°). The activation wave fronts were located midway between the endocardium and the epicardium ($r_e=2.25$ cm). The double-layer strength of the wave fronts was taken to be unity. Potentials produced by tangential sources were computed for two pairs of dipoles, representing two distinct and separate ectopic events. The separation between the two dipoles for each ectopic event was set equal to 5 mm, representative of the major axis of the activation front early after the stimulus.\textsuperscript{14} Dipole strength was taken to be unity. A
probe size of 1.5 cm in diameter ($r_i=0.75$ cm), concentrically located within the blood cavity ($d=0$ cm), was used in the simulations as a reference for evaluating the effects of geometric changes. In studying the effects of variations in an individual parameter, the remaining parameters were held constant at the typical values given above. Peak potential magnitude was defined as the difference between the minimum and the maximum (peak-to-peak amplitude; see A in Figure 4).

Although the expressions obtained for endocardial and probe potentials are analytic, their functional dependence on the geometric and conductive parameters is complicated; therefore, numerical computations must be conducted to investigate the effects of variations in each of these parameters. Summations over an infinite number of terms appear in the expressions for the potentials (Equations 1 and 2). These summations were truncated and tested for convergence.

**Results**

*Definition of peak magnitude (A) of typical potential distributions. Top panel: Two radial activation fronts. Bottom panel: Two “ectopic foci” (see text).*

*Comparison between endocardial (tracing 1) and intracavitary probe (tracing 2) potentials originating from two radial activation wave fronts located within the myocardium. The central angle (extent) of each wave front is $10^\circ$. The separation between the waves is $40^\circ$ (panel A), $80^\circ$ (panel B), and $120^\circ$ (panel C) (equivalent to 1.57, 3.14, and 4.71 cm, respectively). Endocardial and probe potentials are plotted as functions of the angle $\theta$ with its origin at the center of the cavity and at the center of the probe, respectively. Potentials are normalized for a unity double-layer strength. Insets show cross sections of the probe–heart geometry for different wave front separations.*

The endocardial and probe potential distributions generated by two discrete radial activation wave fronts located midway within the myocardium are shown in Figure 5. The two activation wave fronts have central angles (extent) of $10^\circ$ each and are separated by $40^\circ$, $80^\circ$, and $120^\circ$ (equivalent to a separation of 1.57, 3.14, and 4.71 cm, respectively) as shown in the insets of Figures 5A, 5B, and 5C, respectively. It can be observed that, when the two activation wave fronts are separated by $40^\circ$ or $80^\circ$, two discrete minima appear on the endocardium, whereas a single broad minimum appears on the probe surface. For a separation of $120^\circ$, two low-amplitude discrete minima are quite apparent on the probe surface as well. The minima of the potential distributions (peak magnitudes) resulting from two wave fronts separated by $40^\circ$, $80^\circ$, and $120^\circ$ occur at $\pm24^\circ$, $\pm45^\circ$, and $\pm65^\circ$, respectively, on the endocardium and at $0^\circ$, $0^\circ$, and $\pm53^\circ$, respectively, on the surface of the probe. Even when the two minima appear on the probe surface (Figure 5C), their location does not correspond to the location of the wave fronts (error of $12^\circ$, equivalent to 0.47 cm).

Endocardial and probe potential distributions generated by two subendocardial ectopic foci (each represented by a pair of equal and opposite dipoles) located at $r_e=2.1$ cm and separated by $20^\circ$, $45^\circ$, and $90^\circ$ (equivalent to a separation of 0.73, 1.65, and 3.3 cm, respectively) are shown in Figure 6. Two discrete minima appear on the endocardial surface at $\pm10^\circ$, $\pm22.5^\circ$, and $\pm45^\circ$ for the separations shown in Figures 6A, 6B, and 6C, respectively. These endocardial minima reflect accurately the position of the ectopic foci.
and their separation. In contrast, the probe potential exhibits a single broad minimum for ectopic events that are separated by 20° and 45° (Figures 6D and 6E, respectively), but two distinct minima appear on the probe surface at ±46° for widely separated events (90°) as shown in Figure 6F.

The results demonstrate that activation details seen at the endocardium tend to be smoothed out at the probe surface. Furthermore, unlike probe potentials, the endocardial potential distribution accurately depicts the location of the underlying myocardial sources. In particular, the position of subendocardial foci is reflected accurately in potential minima on the endocardium. The results of these simulations are consistent with the experimentally determined patterns of endocardial potentials and intracavitary potentials during myocardial pacing. It was demonstrated that endocardial and intracavitary primary potential minima develop in the electrodes closest to the pacing site. Moreover, in studying the feasibility of detecting the origin of multiple pacing sites within the myocardium from intracavitary multielectrode-probe potential, it was shown that endocardial stimuli 1.2 cm apart were generally distinguished. However, there was a slight error in determining the origin of the pacing site, a result similar to that of Figures 5C, 6C, and 6F, with a smaller error associated with tangential activation than with radial activation.

![Figure 6](http://circres.ahajournals.org/)

**Figure 6.** Comparison between endocardial (panels A, B, and C) and intracavitary probe (panels D, E, and F) potentials originating from two ectopic foci located subendocardially at a radius of 2.1 cm. The separation between the foci is 20° (panel A), 45° (panel B), and 90° (panel C) (equivalent to 1.57, 3.14, and 4.71 cm, respectively). Potentials are normalized for a unity dipole strength. Partial cross sections of the probe–heart geometry are shown for different separations between the ectopic foci.

![Figure 7](http://circres.ahajournals.org/)

**Figure 7.** Effects of intracavitary probe diameter and position on probe potential distributions produced by radial activation fronts. Each panel shows probe potentials for three different probe radii (1, r1=0.25 cm; 2, r1=0.75 cm; and 3, r1=1.25 cm). Endocardial potential distribution is also shown for reference (bold trace). Each panel relates to different distance of the probe from the endocardium (panel A: p=0.75 cm; panel B: p=0.5 cm; and panel C: p=0.25 cm). Potentials are normalized for a unity double-layer strength. Insets show cross sections of the probe–heart geometry, with the probes located at the different positions.

**Dependence of the Probe Potential on Its Size and Eccentricity**

The effects of the intracavitary probe diameter and its position within the blood cavity on the probe potential for radial activation fronts is demonstrated in Figure 7. Each panel shows three traces of probe potential distributions associated with three different probe radii: tracing 1, r1=0.25 cm; tracing 2, r1=0.75 cm; and tracing 3, r1=1.25 cm (endocardial potential distribution is in bold line for reference). In each panel, p, the minimum distance between probe surface and endocardium at θ=0°, is kept constant for all probe sizes. Probe position within the cavity is different for each panel (panel A, p=0.75 cm; panel B, p=0.5 cm; and panel C, p=0.25 cm).

The probe potential distribution is greatly affected by the probe size and position within the cavity. A large-
diameter probe demonstrates large potential magnitudes. For example, relative to a probe with a radius of \( r_1 = 0.25 \) cm, probe potential magnitude increases by 117% and 198% when measured with probes of radii \( r_1 = 0.75 \) cm and \( r_1 = 1.25 \) cm, respectively, at a constant \( p = 0.75 \) cm (Figure 7A). In addition, the closer the probe to the endocardium, the larger the potential magnitude and the better the resolution (i.e., the ability to separate discrete patterns such as potential maxima and minima). Relative to a probe with a radius of \( r_1 = 0.75 \) cm located at \( p = 0.75 \) cm (tracing 2 in Figure 7A), probe potential magnitude increases by 18% and 44% as the probe approaches the endocardium (\( p = 0.5 \) cm and \( p = 0.25 \) cm; tracing 2 in Figures 7B and 7C, respectively). Similar dependence of probe potential on probe size and position is observed for tangential dipole sources.

**Variations in the Depth of the Myocardial Sources**

The effect of variations in the depth of radial activation wavefronts and of ectopic foci within the myocardium on endocardial and probe potentials are illustrated in Figures 8 and 9, respectively. The figures show endocardial (panels A) and intracavitary probe (panels B) potential distributions generated by two myocardial sources situated at three different locations within the myocardium: very close to the endocardium (subendocardium; tracing 1, \( r_o = 2.1 \) cm), midway within the myocardium (tracing 2, \( r_o = 2.25 \) cm), and very close to the epicardium (subepicardium; tracing 3, \( r_o = 2.4 \) cm). For radial activation wavefronts (Figure 8), endocardial potential magnitude decreases by 44% when the wave fronts are midway within the myocardium and decreases by 66% when the wave fronts are subepicardial, as compared with the subendocardial location. However, probe potential magnitude is less sensitive to the depth of the activation wave fronts than endocardial potential. Probe potential magnitude decreases by 20% when the wave fronts are midway within the myocardium and decreases by 38% when the wave fronts are subepicardial, as compared with the subendocardial location. For myocardial sources representing two ectopic foci separated by \( 45^\circ \) (Figure 9), endocardial potential magnitude decreases by 80% when the sources are midway within the myocardium (\( r_o = 2.25 \) cm), and by 91% for subepicardial sources (\( r_o = 2.4 \) cm), as compared with the subendocardial location (\( r_o = 2.1 \) cm). Similarly, the less sensitive probe potential decreases in magnitude by 13% and 23%, respectively, as the sources move away from the endocardium toward the epicardium.

The dependence of the endocardial potential magnitude on the location of the activation site was experimen-
tally demonstrated by Harada et al.23 Left and right ventricular endocardial potentials were simultaneously recorded at multiple sites during septal pacing at three different locations: the left ventricular side, midseptum, and right ventricular side. It was observed that the magnitude of the primary potential minimum decreased with increasing distances of the pacing site away from the endocardium. Furthermore, the morphology of the endocardial potential depended on the location of the pacing site. In another set of studies by Taccardi et al.,4 intracavitary potentials were recorded with a multielectrode probe during endocardial, midwall, or epicardial pacing. It was shown that intracavitary potential magnitudes progressively decreased when the stimulus was delivered at increasing distances away from the endocardium.

**Effects of Variations in Conductivity of Individual Compartments**

**Variations in blood conductivity.** Endocardial and probe potentials, produced by radial and tangential sources, as a function of the intracavitary blood conductivity ($\sigma_2$) are shown in Figure 10. The potentials, calculated at the location of peak magnitudes, are normalized by the corresponding potential values at normal physiological blood conductivity ($\sigma_2=0.006$ mho/cm). It can be observed that both endocardial and probe potential magnitudes decrease monotonically with increasing blood conductivity (decreasing hematocrit); potentials produced by a tangential dipole are more sensitive to variations in blood conductivity. Figure 11 shows endocardial (panel A) and probe (panel B) potential distributions, produced by two radial activation wave fronts, for selected blood conductivities: tracing 1, $\sigma_2=0.002$ mho/cm (normal conductivity of myocardium); tracing 2, $\sigma_2=0.006$ mho/cm (normal conductivity of blood); and tracing 3, $\sigma_2=0.0198$ mho/cm (conductivity of physiological saline). Compared with the potential obtained at normal blood conductivity, endocardial and probe potential magnitudes increase by 74% and 57%, respectively, when the blood conductivity is decreased to 0.002 mho/cm (homogeneous with the surrounding myocardium), whereas endocardial and probe potential magnitudes decrease by 56% and 49%, respectively, when the blood conductivity is increased to 0.0198 mho/cm (saline conductivity). Endocardial and probe potentials produced by two subendocardial ectopic foci ($r_o=2.1$ cm) separated by 45° (Figure 12) increase in magnitude by 84% and 104%, respectively, at blood conductivity of 0.002 mho/cm, whereas the potentials decrease in magnitude by 63% and 58%, respectively, at blood conductivity of 0.0198 mho/cm, as compared with potentials at the normal blood conductivity of 0.006 mho/cm.

The attenuation in the endocardial and intracavitary potential magnitudes as a consequence of an increase in blood conductivity (Figure 10) is a manifestation of the “short-circuiting” effect of the highly conductive blood region.24 In addition to this effect, probe potential is also attenuated as a result of its distance from the myocardial sources. The isolated effect of the high blood conductivity can be separated from the distance falloff effect by comparing tracing 1 in Figure 11B for two radial activation wave fronts (or tracing 1 in Figure

**FIGURE 10.** Graph showing effect of variations in intracavitary blood conductivity on endocardial and probe potential magnitudes produced by radial and tangential dipole sources. Potentials are normalized to one at the normal blood conductivity of 0.006 mho/cm. Bottom panel: Hematocrit values corresponding to the blood conductivity values are also shown.

**FIGURE 11.** Endocardial (panel A) and intracavitary probe (panel B) potential distributions, produced by two radial activation fronts, for three different blood conductivities: tracing 1, blood cavity sphere ($\sigma_2$)=0.002 mho/cm; tracing 2, $\sigma_2=0.006$ mho/cm (normal); and tracing 3, $\sigma_2=0.0198$ mho/cm. Potentials are normalized for a unity double-layer strength.
myocardial conductivity. The effect of variations in
myocardial conductivity is more pronounced for
tangential than radial sources. Figure 14 shows endocardial
(panel A) and probe (panel B) potential distributions,
produced by two radial activation wave fronts, for

FIGURE 13. Graph showing effect of variations in myocardial
costivity on endocardial and probe potential magnitudes produced by radial and tangential dipole sources. Potentials are normalized to one at the normal myocardial conductivity of 0.002 mho/cm.

FIGURE 14. Endocardial (panel A) and intracavitary probe (panel B) potential distributions, produced by two radial activation fronts, for three different myocardial conductivities: tracing 1, myocardial sphere ($\sigma$) = 0.002 mho/cm; tracing 2, $\sigma$ = 0.006 mho/cm; tracing 3, $\sigma$ = 0.0198 mho/cm. Potentials are normalized for a unity double-layer strength.

FIGURE 12. Endocardial (panel A) and intracavitary probe (panel B) potential distributions, produced by two ectopic foci, for three different blood conductivities: tracing 1, blood cavity sphere ($\sigma$) = 0.002 mho/cm; tracing 2, $\sigma$ = 0.006 mho/cm (normal); and tracing 3, $\sigma$ = 0.0198 mho/cm. Potentials are normalized for a unity dipole strength.

12B for two ectopic foci), where blood conductivity is
the same as that of the myocardium (homogeneous),
with tracing 2 in Figure 11B (tracing 2 in Figure 12B),
where blood conductivity is normal. In the presence of
blood, the magnitude of probe potential, at a fixed
probe position, decreases by 37% for radial activation
wave fronts and decreases by 51% for ectopic foci, as
compared with a homogeneous medium. The clinical
relevance of variations in blood conductivity is in its
correspondence with changes in hematocrit.25 Patients
with anemia (low hematocrit and high blood conductivity)
are expected to have endocardial potentials of low
magnitude and low-amplitude intracavitary potentials,
whereas patients with polycythemia (high hematocrit
and low blood conductivity) are expected to have
endocardial potentials of high magnitude and high-ampli-
tude intracavitary potentials.

Variations in myocardial conductivity. Figure 13 de-
scribes the effect of variations in myocardial conductivity ($\sigma$) on endocardial and probe potentials produced
by radial and tangential sources. The potentials, cal-
culated at the locations of peak magnitudes, are normal-
ized by the corresponding potential values at normal
myocardial conductivity ($\sigma$ = 0.002 mho/cm). In con-
text to the decrease of potentials due to an increase in
blood conductivity, both endocardial and probe poten-
tial magnitudes increase monotonically with increasing

Endocardial (panel A) and intracavitary probe (panel B) potential distributions, produced by two ectopic foci, for three different blood conductivities: tracing 1, blood cavity sphere ($\sigma$) = 0.002 mho/cm; tracing 2, $\sigma$ = 0.006 mho/cm (normal); and tracing 3, $\sigma$ = 0.0198 mho/cm. Potentials are normalized for a unity dipole strength.
selected myocardial conductivities: tracing 1, \(\sigma_t=0.0005\) mho/cm (normal conductivity of lung region); tracing 2, \(\sigma_t=0.002\) mho/cm (normal conductivity of myocardium); tracing 3, \(\sigma_t=0.006\) mho/cm (normal conductivity of blood). Compared with the potential obtained at normal myocardial conductivity, endocardial and probe potential magnitudes decrease by 43% and 21%, respectively, when the conductivity of the myocardium is reduced to 0.0005 mho/cm (homogeneous with the surrounding lung medium), whereas endocardial and probe potential magnitudes increase by 47% and 11%, respectively, when the conductivity of the myocardium is increased to 0.006 mho/cm (homogeneous with the interior blood region). Endocardial and probe potentials produced by two subendocardial ectopic foci (\(r_o=2.1\) cm) separated by 45° (Figure 15) decrease in magnitude by 69% and 72%, respectively, at myocardial conductivity of 0.0005 mho/cm and increase in magnitude by 99% and 133%, respectively, at myocardial conductivity of 0.006 mho/cm compared with normal myocardial conductivity. The conductivity of the myocardium could be modulated by physiological changes in the myocardial tissue, such as the effects of fibrosis, drugs, and blood flow pattern. The volume conductor influence of such factors would be expected to correspond, qualitatively, with the results of the simulations regarding the endocardial and probe potentials.

The volume conductor effects of myocardial anisotropy on endocardial and intracavitary probe potentials were assessed. The boundary extension technique was used to examine four different cases of myocardial anisotropy. The potentials were computed for longitudinal (along fiber axis) to transverse (perpendicular to fiber axis) conductivity ratios of 9:1, 3:1, 2:1, and 1.5:1. Myocardial sources used in the simulations were two radial wave fronts extending from –30° to –20° and from 20° to 30° (i.e., separated by 40°) at \(r_o=2.25\) cm and two ectopic foci at \(r_o=2.25\) cm and separated by 45°. As shown in Figure 16, endocardial potential magnitude and resolution (i.e., the ability to separate discrete patterns such as potential maxima and minima) are attenuated relative to the isotropic case with \(\sigma_t=0.002\) mho/cm. Quantitatively, the respective attenuation in magnitude for radial activation fronts and ectopic foci is 60% and 96% for a 9:1 ratio, 32% and 73% for a 3:1 ratio, 20% and 54% for a 2:1 ratio, and 12% and 34% for a 1.5:1 ratio. The two discrete minima appearing on
the endocardial surface tend to disappear at high ratios of anisotropy. Intracavitary probe potentials are also attenuated in magnitude with respective attenuation for radial activation fronts and ectopic foci of 71% and 85% for a 9:1 ratio, 24% and 42% for a 3:1 ratio, 12% and 25% for a 2:1 ratio, and 6% and 13% for a 1.5:1 ratio.

Variations in lung conductivity. The effect of variations in lung conductivity ($\alpha_L$) on endocardial and probe potentials is shown in Figure 17. The potentials, calculated at locations of peak magnitudes, are normalized by the corresponding potential values for normal lung conductivity ($\alpha_L=0.0005$ mho/cm). Endocardial and probe potentials, produced by radial activation wave fronts, increase monotonically in magnitude with increasing lung conductivity, whereas the potentials produced by a tangential dipole decrease in magnitude but with less sensitivity. Figure 18 shows endocardial (panel A) and probe (panel B) potential distributions for selected lung conductivities: tracing 1, $\alpha_L=0.0001$ mho/cm (low conductivity of lung region); tracing 2, $\alpha_L=0.0005$ mho/cm (normal conductivity of the lung region); tracing 3, $\alpha_L=0.0198$ mho/cm (conductivity of physiological saline). Compared with the potential obtained at normal lung conductivity, endocardial and probe potential magnitudes decrease by 22% and 38%, respectively, when the conductivity of the lung region is decreased to 0.0001 mho/cm, whereas endocardial and probe potential magnitudes increase by 88% and 151%, respectively, when the conductivity of the lung region is increased to 0.0198 mho/cm.

The isolated effect of lung conductivity is clinically related to lung diseases that result in variations in the conductivity of the lungs. Disease conditions such as pulmonary emphysema and cystic fibrosis result in low conductivity of the lung region due to the entrapment of nonconducting air. According to the simulations, decreased endocardial and probe potential magnitudes are expected. On the other hand, in pulmonary edema, a condition that results in increased conductivity of the lung region, augmented endocardial and probe potential magnitudes are expected.

Variations in skeletal muscle conductivity. Results of the simulations (not shown) demonstrate that variations in skeletal muscle conductivity do not have a significant effect on either endocardial or probe potentials. Therefore, disease conditions that modulate skeletal muscle conductivity, such as Pompe’s disease (glycogen storage disease), would not be expected to have any significant influence on endocardial or probe potentials.

Discussion

Accurate localization of the arrhythmogenic site in patients with ventricular arrhythmias is critical to the success of nonpharmacological approaches, such as surgical and catheter-based nonsurgical ablation methods, in the management of these arrhythmias. Since the origin of most ventricular arrhythmias is subendocardial,1 endocardial potential mapping has become an important tool in the study of arrhythmias and a method for localizing the arrhythmogenic site before treatment.2,3 Current techniques of mapping the potentials directly from the endocardium present certain difficulties. With intravascular catheter techniques, the number of recording sites is limited (typically 10–15 points), and the procedure is very time consuming. The multielectrode endocardial balloon technique2,3 requires an open-heart surgery with a cardiopulmonary bypass. It is not always possible to induce sustained tachycardias in the operating room; furthermore, it is difficult to induce all types of clinical arrhythmias, during surgery, that were observed preoperatively. Recently, Taccardi et al4 developed multielectrode probes...
with 41 evenly distributed electrodes on their surfaces. These probes do not occlude the ventricular cavity and thus can be used in the blood-filled ventricle to measure, simultaneously, the spatial distribution of cavity potentials in all directions relative to the entire endocardial surface. This approach was shown to provide information on the site of origin of ectopic paced beats in the canine heart.\textsuperscript{4–6} The objective of the present study was to analyze the effects of variations in conductive and geometric parameters of the heart–torso system on the magnitudes and distributions of endocardial and intracavitary probe potentials. A mathematical model was developed to study these volume conductor effects for a given myocardial source configuration (forward problem). Myocardial sources were represented by two different configurations of equivalent sources: 1) double-layer surfaces oriented radially and 2) pairs of equal and opposite dipoles on a line tangential to the endocardial surface. Geometric and conductive parameters were varied in the model, and the effects on endocardial and probe potential distributions were investigated. The analytical model and the simulations presented in this study help relate endocardial and probe potentials to myocardial activation and assist in the interpretation of these potentials in the presence of complicating volume conductor effects. The simulations also compare endocardial and probe potentials in terms of their sensitivity to volume conductor effects and help define the limitations of probe potentials as a mirror of local myocardial activity.

The focus of this study is on the effects of the volume conductor on both endocardial and intracavitary probe potentials for a given source configuration; i.e., the study focuses on the global effects of the volume conductor and not on the nature of the cardiac sources. Although the simulations can provide answers to global questions such as the ability to detect and localize activation fronts and foci of electrical activity, they cannot address characteristics of potential waveforms that reflect details of the underlying activity at the cellular and tissue levels (for example, fractionated electrograms and their cause\textsuperscript{26,27}). Such phenomena can only be investigated using a model that incorporates cellular processes (the action potential) and details of the myocardial structure\textsuperscript{28} on a much smaller scale than the heart–torso model used here. It should be added that understanding the isolated effect of the volume conductor is important for a correct interpretation of measured potentials in terms of underlying processes. In this study, we fix the source and study the effects of volume conductor properties separately. We use two configurations of equivalent myocardial sources to represent two important patterns of cardiac activation. Radial double-layer fronts are representative of endocardium-to-epicardium propagation during normal activation. A pair of equal and opposite dipoles oriented along the fiber direction is representative of point stimulation (ectopic focus). The latter source configuration is particularly relevant to the present study since detection and localization of arrhythmogenic foci are important objectives of intracavitary probe measurements.

The results of the simulations demonstrate that the endocardial potential distribution provides an accurate reflection of the underlying cardiac source configuration, whereas intracavitary probe potential exhibits a smoothed-out, low-amplitude distribution. Unless widely spaced, separate local events within the endocardium (e.g., two arrhythmogenic foci) cannot be distinguished from the probe. Furthermore, once localized, distinct events reflected in the probe potential distribution tend to be out of phase with respect to the location of the sources within the myocardium. In addition, probe potential is dependent on probe size and location within the cavity; the potential distribution is sensitive to variations in the location of the probe, which may be caused by the movement of the blood within the cavity.

In addition to the geometric parameters, endocardial and probe potentials are influenced by variations in the conductivity of different regions. Simulations of variations in conductive parameters provide insight into the effects of various volume conductor inhomogeneities on the potentials. The low-amplitude distribution of intracavitary probe potential is a consequence of the “short-circuiting” effect of the highly conductive blood medium as well as the effect of the probe distance from the myocardial sources. An increase in cavitory blood conductivity results in a decrease in endocardial and probe potential magnitudes.

The anisotropic electrophysiological properties of the myocardium affect the potentials generated by cardiac excitation at three levels: 1) anisotropy plays an important role in determining the shape of the activation front (the isochron),\textsuperscript{29} 2) anisotropy significantly influences the strength and distribution of the electrical sources that arise from excitation, and 3) anisotropy affects the potential field generated by these sources through its volume conductor effect. Since radial wave fronts propagate transverse to fiber orientation throughout their extent, a uniform source-strength representation (uniform double layer) is adequate in this case. The electrical sources arising from excitation at a point are strongly influenced by the anisotropy.\textsuperscript{30} The oblique dipole-layer model, introduced by Colli-Franzone et al,\textsuperscript{14} incorporates myocardial anisotropy into the source representation. This model consists of an oblique dipole layer on the wave front, which can be viewed as the superposition of an axial (along fiber direction) and a transverse (perpendicular to fiber direction) component. The results of Colli-Franzone’s study clearly demonstrate the dominant role played by the axial component in determining the potential field. This observation provides the basis for our representation of the electrical sources generated by point stimulation as a pair of axial dipoles on the long axis of an ellipsoidal wave front. This model represents (at least qualitatively) the effect of anisotropy on the geometry of the isochron (ellipsoidal with long axis parallel to fiber direction) and on the distribution and strength of the equivalent sources on the isochron (dominant role of the axial component). In the present study, we examine the third factor, namely the effect of anisotropy as a property of the myocardial volume conductor. The results demonstrate that the effect is an attenuation of both endocardial and probe potentials generated by either radial activation fronts or by sources associated with ectopic stimulation.

An increase in lung conductivity results in an increase in endocardial and probe potentials produced by radial sources but a slight decrease in the potentials produced by tangential sources. The increase in the magnitude of endocardial potentials produced by radial sources with increasing lung conductivity seems, at first, contradictory to the attenuation of epicardial potential\textsuperscript{5} caused
by the same conductivity change. The attenuation of epicardial potential reflects less confinement of current to the “heart” compartment when the lung conductivity is increased (theoretically, the potential on the surface of a sphere, generated by a dipole located at the center of the sphere, is augmented by a factor of 3 when the sphere is “cut out” of an infinite homogeneous medium and placed in a nonconducting medium). However, in the model, the activation front is represented as an equivalent double layer that is characterized by a discontinuity of potential across it (boundary condition 4 in Table 1) that reflects the amplitude of the action potential. A decrease in the magnitude of (the positive) epicardial potential is necessarily accompanied by an increase in the magnitude of (the negative) endocardial potential, so that the constant discontinuity of potential across the double-layer source (activation front) is maintained. The increase of endocardial potential magnitude is reflected, in turn, in an accompanying increase of the magnitude of the intracavitary probe potential.

We used the method of images16 in the past to describe the isolated effect of an inhomogeneous single compartment on the potential field. This simple approach lends itself to easy and intuitive interpretations and explanations. We used image systems in infinite planes31 and, to better represent the heart geometry, in spheres.32 However, this simple approach can only be used to study the isolated effect of the conductivity of a single compartment on the volume conductor potential (e.g., the isolated effect of the intracavitary blood). One of the most important results of our previous work was that interactions between the various compartments play an important role in determining the potential distribution. Combined models, rather than models that isolate portions of the torso volume conductor, should be used in the investigation of the effects of variations in conductivity and/or geometric parameters on the potential. Since the complex interactions between the various compartments cannot be accounted for by using simple image systems, using such oversimplified systems should be reserved for the purpose of illustrating principles rather than for the simulation of actual effects, which is the purpose of this study.

The intracavitary probe potential distribution is a projection of the electrical activity within the myocardium through the intervening blood to the probe surface. In view of the results regarding the loss of resolution, amplitude, and exact location of wave fronts in the probe data, a mathematical method for reconstructing the potential distribution on the endocardial surface from the probe potentials would be necessary to better characterize the underlying myocardial events with acceptable spatial resolution and accuracy. The problem of reconstructing endocardial potentials from potential information obtained on the probe surface is known as the “inverse problem.” Attempts to characterize endocardial potential from measured intracavitary potentials were presented earlier.5,6,33 Previous work on the inverse problem (see reference 10 for review) demonstrated that epicardial potentials can be computed with good accuracy from body surface potential data. The application of similar methods to the inverse reconstruction of endocardial potentials from intracavitary probe data will permit detailed examination of regional cardiac events (site of origin and type of arrhythmia, location and size of ischemic and infarcted regions, nature of conduction abnormalities, or effects of drugs) in the clinical electrophysiology laboratory and in the intact experimental animal. The present study can facilitate the development of solutions to the probe-endocardium inverse problem in several ways: 1) The results suggest that certain simplifications can be used in solving the inverse problem; for example, the skeletal muscle layer can be neglected. 2) The analytical model used here for the forward problem is ideal for the development of methodology for solving the inverse problem before its application to measured data contaminated by noise and to models of realistic geometry that are subject to geometric error. 3) The analytical approach allows for a continuous change of all parameters (geometric and conductive) over a wide range of values without any difficulty. Such an approach will permit for detailed and systematic studies of the sensitivity of the inverse procedure to determine errors in these parameters.

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Circ Res. 1992;71:511-525
doi: 10.1161/01.RES.71.3.511

Circulation Research is published by the American Heart Association, 7272 Greenville Avenue, Dallas, TX 75231
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Print ISSN: 0009-7330. Online ISSN: 1524-4571

The online version of this article, along with updated information and services, is located on the World Wide Web at:
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