Unipolar Potential Measurements in the Electric Field Produced by an Arbitrary Dipole in a Circular Homogeneous Lamina

By Robert H. Bayley, M.D.

Unipolar potential measurements are carried out on a circular homogeneous conducting lamina. One set of potential measurements is made using an averaging network for the zero potential reference. A second set of potential measurements is made with reference to the Frank-Kay “zero” potential reference on the outside ground of the dipole generating circuit. Both sets of unipolar potential measurements are then compared with the theoretically predicted values. The first set is found to be highly accurate. The second set is found to contain errors which make this method unacceptable.

In the past several years two quite different methods have been used to evaluate unipolar potential measurements in man. In one instance the potential of the three or four arm Wilson central terminal was compared to that of an averaging network in form of a large (copper screen) immersion sphere which contained the living human subject and a poorly conducting homogeneous medium (tap water). Other investigators have used torso models of homogeneous content in which the electric field is created by a dipole electrode. The electric dipole is positioned in the “heart region” and a reference potential is taken from an outside ground on the dipole generating circuit. The results obtained by the two methods differ in many important respects. Studies on the living human subject with the immersion sphere indicate that the electrical center of the heart lies in or very near the potential plane of the Burger triangle. The experiments herein reported are designed to test the two methods of unipolar potential measurements against the known theoretical values of a simple model.

Methods

Theory. The solution for the potential at any point in or upon the boundary of a homogeneous circular (plane) conducting lamina produced by an arbitrary dipole has recently been reported from this laboratory. The great advantages of using this model are due to the simple form of the potential equations and the low cost of construction of a circular plastic tray for containing the conducting lamina (tap water).

If the centric dipole is given an axis direction along the positive axis of X and is made eccentric along the positive or the negative axis of X (fig. 1) the potential at any point on the circular boundary of the lamina is defined by the equation,

\[ V_r = \frac{M}{\pi R} \left( \frac{\gamma - f}{1 + f - 2 f \gamma} \right) \]  

(1)

Here \( V_r \) is the potential, \( M \) is the dipole moment, \( R \) (\( = 24.13 \) cm.) is the radius of the circular lamina, \( fR \) is the eccentricity measured in centimeters from the center of the lamina to the midpoint of the dipole, and \( \gamma \) is the cosine of the angle \( \Phi \) made by the radius vector to the dipole when on the positive X axis and the radius vector to the boundary point where \( V_r \) is measured. It was decided to compute the values of the eccentricities \( fR (\gamma) \) (table 1) in terms of angle \( \Phi \) made by \( fR \) along the positive axis of X and \( R \) drawn to the electrode posi-
sion for the potential on the boundary due to a
unaltered provided
d shown by potential theory that a finite distance d
on the boundary of the circular lamina for arbi-
trary values of the eccentricity. Moreover, finite
does not alter the position of the zero of potential
distance d between the poles of the dipole is vei'y
small in comparison with

pole potential for the circular lamina wherein the
equations are based on a general solution of the di-

relations are valid for spherical coordinates sub-
ject to the condition that the electric field is
directed along the positive or negative axis of X
and consequently if the unipolar potentials are
measured at the boundary electrodes positioned at
*** = ±5°.
The dipole was
placed on the boundary electrode indicated

by the angle $ (p)$ (table 1). The dipole was
in a direction of the positive axis of $y$, the result is

\[ V_s = \frac{2M}{R} \left( \frac{1 + \mu}{1 - \frac{1 + \mu}{2 \mu}} \right) \]

(4)

Where $\mu$ is the direction cosine with respect to
$y$ of the radius vector to the point on the bound-
ary where the potential $V_s$ is measured. If this
equation is solved for the eccentricity $fR$ in terms
of $\Phi$ for the maximal (positive or negative) value
of $V_s$ on the boundary, the result is

\[ fR = R (1 - \sin \Phi) \]

(5)

In table 3 are the theoretically predicted values
of the eccentricity $fR$ (p) due to maximal poten-
tials at the boundary electrodes positioned at $\Phi =
90^\circ$, $\Phi = 85^\circ$, $\Phi = 80^\circ$ . . . $\Phi = 50^\circ$ .
The measurements for $V_s$ maximal are not critical
($= 5^\circ$).

Let the general solution be reduced to the ex-
pression for the potential $V_s$ on the boundary
when the eccentricity $fR$ is along the positive or
negative axis of X and when the dipole axis makes
an angle of 45° with the radius vector to the di-
pole. $V_s$ may then be equated to zero and the
resulting equation may then be solved for theoreti-
cal values of the eccentricity $fR$ (p) (table 4) in
terms of the angle made by the radius vector to
the dipole and the radius vector to the boundary
electrode where $V_s = 0$. The result is

\[ fR = R (\cos \Phi - \sin \Phi) \]

(6)

Finally, it can be shown that the average value
$\bar{V}_s$ of the potential $V_s$ over the circular boundary
of the conducting lamina is given by

\[ \bar{V}_s = \frac{1}{3} \int \frac{V_s}{R} \, ds = 0, \]  

(7)

and consequently if the unipolar potentials are
measured at each of 72 boundary electrodes spaced
5° apart or at each of 37 of the 73 electrodes on
half of the boundary for the eccentric radial di-
pole from $V_s$ maximal through $V_s$ minimal the
average should be nearly zero. An arbitrary ec-
centricity of 11.85 cm. was chosen and 37 poten-
tials measured. A set of satisfactory potential
measurements should, according to (7), have an
average value which is very close to $\bar{V}_s = \Sigma V_s / 37 \approx 0$.

Measurements. Measurements for table 1 were
made entirely with detector 1 (figs. 1 and 2). This
detector records the potential difference $V_s - V_I$
where $V_s$ is at any electrode $E$ on the boundary.
Detector $D_I$ operates balanced-to-ground with very
high common-mode rejection. Its exploring probe
$A$ was placed on the boundary electrode indicated
by the angle $\Phi$ (p) (table 1). The dipole was
then moved along the x-axis, as shown in figure 1, until \( V_A - V_B = 0 \), the distance \( fR \) was then measured. The result is indicated in table 1 under \( fR \) \((m)\) and is to be compared with the theoretically predicted values \( fR \) \((p)\) of this table. In measuring the ratios \( V_A/V_B \) \((m)\) of table 1, probe \( A \) of detector 1 was placed first on electrode \( A \) (fig. 1) and then on electrode \( B \) (fig. 1). The various eccentricities used are those under \( fR \) \((p)\) of this table. The results \( V_A/V_B \) \((m)\) are to be compared with the theoretically predicted values under \( V_A/V_B \) \((p)\). During these measurements the detector \( D_2 \) (fig. 2) was on pole 1 of switch \( S \) (fig. 2) and thus recorded the potential differences \( V_A - V_B \) and the Wagner ground device \( W \) was adjusted to keep this potential difference at zero.

In obtaining the data under \( \Phi' \) \((m)\) of table 2 detector \( D_2 \) (fig. 2) was connected to pole 2 of switch \( S \). The Wagner ground device was set at the Frank-Kay "zero" reference potential. This is \( V_A - V_B = 0 \) with the dipole centric and with \( V_A \) indicated by \( \Phi = 90^\circ \) (fig. 1). This balance is always checked before and after a potential measurement \( V_A' - V_B' \) with the dipole at the eccentric positions shown under \( fR \) \((p)\) (table 2). Exploring probe \( B \) of \( D_2 \) was then moved along the boundary in search of the position indicated

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**Fig. 1.** View of the homogeneous circular lamina \( C-L \) (tap water) looking down from above. A plastic circular tray with a vertical side wall contains the tap water about 2.25 cm. deep. \( E \) is one of 72 (only 24 shown) electrodes mounted 5° apart on the circular boundary of the lamina mid-way from the surface to the bottom of the tank. Each electrode \( E \) is connected to the central terminal \( C-T \) through a resistance of 1 meg. \( C-T \) is a copper wire which averages the potentials of the 72 electrodes. The ring \( C-T \) is not closed in order to avoid an inductive loop. The potential of \( C-T \) is denoted by \( V_T \). \( R \) is the radius of the circular lamina. The dipole \( (+, -) \) is shown along the positive or negative axis of \( X \). The potential at electrode \( A \) is denoted by \( FA \) and \( F_A \) is the radius \( = 24.13 \) cm.) which contains the dipole \( [0 - 0' = 2.5 \text{ cm.}] \). \( C-T \) is the central terminal of the averaging network connected to 72 electrodes on the boundary of \( C-L \) through 72 equal 1 meg. ohm resistances. \( D_1 \) is a second detector similar to \( D_2 \) connected to the exploring probe \( A \) and to the central terminal \( C-T \). Each of the detectors \( D_1 \) and \( D_2 \) are fed through cathode followers and consist of differential preamplifiers which feed their output into two cathodery oscilloscopes (H.P. type 130 BR) through 60 cycle rejection filters, balanced to ground. \( D_1 \) is necessarily operated unbalanced to ground with a cathode follower on the high-level input. (Further circuit details will be supplied upon request.)

**Fig. 2.** The primary of the transformer \( T \) is energized by a general radio beat frequency oscillator (type 130 1A). The transformer \( T \) is double shielded (G.R. type 578-B). \( R_1 \) and \( R_2 \) are a.c. resistance decade \( L \) (and \( X \) ) operated in the immediate range of 10 K ohms each. \( C \) is a differential capacitor (G.R.) which is connected in parallel with \( R_1 \) and \( R_2 \) and together with the resistances makes up the Wagner ground. \( D_1 \) is a detector which, on position 1 of switch \( S \), acts as a secondary null balance of the Wagner ground. On position 2 of \( S, D_2 \) is used for measuring the potential of probe \( B \) when the ground potential of \( W \) has been set at the Frank-Kay reference value. \( C-L \) is the homogeneous circular conducting lamina of radius \( R \) \((= 24.13 \text{ cm.}) \) which contains the dipole \([0 - 0' = 2.5 \text{ cm.}] \). \( C-T \) is the central terminal of the averaging network connected to 72 electrodes on the boundary of \( C-L \) through 72 equal 1 meg. ohm resistances. \( D_1 \) is a second detector similar to \( D_2 \) connected to the exploring probe \( A \) and to the central terminal \( C-T \). Each of the detectors \( D_1 \) and \( D_2 \) are fed through cathode followers and consist of differential preamplifiers which feed their output into two cathodery oscilloscopes (H.P. type 130 BR) through 60 cycle rejection filters, balanced to ground. \( D_1 \) is necessarily operated unbalanced to ground with a cathode follower on the high-level input. (Further circuit details will be supplied upon request.)
Predicted (p) and Measured (m) Values of the Zero of Potential on the Boundary for Arbitrary Values of the Eccentricity Using the Frank-Kay Reference potential*  

<table>
<thead>
<tr>
<th>φ (p)</th>
<th>φ’ (m)</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>90°</td>
<td>set fR?</td>
</tr>
<tr>
<td>2.10</td>
<td>85°</td>
<td>80° (4.19)</td>
</tr>
<tr>
<td>4.19</td>
<td>80°</td>
<td>72.5° (7.25)</td>
</tr>
<tr>
<td>6.24</td>
<td>75°</td>
<td>63° (10.95)</td>
</tr>
<tr>
<td>12.06</td>
<td>60°</td>
<td>37° (19.27)</td>
</tr>
<tr>
<td>15.51</td>
<td>50°</td>
<td>27° (21.56)</td>
</tr>
</tbody>
</table>

*Under fR (p) and φ (p) are the theoretically predicted values of the eccentricity fR and of the associated central angles φ which locate the boundary electrodes of potential zero. φ’(m) is the position found to have 'zero' potential with reference to the Frank-Kay potential at W(fig. 2). The error of position for the zero potential increases with increasing eccentricity. If the dipole position is theoretically predicted from these 'zeros' it's eccentric position is falsely exaggerated, see values in parentheses under Φ (m).

by (V₄',-V₃) = 0. Those positions are indicated under Φ (m) for the particular eccentricities under fR (p) of this table. The measured positions under Φ’ (m) are to be compared with the theoretically predicted values under Φ (p) of this table. The percent error is shown in the right hand column. In addition, the associated values of fR (p)? are computed from the measured values of Φ’ (m) for comparison with the actual values under fR (p) of this table.

Measurements for table 3 involve dipole displacements along positive x-axis with the direction of the dipole axis maintained collinear with the positive y-axis. The dipole is moved to the indicated values of the eccentricities fR (p) and the potentials are measured first at the boundary electrode position Φ (p) where maximal (or minimal) values are predicted. These measurements are then compared with those on adjacent boundary electrodes 5° on either side of the expected maximal. A check under V₄ (m) and V₄' (m) indicates that the maximal value is on the expected electrode, otherwise the magnitude of the error is indicated.

Measurements fR (m) and fR' (m) (table 4), are very critical. Here the probe electrode is placed at a position on the boundary indicated by the angle Φ, and the dipole with axis at 35° with respect to the positive axis of Y, is moved along the negative or the positive axis of X until the detector D₁ shows the null (V₃'-V₄) = 0, and detector D₂ is made to show the null (V₄'-V₃) = 0 by adjusting the Wagner ground W. These electrode positions were in the fourth quadrant and the value of Φ in the table must therefore be subtracted from 2π. The value of the eccentricity fR (m) is then measured. The procedure is then repeated using the Frank-Kay reference potential (V₄'-V₃) = 0 on detector D₂ with probe B on the boundary electrode of position Φ = 2π--45° toward the negative axis of Y with the dipole centric. Detector D₁ is disconnected from the circuit. Probe B (fig. 2.) is then moved to the other boundary electrodes indicated by the angle Φ and the dipole is moved along the positive or negative axis of X for a null on D₂. The measured values under fR' (m) were not accepted unless a null after measurement checked for the centric dipole at boundary electrode Φ = 2π--45°. All measurements in table 4 represent an average of three separately determined values of fR (m) and fR' (m). Repeated measurements under fR (m) checked within 0.03 to .04 cm., and those under fR' (m) checked within 0.2 cm.

The potential measurements in table 5 were made after arbitrarily choosing an eccentricity of fR (p) = 11.85 cm. with the dipole axis along the positive axis of X (fig. 1). This arbitrary choice gave a precise null (V₄'-V₃) = 0 on detector 1 for the boundary electrode of position Φ = 60°. The potentials V₄' (m) on the 57 indicated electrodes were measured without altering the dipole position.

<table>
<thead>
<tr>
<th>Φ (p)</th>
<th>fR (p)</th>
<th>V₄ Max.</th>
<th>V₄' Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>90°</td>
<td>0</td>
<td>√</td>
<td>5° Error</td>
</tr>
<tr>
<td>85°</td>
<td>1.05</td>
<td>√</td>
<td>5° Error</td>
</tr>
<tr>
<td>80°</td>
<td>2.11</td>
<td>√</td>
<td>5° Error</td>
</tr>
<tr>
<td>75°</td>
<td>3.18</td>
<td>√</td>
<td>5° Error</td>
</tr>
<tr>
<td>70°</td>
<td>4.25</td>
<td>√</td>
<td>5° Error</td>
</tr>
<tr>
<td>65°</td>
<td>5.44</td>
<td>√</td>
<td>5° Error</td>
</tr>
<tr>
<td>60°</td>
<td>6.47</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td>55°</td>
<td>7.60</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td>50°</td>
<td>8.78</td>
<td>√</td>
<td></td>
</tr>
</tbody>
</table>

*Under fR (p) are the theoretically predicted eccentricities along the X-axis for arbitrary boundary electrode positions of potential V₄ = maximal by the angle Φ (p). Under V₄ (max.) are checks for values less than +5° error for the position of maximal (positive or negative) potential on the boundary with the dipole axis collinear with the positive axis of Y. These potentials are measured with reference to Vₑ of the averaging net work. The potentials Vₑ', are measured for similar dipole positions using the Frank-Kay reference potential Vₑ at the Wagner ground (fig. 2.). The error for the centric position appeared with rotation of the dipole through 90° from 'zero set' at Φ = 90°.
UNIPOLAR POTENTIAL MEASUREMENTS

In obtaining the potentials \( V'_e \) (m) by the Frank-Kay method it was required to move the dipole to centric position for “zero check” after each measurement. If the null \( (V_e - V_w) = 0 \) on \( D_2 \) did not check for the boundary electrode \( \Phi = 90^\circ \) with centripole dipole the measurement was not accepted. The potential differences \( (V'_e - V_w) \) are unaltered by the presence of the averaging network \( C-T \). This can be shown by using 10 meg. ohm resistances or by removing \( C-T \).

Careful positioning and leveling of the circular lamina under the dipole suspension track is initially carried out. The suspension track is carefully leveled. In this way the dipole suspension platform enables quick and accurate positioning along the track on the \( \pm \) axis of \( X \). The dipole electrode proper is damped in a brass collar which permits accurate angles of dipole axis rotation in the suspension platform.*

**DISCUSSION**

An examination of tables 1-5 shows highly accurate agreement between measured values on the one hand and predicted theoretical values on the other in each instance in which the reference potential \( V_T \) on the averaging network \( C-T \) is used. Moreover the result obtained in table 5 for the average value of \( V_S \) over 17 electrodes indicates why \( V_T \) may be used as a highly accurate zero reference potential. This experiment serves as an entirely satisfactory demonstration of the mathematical truth of equation (7) for the definition of the zero of potential offered in this and other articles. Table 5 and equation (7) also contradict Burger’s assertion that “we can connect any point on the body to ground and consider this a zero of potential.” 15 If the right leg is grounded for Burger’s “zero” the potential on the right of equation (7) fluctuates at the unipolar potential of the right leg, and every other unipolar potential of the body has that of the right leg superimposed upon it. Exception must also be taken to Burger’s statement that “a dipole is not two poles, however, but is a singularity.” 15 A singularity is, in fact, a general mathematical term which, for example, may be used for a multipole with a finite distribution that is enclosed by a sphere of finite radius outside of which the potential function due to the multipole is harmonic. 16 Here, Burger confuses the term doublet, in which the two poles of the dipole approach each other until the finite distance \( d \) which separates them is very small in comparison.

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*The nonradial dipole makes an angle of 45° with the direction of the eccentricity along the \( X \)-axis. Under \( fR (p) \) are the theoretically computed values of the eccentricities for arbitrary positions \( \Phi (p) \) of boundary electrodes in the fourth quadrant at the potential \( V_s = 0 \). Under \( fR (m) \) are the measured eccentricities for positions of zero of potential on the boundary. The reference potential is \( V_w \) of the averaging network. If the values under \( \Phi (p) \) are subtracted from 180° for a new value of \( \Phi \) and equated to the values under \( fR (p) \) taken from above down with the sign changed, the former indicates the position on the boundary of the zeros of potential in the second quadrant. The values under \( fR (m) \) are not symmetrical, however, and this column must therefore be inverted in order to be equated to 180° - \( \Phi (p) \). Under \( fR (m) \) are the measured eccentricities for positions of “zero” of potential on the boundary. The reference potential \( V_w \) at the Wagner ground (fig. 2) was used in these measurements in accordance with the method introduced by Frank and Kay. Here \( V_w \) is adjustol (set) equal to the potential \( V_s \) of the boundary electrode indicated by the central angle \( \Phi = 2\pi - 45^\circ \) when the eccentricity \( fR \) is zero. Values under \( fR (p) \) and \( fR (m) \) are to be compared with those under the theoretically predicted values \( fR (p) \). All measured values are the average of three sets of measurements.

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**Table 4.—Predicted (p) and Measured (m) Values of the Eccentricity \( fR \) of the Non-Radial Dipole with Respect to the Reference Potential \( V_w \) of the Averaging Network and the Eccentricity \( fR \) of the Non-Radial Dipole with Respect to the Reference Potential \( V_w \) of Frank and Kay**

<table>
<thead>
<tr>
<th>( \Phi (p) )</th>
<th>( fR (p) )</th>
<th>( fR (m) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>70°</td>
<td>-14.42</td>
<td>-14.35</td>
</tr>
<tr>
<td>65°</td>
<td>-11.67</td>
<td>-11.68</td>
</tr>
<tr>
<td>60°</td>
<td>-8.83</td>
<td>-9.24</td>
</tr>
<tr>
<td>55°</td>
<td>-5.93</td>
<td>-5.97</td>
</tr>
<tr>
<td>50°</td>
<td>-2.97</td>
<td>-2.95</td>
</tr>
<tr>
<td>45°</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>40°</td>
<td>+2.87</td>
<td>+2.73</td>
</tr>
<tr>
<td>35°</td>
<td>+5.93</td>
<td>+5.62</td>
</tr>
<tr>
<td>30°</td>
<td>+8.83</td>
<td>+8.70</td>
</tr>
<tr>
<td>25°</td>
<td>+11.67</td>
<td>+11.78</td>
</tr>
<tr>
<td>20°</td>
<td>+14.42</td>
<td>+14.44</td>
</tr>
</tbody>
</table>

*Values of \( V_s \) are given in centimeter peak-to-peak. The detector circuit was operated at a sensitivity of 20 M.V./cm. differentially and 10 M.V./cm. unbalanced to ground.
poor measurements for \( V_s' (m) \) where \( V_s = 0 \), for \( V_s' (m) \) where \( V_s \) maximal, for values of \( fR' (m) \) and finally for the values of \( V_s' \). In our hands the Frank-Kay reference potential\(^5\) has proved unreliable for unipolar potential measurements. This result can not be ascribed to differences in the quality of generating and detecting equipment.

The fundamental error in the Frank-Kay reference potential \( V_w \) appears when the centric dipole is rotated through 90° and whenever the dipole is made eccentric. The zero potential set \( (V_s' - V_w) = 0 \) and the null \( (V_T - V_w) = 0 \) move grossly out of balance under these circumstances. Inasmuch as \( V_T \) is demonstrated as a sound, and \( V_w \) an unsound, reference potential for unipolar potential measurements, it is clear that these motions of the dipole impose a common-mode potential-of-error upon every point of the circular lamina and therefore upon \( C-T \) also. This error vanishes in the differential detector \( D_1 \) which records the potential difference \( (V_s - V_T) \) and the reference potential \( V_T \) is therefore in agreement with equation (7) with no appreciable error in unipolar potential measurements. The unbalanced-to-ground detector circuit of Frank and Kay is sensitive to the superimposed common-mode error-voltage in the leads \( (V_s' - V_w) \) and \( (V_T - V_w) \) and accurate unipolar potential measurements can not be made with the Frank-Kay reference \( V_w \) unless this potential is first made equal to \( V_T \) by adjustment of \( R_1 \) or \( R_2 \) and \( C \) on the Wagner ground. Bridge theory predicts why \( (V_T - V_w) \neq 0 \) when the dipole is made eccentric. When this error appears with rotation of the centric dipole after zero set, the maximal error occurs at 90° rotation and is equal in magnitude to \( fR = 0.25 \). Being common-mode at all points of \( C-L \) the error is detected only on detector \( D_2 \). Its cause appears to be within the dipole contact surfaces in the form of an impedance decrease in one pole with an opposite equal increase in the other. It can be completely eliminated by adjusting \( W \) for the null \( (V_T - V_w) = 0 \). A similar effect

### Table 5.—Potentials Measured with Respect to the C-T Net Work and with Respect to the Frank-Kay Reference

<table>
<thead>
<tr>
<th>( \phi )</th>
<th>( V_s (m) )</th>
<th>( V_T (m) )</th>
<th>( \phi )</th>
<th>( V_s (m) )</th>
<th>( V_T (m) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>3.98</td>
<td>2.81</td>
<td>95°</td>
<td>-0.95</td>
<td>-1.55</td>
</tr>
<tr>
<td>5°</td>
<td>3.80</td>
<td>2.65</td>
<td>100°</td>
<td>-1.00</td>
<td>-1.60</td>
</tr>
<tr>
<td>10°</td>
<td>3.58</td>
<td>2.20</td>
<td>105°</td>
<td>-1.02</td>
<td>-1.61</td>
</tr>
<tr>
<td>15°</td>
<td>3.20</td>
<td>1.82</td>
<td>110°</td>
<td>-1.10</td>
<td>-1.68</td>
</tr>
<tr>
<td>20°</td>
<td>2.80</td>
<td>1.57</td>
<td>115°</td>
<td>-1.15</td>
<td>-1.70</td>
</tr>
<tr>
<td>25°</td>
<td>2.40</td>
<td>1.29</td>
<td>120°</td>
<td>-1.20</td>
<td>-1.73</td>
</tr>
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<td>30°</td>
<td>2.05</td>
<td>1.00</td>
<td>125°</td>
<td>-1.21</td>
<td>-1.76</td>
</tr>
<tr>
<td>35°</td>
<td>1.50</td>
<td>0.68</td>
<td>130°</td>
<td>-1.23</td>
<td>-1.80</td>
</tr>
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<td>40°</td>
<td>1.15</td>
<td>0.29</td>
<td>135°</td>
<td>-1.28</td>
<td>-1.82</td>
</tr>
<tr>
<td>45°</td>
<td>0.85</td>
<td>-0.18</td>
<td>140°</td>
<td>-1.30</td>
<td>-1.85</td>
</tr>
<tr>
<td>50°</td>
<td>0.60</td>
<td>-0.30</td>
<td>145°</td>
<td>-1.32</td>
<td>-1.97</td>
</tr>
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\[ V_T = \frac{\sum V_s}{37} = 0.007 \text{ cm.} \quad V_T = \frac{\sum V_s'}{37} = -0.732 \text{ cm.} \]

*Under \( V_s (m) \) are the measured potentials on the boundary electrodes positioned from where \( \phi = 0° \) to where \( \phi = 180° \) with reference to the potential \( V_T \) of the averaging net work \( C-T \) for the eccentricity \( fR (m) = 11.85 \text{ cm.} \). Under \( V_s' \) are the measured potentials on the boundary electrodes from where \( \phi = 0° \) to where \( \phi = 180° \) for the eccentricity \( fR (m) = 11.85 \text{ cm.} \) with reference to the Frank-Kay ground potential at \( W \) (fig. 2) adjusted to the potential of the boundary electrode \( \phi = 90° \) when the dipole was centric. \( fR = 0.007 \) indicated that \( V_T (m) \) the potential on \( C-T \) is a highly accurate "zero" reference. The average error of the Frank-Kay reference is 208 times greater than the measured value of \( V_T \). Moreover the ratio of the number of negative to the number of positive boundary potentials is 2.41 times the proper value of this ratio.

with distances from the doublet at which observation of the potential is made.\(^1\) The two poles must not, however, be superimposed at a point for in this case the field vanished experimentally in accordance with equation (7).

In table 5, the value \( V_T = 0.007 \text{ cm.} \) is too small to be observed on the oscilloscope. In striking contrast to these results the Frank-Kay reference potential shows an average error which is 208 times as great. Tables 2-5 show what may be regarded as very
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is much more striking when the dipole electrode is inserted into biological tissues. This common-mode voltage error that occurs with moving the dipole is not eliminated by returning the dipole to centric position for "zero check" as suggested by Frank. The error promptly reappears when the dipole is returned to the eccentric position. It is a curious fact that Frank and Kay measured boundary potentials in the hemisphere with an eccentric dipole. These potentials were measured with respect to an electrode placed on the boundary at a point which was theoretically computed to be of potential zero for the dipole position used. Their reference potential on the dipole generating circuit was thereby left unevaluated against known unipolar potentials in a simple model.

These experimental results were not entirely unexpected for it has been pointed out elsewhere that the torso model measurements which have been reported by Frank were not in agreement with appropriate theoretical considerations. The errors in torso model measurements are striking and are of the same general nature as observed here in circular plane lamina. The position may safely be taken that any lead system which has its foundation "unipolar" potential measurements on the torso model should not at this time be projected to the living human subject.

These experimental findings lend support to the averaging type network for a zero reference potential for unipolar leads. The immersion sphere and the weighted Wilson central terminal are averaging networks for "zero" reference potential similar in theory to the kind used in these satisfactory experiments on a simple model. The author has repeatedly emphasized the importance of the averaging network for a central terminal reference potential in making unipolar potential leads.

SUMMARY

The theoretical solution for the unipolar potential at any point in or upon the boundary of the homogeneous circular lamina is justified by a highly accurate method of unipolar potential measurement. The appropriate zero of potential reference for unipolar leads or unipolar potential measurements in a circular conducting lamina is obtained by an averaging network. Its central terminal potential is the proper reference potential. The "outside" ground potential on the Wagner-ground device is brought to the potential of the central terminal (which is not actually connected to ground) by a secondary adjustment on the Wagner ground whenever the dipole position is changed. This adjustment is not critical. The Frank-Kay type of reference potential on the dipole generating circuit is in effect the potential of the Wagner-ground device. It is adjusted to the known zero of potential when the dipole is centric.

When the dipole is made eccentric in the radial or the nonradial positions, this reference potential is unsound for making unipolar potential measurements in a simple model where the theoretical values of the potential are known. Consequently, the Frank-Kay reference potential is unsound also in the torso model and in either case its use in our hands appears to introduce serious errors in unipolar potential measurement.

SUMARIO IN INTERLINGUA

Le solution theoretica pro le potential unipolar a un puncto qualcunque intra o super le limite de un homogenee lamina circular es justificate per un accuratissime metodo de mesurar le potential unipolar. Le appropriate zero de potentia como referentia pro derivaciones unipolar o mesurationes de potential unipolar in un conductee lamina circular es obtenite per un rete a medias. Su potential terminal central es le appropriate potential de referentia. Le "externe" potential de terra in le dispositivo Wagner es apportate al potential del termino central (que non es vermente connectita con le terra) per un adustation secundari in le dispositivo Wagner quandocunque le position del dipolo es alterate. Iste adustation non es critic. Le
typo Frank-Kay\textsuperscript{3} de potential de referentia in le circuito generatori dipolar es de facto le potential del dispositivo de Wagner. Illo es adjustate al cognoscite zero de potential quando le dipolo es centric. Quando le dipolo es eccentric in positiones radial o non-radial, iste potential de referentia es inadequate pro facer mesuraciones de potential unipolar in un simple modello ubi le valores theorie del potential es cognoscite. Per consequente, le potential de referentia Frank-Kay es inadequate etiam in le modello-torso. In ambe casos, su uso in nostre manos pare introducer serie errores in le mesuración de potential unipolar.

REFERENCES
Unipolar Potential Measurements in the Electric Field Produced by an Arbitrary Dipole in a Circular Homogeneous Lamina

ROBERT H. BAYLEY

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