Flow of Liquids Through "Collapsible" Tubes

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Liquid flowing through a thin-walled "collapsible" tube similar to a vein, inclined to any angle between vertical and slightly above horizontal, differs from the flow in rigid tubes in that: a "collapsible" tube accepts any flow presented to it without affecting the rate of flow through the system; the lateral pressure at all cross sections throughout the tube is zero and does not change as the flow changes; the cross section of the tube and the mean velocity of flow increase as the flow increases; for any particular flow, the cross section increases and the mean velocity of flow diminishes as the potential energy difference between the upper and lower ends of the tube is decreased; for any given flow and potential energy difference, an increase in the length of the tube causes an increase in the cross section and a decrease in mean velocity; also, an increase in the perimeter of the tube causes an increase in cross section and a decrease in mean velocity, and an increase in the viscosity of the liquid causes an increase in cross section and a reduction in mean velocity. In such tubes there is no lateral pressure gradient between any two cross sections as there is in rigid tubes; instead there is a potential energy gradient which is proportional to the energy loss along the tube. Under certain conditions the flow of liquid through a collapsible tube is described with a reasonable degree of accuracy by a modification of Poiseuille's law for tubes of elliptical cross section.

In the flow of liquids through "collapsible" tubes, such as veins, the cross section of the tube is free to change as the lateral pressure within the tube changes. This differs from the flow through rigid tubes where the cross section remains fixed regardless of the lateral pressure, and differs from the flow through elastic tubes of circular cross section in which the cross section increases as the lateral pressure increases; it differs from the flow in open channels where there is a liquid-solid interface on part of the boundary and a liquid-gas interface where the surface is in contact with air; and it differs from a viscous liquid which, when poured from a container, falls freely in air and has a liquid-gas interface extending over the entire surface. In a collapsible tube there is a liquid-solid interface extending over the entire surface, even though the tube is free to change its cross section. When a collapsible tube becomes distended to the point that its cross section is circular, and its wall is stretched, it no longer behaves as a collapsible tube but functions as a rigid tube, and the flow through it is described by the classical laws of hydrodynamics for circular tubes.

Following our earlier studies1-3 of the flow of fluids through collapsible tubes, numerous investigations have been carried out concerning such tubes. These include studies involving venous return to the chest,4 energy relationships,5 the flow in particular organs,6 and a study of the flow through long, vertical collapsible tubes by Duomareo et al.,7 in which he found that there is zero lateral pressure throughout the length of such a tube. A comprehensive review of the literature is given by Brecher.4

In our earlier studies we were concerned only with the flow in collapsible tubes that were in the collapsed state for a short length of the tube, the remaining part being distended to the point where it closely approximated a rigid tube with a circular cross section. In our present studies we have been concerned with the flow through long tubes collapsed throughout their entire length; that is, either a vertical collapsible tube, or one in-
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In mammals the flow through veins that are above heart level is generally of this nature; an extreme example is the flow of blood through the external jugular vein of the giraffe, when the head is above heart level. In these studies we have been interested in learning the inter-relationships between the following factors: rate of flow (milliliter per second), cross section, mean velocity (centimeter per second), viscosity, length of tube, resistance, potential energy difference between any two cross sections of the tube, and the perimeter of the tube.

Brecher has suggested that the modified Poiseuille equation for elliptical tubes, \[ F = \frac{\pi (P_1 - P_2)}{4 \eta L} \left( \frac{a^3 b^3}{a^2 + b^2} \right) \] given by Milne-Thompson, describes the flow through collapsible tubes. It should be noted that the above \( \frac{a^3 b^3}{a^2 + b^2} \) term, where \( a \) is the major semi-axis and \( b \) the minor semi-axis of an ellipse, replaces the radius to the fourth power, \( r^4 \), in the classical Poiseuille equation, and that the constant \( \frac{\pi}{8} \) in the Poiseuille equation for circular tubes is replaced by \( \frac{\pi}{4} \) in the above elliptical modification. Since there have been no experimental studies concerning the applicability of this equation to the flow through long collapsible tubes, collapsed throughout their entire length, we have attempted to ascertain how the equation applies to the flow through such tubes; and also to learn how the flow through these tubes is related to the classical hydrodynamics based on Bernoulli’s theorem. It will be shown that the flow of viscous liquids through such collapsible tubes is a special case of Bernoulli’s theorem in which the lateral pressure and kinetic energy functions become zero, leaving only the potential energy and heat energy functions.

Methods

The apparatus, figure 1, consisted of a Mariotte bottle from which 0.9 per cent NaCl solution containing different concentrations of sucrose* flowed through heavy-walled, semi-rigid rubber tubing into the upper end of the collapsible tube, \( P_1 \), through the tube to \( P_2 \) where the collapsible tube was connected to a rigid tube, and through semi-rigid tubes to the outflow, \( Z \), located several centimeters below \( P_2 \). The flow in milliliters per second through the collapsible tube was changed by varying the resistance between the Mariotte bottle and \( P_1 \), and was measured by collecting the outflow in a graduate cylinder in a given time. The collapsible tube was placed within a plethysmograph which was connected to a volume recorder consisting of a paraffin-coated 120 cm. long glass tube, having a radius of 0.15 cm., containing a small bubble of water. With zero flow through the collapsible tube the position of the water bubble in the volume recorder was read. A few seconds after a particular flow had been established the volume recorder was read again. The difference between the two readings gave the internal volume of the collapsible tube for the particular flow. The volume recorder was gently vibrated at the time of reading. Since, as will be shown, the cross section of the collapsible tube was the same at all points throughout the length of the tube (except for a few centimeters at the upper and lower ends) the cross section of the tube for a particular flow was calculated by dividing the internal volume of the tube by its length. Since we knew the perimeter of the collapsible tube and the experimentally determined cross section (for each flow studied), it was possible to calculate, from the well known equations for the perimeter and cross section of an ellipse, the major and minor axes, \( a \) and \( b \), of the ellipse having the same cross section as that determined experimentally, The apparatus was hinged at the level \( P_2 \), so that the collapsible tube and plethysmograph could be rotated about \( P_2 \) and thus inclined to any desired angle with the horizontal. In this way the vertical distance between the upper and lower ends of the tube could be changed, thus varying the potential energy difference between the inflow and the outflow ends of the tube. The potential energy difference between the two ends is equal to \( \rho g h_1 - \rho g h_2 \), where \( \rho \) is the density of the liquid, \( g \) the gravitational constant, \( h_1 \) the vertical height of the upper end of the tube above a fixed reference plane, and \( h_2 \) the vertical height of the outflow end above the same reference plane. The mean velocity of flow in centimeters per second through the collapsible tube was calculated by dividing the flow in milliliters per second by the cross section. The viscosity of the different solutions studied was measured with an Oswald viscosimeter, and expressed as relative viscosity to 0.9 per cent NaCl solution, which was taken as 1. More than five different liquids having relative viscosities ranging from 1 to 14 have been studied. The temperature in the plethysmograph was maintained constant. Pres-

*Clear, Red Label Karo Syrup.
sizes were measured by means of water manometers or Statham strain gages, and recorded with a Brush electromagnetic oscillograph. The pressure at different points within the collapsible tube was measured by passing a small-bore polyethylene tube through a side arm near $P_1$ to different points throughout the length of the collapsible tube. The conformation of the collapsible tube at its upper and lower ends differs from that of the rest of the tube, and was studied by photographing the ends of the tube. The volume of the collapsible tube at its ends was calculated by planimetering the area on the photographs, and in some cases the measured volume of the tube was corrected by subtracting the excess volume at the two ends from the total. This correction was generally small.

In an experiment all of the factors (viscosity, length, perimeter, and potential energy difference between the ends of the tube) were maintained constant, and the volume and cross section of the vertical collapsible tube determined for flows ranging from zero to 20 ml./sec. This was carried out with the tube inclined to angles of $9^\circ$, $18^\circ$, $30^\circ$, $57^\circ$ and $90^\circ$; in this way the potential energy difference between the ends of the tube was changed. This entire procedure was carried out for different viscosity liquids, tubes of different length, and tube of different perimeter. Thus the effect of each of the above factors on the flow–cross-section curve was studied. All of the studies have been carried out on thin-walled Penrose rubber tubing having lengths from 9.75 to 78.5 cm., and perimeters of 2.0, 3.85 and 6.0 cm.

Two other series of similar experiments were carried out which differed from the above in that the plethysmograph was not used; instead an indicator was injected instantaneously at $P_1$, and its concentration curve recorded at $P_2$. This method gave inconsistent results, as shown by the fact that the flow calculated from the indicator dilution curve frequently differed greatly from the true flow. This may be explained by the fact, as shown by Rossi et al. that with streamline flow the indicator dilution technic does not give an accurate measure of flow.

RESULTS

In a liquid-filled system such as shown in figure 1, when there is zero flow through a 78 cm. long collapsible tube inclined to some angle with the horizontal, the tube has a uniform shape extending throughout its length. Its cross section is of the shape shown in stage 1, figure 2. When a small constant flow is
established through the tube it assumes throughout its length, except for the upper 3 cm. and the lower 3 cm., the cross section shown in stage 2, figure 2. At the upper end the tube decreases in cross section more or less uniformly from the rigid circular tube to which it is attached to a point approximately 3 cm. below the upper end, where it assumes the cross section shown in stage 2. The lower 3 cm. of the tube assumes the dilated hour-glass shape shown in figure 3. This hour-glass shape is more pronounced with large flows. With the establishment of a greater flow, the tube takes on throughout its length, except for the upper and lower 3 cm., the elliptical cross section shown in stage 3, figure 2. With a still greater flow, the elliptical shape shown in stage 4 is assumed throughout its length, except for the upper and lower ends. If a very large flow is established the tube becomes circular in cross section throughout its entire length. It develops a lateral pressure inside the tube which is greater than the pressure outside, and the tube no longer behaves as a "collapsible" tube but as a rigid or elastic circular tube.

"Collapsible" Tube Does Not Affect Flow. The presence of a vertical collapsible tube in a system such as that shown in figure 1 causes no change in the amount of liquid flowing from the Mariotte bottle system. This is shown in figure 4A where the pressure, \( Y \) (figure 1), was maintained constant and a 78 cm. long, 3.85 perimeter collapsible tube, having a 4.5 viscosity liquid flowing through it, was rotated about \( P_2 \) changing the flow and the pressure head \( Y - P_1 \). This was done with the collapsible tube in the system and with the collapsible tube removed from the system; in the latter case the outflow was measured from \( P_1 \). The flows were nearly the same but were always a little larger with the collapsible tube in the system. The reason for this is that the thin-walled rubber collapsible tube is not perfectly collapsible, as are veins. Because of its elasticity the upper and lower few centimeters of the tube do not collapse perfectly, and with small flows the pressure 3 cm. below \( P_1 \) is sub-atmospheric, as shown in figure 4B. When the data in figure 4A were corrected for this factor the flows became approximately equal. Also, when the tube is in the extremely collapsed state, two small capillary tubes form along either side of the collapsible tube and serve as more or less rigid tubes in parallel with the collapsible tube. Therefore, with small flows the flow is slightly greater with the collapsible tube in the system. It is concluded that a perfectly collapsible tube accepts whatever flow is presented to it without affecting the flow through the system.

Lateral Pressure Is Zero at All Cross Sections. The lateral pressure at all cross sections throughout a vertical collapsible tube, except for a few centimeters at the upper and lower ends, is zero and does not change as the rate of flow through the tube changes. This is shown in figure 4B, in which the lateral pressure, at a point 3 cm. below the upper end of the collapsible tube, is plotted against the flow when a 7.6 viscosity liquid flowed through a
78 cm. long, 3.85 cm. perimeter tube inclined 90° to the horizontal. In this experiment the zero position was taken as the point of pressure measurement in the tube 3 cm. below the upper end of the tube. As the flow increased from zero to 2 ml./sec. there was a rapid rise in pressure from 107.8 cm. water below atmospheric to 3 cm. below atmospheric. Further increase in flow was associated with a slight increase in pressure, until it reached zero, where it remained for large flows. Measurement of the lateral pressure at all points
throughout the length of the vertical tube, taking the point of pressure measurement in the tube to be the zero level, gave similar results to the above, except at the lower 3 cm. of the tube, where the pressure was always sub-atmospheric. Similar results were obtained when the tube was inclined to various angles with the horizontal instead of being in the vertical position. The reason that the pressure differed from zero when the flow was small is that the tube possessed certain elastic properties and several centimeters of water pressure was required to maintain the deformation of the tube. As a result, with zero flow through the system two small, more or less rigid, capillary tubes form along the sides of the collapsible tube, and because of these the pressure, $P_1$, registers the level of the outflow from the system where the liquid passes into the atmosphere. As the flow increases from zero to 2 ml./sec, the pressure, $P_1$, rises rapidly, because the effect of the two small capillary tubes that form along the lateral edges of the collapsible tube is largely overcome; as the flow increases beyond 2 ml./sec. there is little change in $P_1$ because the effect of the two small rigid tubes has been overcome and the tube acts as a collapsible tube with the lateral pressure being independent of the rate of flow.

Cross Section Same Throughout Tube. As described above, the cross section of a collapsible tube through which liquid is flowing at a constant rate appears on visual inspection to be the same throughout the length of the tube, except for the upper and lower ends. Measurements, taken by means of calipers throughout the length of a 78 cm. long collapsible tube inclined to some angle with the horizontal, through which liquid was flowing at a constant rate, showed no measurable difference in the minor axis of the more or less elliptical cross section shown in stage 3 (fig. 2). Further evidence in support of this was obtained by injecting an indicator at different points throughout the length of the "collapsible" tube, when there was a constant flow through the tube, and recording the indicator time-concentration curve at $P_2$, thus measuring the mean flow time. Although there was considerable scatter of the data in some experiments, for a given rate of flow throughout the collapsible tube a linear relationship was found between the mean flow time and the length of the collapsible tube traversed by the indicator, indicating that the cross section was uniform throughout the tube. Since the cross section is the same throughout the tube and the same amount of liquid is flowing through each cross section at any given time, it follows that the mean velocity of the flow is the same at all cross sections.

Flow Increase Causes Cross Section Increase. In figure 4C is shown the relationship between flow and cross section of the collapsible tube, when liquid having relative viscos-
Fig. 6. A, Relationship between the elliptical cross section function, $\frac{a^2 b^3}{a^2 + b^2}$, and flow for liquids of different viscosities flowing through a 78 cm. long, 3.85 cm. perimeter tube inclined to an angle of 30°. This data was taken from figure 4C. B, Relationship between viscosity and the elliptical cross section function, $\frac{a^2 b^3}{a^2 + b^2}$, for the data shown in figure 4C. C, Relationship between the elliptical cross section function, $\frac{a^2 b^3}{a^2 + b^2}$, and the product (flow divided by the potential energy difference between the two ends of the tube) for liquid flowing through a collapsible tube inclined to different angles with the horizontal. D, Relationship between the elliptical cross section function, $\frac{a^2 b^3}{a^2 + b^2}$, and the length of the collapsible tube for tubes of different length. Data from figure 4E. E, Relationship between the elliptical cross section function, $\frac{a^2 b^3}{a^2 + b^2}$, and the flow for tubes of different perimeters. Data from figure 4F.

Flows of 1, 2.9, 5.7, and 12.5 flowed through a 78 cm. long, 3.85 cm. perimeter tube inclined 30° to the horizontal. It will be noted that as the flow increases the cross section increases in a curvilinear manner, and that for any particular flow the greater the viscosity the larger the cross section. It is assumed that the cross section of the tube is of elliptical shape, then the function of the elliptical cross section, $\frac{a^2 b^3}{a^2 + b^2}$, can be calculated. When this was done for the data in figure 4C, results shown in figure 6A were obtained. The relationship is approximately linear for flows greater than 2 ml./sec. with the elliptical function increasing as the flow increases.

Velocity Increases As Flow Increases. In figure 5A is shown the relationship between the flow in milliliters per second and the mean velocity of flow in centimeters per second for the data in figure 4C. As the flow increases the mean velocity increases in a curvilinear
manner, and for a given flow the velocity decreases as the relative viscosity increases.

**Cross Section Increases As Velocity Increases.** The relationship between mean velocity of flow and the cross section of the collapsible tube for the data in figure 4C is shown in figure 5B. As the velocity increases the cross section increases in a curvilinear manner.

**Cross Section Directly Related to Viscosity.** When for a particular flow, for the data shown in figure 4C, the cross section is plotted against the viscosity, a curvilinear relationship is obtained with the cross section increasing as the viscosity increases. When the elliptical function of the cross section, \(\frac{a^3b^3}{a^2 + b^2}\), is plotted against the viscosity for this data the results shown in figure 6B are obtained. As the viscosity increases, the elliptical function of the cross section increases in a linear fashion for flows greater than 2 ml/sec.

**Cross Section Inversely Related to Potential Energy Difference Between Upper and Lower Ends of the Tube.** Figure 4D shows the relationship between the flow per unit time and cross section of a 78.5 cm. long, 3.85 cm. perimeter collapsible tube when the potential energy difference between the two ends of the tube was changed by inclining the tube to angles of 9°, 18°, 30°, 57° and 90° for a liquid having a relative viscosity of 12.5. For a given flow, as the angle of inclination to the horizontal decreases the cross section increases; and there is a family of curves which describes the relationship between flow and cross section for all angles of inclination. When the elliptical function of the cross section, \(\frac{a^3b^3}{a^2 + b^2}\), is plotted against the length, from the data in figure 4E, a family of curves is obtained as shown in figure 6D. As the length increases the elliptical cross section function increases in a curvilinear manner.

**Perimeter Increase Causes an Increase in Cross Section.** The relationship between flow per unit time and cross section for a 5.6 viscosity liquid in 78 cm. long collapsible tubes of different perimeters, inclined to an angle of 9 degrees with the horizontal, and so having a vertical difference between the two ends of 9.8 cm., is shown in figure 4F. As the flow increases the cross section increases, and for any particular flow the larger the perimeter the larger the cross section. In figure 6F is shown the relationship between the flow and the function of the cross section, \(\frac{a^3b^3}{a^2 + b^2}\), for tubes of different perimeters. These data are taken from curves in figure 4F. There is an approximately linear relationship, and this relationship is the same for tubes of different perimeters. When the potential energy difference between the two ends of the tube was increased, by increasing the angle of inclination with the horizontal, the cross sections were smaller than those in figure 4E, and under these circumstances the elliptical cross section function, \(\frac{a^3b^3}{a^2 + b^2}\), was always larger for the smaller perimeter tubes. The reason for this is not clear.
Energy Relationships. On the basis of the law of conservation of energy, applying Bernoulli's theorem to the flow of viscous liquids, the sum of the total mechanical energy and heat energy at any cross section of a perfectly collapsible tube must be the same as that at any other cross section. This is expressed in the equation:

$$\Delta M g (h_1 - h_2) + \rho a v^2_1 \Delta T - \rho a v^2_2 \Delta T = \frac{1}{2} \Delta M \left( v_1^2 - v_2^2 \right) + \tau L \Delta M$$

where $\Delta M$ Gm. of liquid enter a cross section near the upper end of the tube and $\Delta M$ Gm. simultaneously exit through a cross section near the lower end in the time $\Delta T$. $p_1$ is the lateral pressure, $a_1$ the cross section, $h_1$ the vertical height above an appropriate plane, and $v_1$ the velocity of the liquid particles at a cross section near the upper end; while $p_2$, $a_2$, $h_2$, and $v_2$ are the corresponding values near the lower end of the tube. $\tau$ is the heat energy per unit mass, per unit length, lost by the liquid; $L$ is the length of tube between the two cross sections where the pressure is measured, and $g$ is the acceleration due to gravity.

Since, as shown earlier, the cross section, lateral pressure and velocity are the same at all cross sections of the tube, three of the above terms drop out of the equation leaving:

$$\Delta M g (h_1 - h_2) = \tau L \Delta M$$

That is, the flow through such collapsible tubes is a special case of Bernoulli's theorem for the flow of viscous liquids, in which the potential energy difference between any two cross sections is equal to the heat energy loss due to friction as the liquid moves from the upper to the lower cross section. In a collapsible tube having a certain difference in vertical height between the two ends of the tube, an increased flow through the tube causes no change in the lateral pressure but causes an increase in the cross section. The increased energy difference between any two cross sections of the collapsible tube, that is associated with an increased flow through the tube, manifests itself as an increase in the product: $\Delta M g (h_1 - h_2)$. This is in contrast to the flow through a comparable rigid tube in which case the increase in energy difference between the two ends of the tube, associated with an increase in flow, manifests itself as an increase in the lateral pressure difference between the two ends of the tube in addition to an increase in the above product. Thus, in collapsible tubes, inclined to some angle with the horizontal, there is no lateral pressure gradient between any two cross sections, instead there is a potential energy gradient which is proportional to the energy loss along the tube:

$$\frac{h_1 \rho g - h_2 \rho g}{L} = \tau \rho$$

In the above we were concerned with energy changes within the collapsible tube extending from the point farthest upstream where the tube is collapsed to the farthest point downstream where the tube is collapsed. In the case of a horizontal collapsible tube, or one inclined to some angle below horizontal, at the outflow end of which is applied a subatmospheric pressure, $P_o$, the collapsible tube is open throughout its entire length except a few millimeters of the tube which are collapsed at its outflow end. The pressure, $P_1$, immediately upstream to the collapsed segment in the open part of the collapsible tube, is slightly above atmospheric, while the pressure at some point within the collapsed segment is atmospheric. The pressure difference from immediately above to immediately below the collapsed segment is $(P_1 - P_2)$. The pressure difference between the upper end of the long collapsible segment and the point immediately downstream to the collapsible segment at the outflow end, is the sum of the energy differences from the inflow to the outflow ends of the two collapsed segments.

Resistance. The resistance to flow through a rigid tube system of constant cross sec-
flow through collapsible tubes

section is generally taken to be the product, \( \frac{\Delta P}{F} \), where \( \Delta P \) is the lateral pressure difference between two cross sections along the length of the tube, and \( F \) the flow per unit of time. In laminar flow as the \( \Delta P \) increases the \( F \) increases and the resistance remains constant. Since the lateral pressure is zero at all points in a collapsible tube, inclined to some angle with the horizontal, and does not change as the flow changes, if we substitute the vertical height \((h_1-h_2)\), above a fixed reference plane, of the two cross sections for the lateral pressure \( p_1 \) and \( p_2 \), then the resistance may be taken to be, 
\[
R = \frac{h_1 \rho g h_2 \rho g}{F}.
\]
This resistance has the same dimensions and meaning as the conventional resistance. With zero flow in a collapsible tube, the resistance is infinite, and as the flow increases the resistance decreases.

In the case of a horizontal collapsible tube, collapsed only at its downstream end, the resistance, \( R \), is equal to \( \frac{P_1-P_2}{F} \). As the flow increases the resistance decreases, and with zero flow the resistance is infinite. Thus, in a collapsed tube system in which the above two collapsed segments are in series, as in a collapsible tube inclined to some angle between vertical and horizontal, and to which suction is applied at the downstream end of the collapsed segment, the total resistance is the sum of the above resistances.

**Discussion**

Since the rate of flow through a collapsible tube system, with the tube collapsed throughout its length, is unaffected by the presence of the collapsible tube, it is clear that freely collapsible veins running from a part above heart level, such as the head, back to the heart can exert no siphoning effect on the flow of blood to the part. Thus, the flow is determined solely by the pressure head, from the aorta to the highest point where the vein is collapsed, and the resistance to flow along these arterial and capillary channels.

The fact that the cross section is the same throughout the length of a long collapsible tube is in contrast to the fact that the cross section of a free-falling liquid, poured from a beaker, decreases as the distance from the beaker increases. The reason for this difference would appear to be the fact that there is little or no frictional resistance at the air-liquid interface of the free-falling liquid, while in the collapsible tube there is frictional resistance at the liquid-solid interface at the perimeter of the collapsible tube.

These studies indicate that in a perfectly collapsible tube inclined to angles between horizontal and vertical the lateral pressure is zero at all points regardless of the flow. Our results confirm those obtained by Duromarco et al. in studies on a model, and are in general agreement with the theoretical and experimental results he reported for the venous system of the dog (when held in the vertical, head-up, position).

Although the resistance to flow through veins in the circulatory system is generally small when the veins are not collapsed, it should be pointed out that the resistance is high in collapsed veins. The giraffe is an example; only a relatively small amount of energy is required to move a given quantity of blood from the aorta to the head of a giraffe, with the head at heart level, however, a much larger amount of energy is required to raise the same amount of blood to head level when the head is in the vertical position because of the large potential energy difference between head and heart level. This large amount of potential energy is dissipated in the form of heat when the blood passes from the head back to the heart through the collapsed veins in the neck where the resistance is large and the velocity high. It would appear that the collapse of veins in the neck of the giraffe is a protective mechanism that tends to maintain the pressure in the capillaries of the head above atmospheric, and to dissipate the large amount of potential energy that the blood possesses at head level on its return to the heart; in this way the blood returns to the heart without excessive kinetic energy.
Attempts to obtain a mathematical equation that will accurately describe, for the flow of liquid through a collapsible tube, the relationships between viscosity, flow per unit time, length, perimeter, potential energy difference between the ends of the tube, and the cross section has not been entirely successful because of the following: 1. Thin-walled rubber tubes are not perfectly collapsible but possess elastic properties which cause them to resist deformation. 2. The cross section of a collapsible tube may vary from the extreme of a circle to the other extreme of a line. As a result the Poiseuille equation for circular tubes does not apply. The only other presently-available equation is that for flow through elliptical tubes, and it is not satisfactory for all cross sections of experimental collapsible tubes because such tubes are only approximately elliptical for certain cross sections. 3. Technical difficulties are encountered in measuring the cross section of a collapsible tube. These difficulties are caused by the increase in cross section at the two ends of the tube, as well as unknown “end-effects” where the collapsible tube joins the rigid tube at its upper and lower ends.

The fact that an approximately linear relationship is obtained in our experiments, with flows ranging between 2 and 16 ml/sec, between the elliptical cross section function, \( \frac{a^b b^a}{a^2 + b^2} \), and: flow, viscosity, and

\[
\frac{\text{Flow}}{h_1 \rho g - h_2 \rho g}
\]

suggests that insofar as these parameters are concerned an elliptical modification of Poiseuille’s equation:

\[
F = \frac{\pi (h_1 \rho g - h_2 \rho g)}{4 \eta L} \frac{a^b b^a}{a^2 + b^2}
\]
describes the flow through collapsible tubes that are collapsed throughout their length. In the case of collapsible tubes that are collapsed only at the downstream end, to which negative pressure is applied, the above term, \( (h_1 \rho g - h_2 \rho g) \) is replaced by the lateral pressure, \( (P_1 - P_2) \). The fact that we did not obtain a linear relationship between the elliptical function, \( \frac{a^b b^a}{a^2 + b^2} \), and the length, \( L \), suggests that this equation is not accurate insofar as length is concerned. As mentioned, this discrepancy may be caused by “end-effects.” Although some of the data indicates that the perimeter of the tube has no effect on the flow, other data (when the cross section is small) suggests that the perimeter does have an effect. Further experiments are necessary in order to clarify these questions.

**Summary**

Studies of the flow of different viscosity liquids through long “collapsible” tubes (Penrose tubing) inclined to some angle between horizontal and vertical, and collapsed throughout the length of the tube were carried out in a model. The flow in such a system is similar to the flow in the external jugular vein of the giraffe when the head is above heart level. The inter-relationship between the following factors was studied: rate of flow through the tube (milliliter/second), cross section of the tube, mean velocity of flow (centimeter/second), viscosity of the liquid, length of the tube, resistance to flow through the tube, the potential energy difference between the two ends of the tube, and the perimeter of the tube. It was shown that, except for the length of the tube, the flow through such tubes is described with a reasonable degree of accuracy by a modification, for elliptical tubes, of Poiseuille’s law for circular tubes; and that the flow of viscous liquids through such tubes is a special case of Bernoulli’s theorem in which the kinetic energy and lateral pressure functions become zero, leaving only the potential energy and heat energy functions. There is no lateral pressure gradient in the flow of liquids through such tubes, instead there is only a potential energy gradient.

**Summario in Interlingua**

Esseva effectuate, in un modello, studios relative al fluxo de fluidos de differente viscositate a transverso longe “collabibile”
FLOW THROUGH COLLAPSIBLE TUBES

tubos inclinat a angulos inter horizontal e vertical e collabite ab termino a termino. Le fluxo in un tal sistema es simile al fluxo in le externe vena jugular del girafa quando le capite del animal es supra le nivello de su corde. Esseva studiate e relacione inter le sequente factores: Intensitate del fluxo a transverso le tubo (cm³/sec), section transverse del tubo, velocitate medie del fluxo (cm/sec), viscostitate del fluido, longor del tubo, resistentia al fluxo a transverso le tubo, le differentia de energia potential inter le duo terminos del tubo, e le perimetro del tubo. Esseva monstrate que, con le exception del longor del tubo, le fluxo a transverso tal tubos pote esser describite con grados adequate de accuratia per un modification (pro tubos elliptic) del lege de Poiseuille pro tubos circular e que le fluxo de fluidos viscoso a transverso tal tubos es un caso special del theorema de Bernoulli, con le functiones del energia cinetic e del pression lateral deveniente zero de manera que solmente le functiones de energia potential e de energia caloric remane. Il non existe, in le fluxo de fluido a transverso tal tubos, un gradiente de pression lateral. Il existe solmente un gradiente de energia potential.

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Flow of Liquids Through "Collapsible" Tubes

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