A Figure of Merit For Catheter Sampling Systems

By H. SHERMAN, D.E.E., R. C. SCHLANT, M.D., W. L. KRAUS, M.D., AND C. B. MOORE, M.D.

The distortion of indicator dilution curves by catheter sampling systems is predictable from theoretic considerations based on laminar flow. Over a wide range of catheter and flow parameters, deductions can be made regarding distortion, by the arterial system or by sampling catheters, of indicator dilution curves coming from an idealized pump. A figure of merit for the sampling system is given by the volume of the catheter divided by twice the flow rate. This number should be less than the time in which significant changes in indicator concentration occur, i.e., the interval between pump strokes, if significant distortion of the indicator dilution curve is to be avoided.

INDICATOR DILUTION methods, using dyes, radioactive materials, etc., are widely used for measurement of cardiac output and blood volume between injection and sampling sites (cardiac output \times mean circulation time). For example, the indicator may be injected into the left atrium and a time-concentration curve obtained by sampling by one method or another from a needle and short length of catheter in the brachial or other large artery. The accuracy of the determination of cardiac output is not dependent on the shape of the time-concentration curve, but the quantitation of other important circulatory parameters is dependent on an accurate recording of downstream indicator concentration at each systole.

It has long been recognized that the catheter properties distort the shape of the indicator concentration curve (Lacy et al.,\textsuperscript{1} Newman et al.,\textsuperscript{2} Sheppard\textsuperscript{3}), but the degree of distortion has not been quantitatively stated in the past except in the important special case of a single step function by Peterson et al.,\textsuperscript{4} and Rossi et al.,\textsuperscript{5} who were interested in total flow measurement.

It is the purpose of this paper to identify a source of flow distortion in the measurement of indicator concentration curves, and to give a figure of merit for catheter sampling systems in applications requiring detailed information on the shape of the time-concentration curve, such as slope.

The contents of this paper will be given in the following order: 1. The time response of a catheter (having a parabolic velocity gradient) to an impulse of indicator will be derived to show that the catheter has "memory." 2. Data taken in experiments performed by Lacy et al.,\textsuperscript{1} will be used to verify the theoretically derived catheter impulse response. 3. The effect of catheter "memory" on an idealized indicator concentration curve will be derived and computed for a variety of parameters showing the deviation of the catheter output from the input concentration. 4. The error in...
LAMINAR FLOW OF A VISCOUS FLUID

Fig. 1. AA' are the rigid walls of a cylindrical catheter of radius R. Injunctate position at t = 0, time = 0 occurs at point AA. The motion of a thin cylinder of indicator at radius r having the velocity v(r) traverses the distance v(r)t. The lined area indicates the position of injectate at various times t = 0, t = t1, and t = t2 and the corresponding time-concentration curve is shown below as it would appear at the output of the catheter.

measuring the slope of the concentration curve after catheter distortion will be calculated for a variety of cases and shown to be most significantly affected by the volume of the catheter divided by the flow rate. Methods for improving such measurements will be discussed.

THE IMPULSE RESPONSE OF A CATHETER

The velocity of fluid flow at various radii in a tube varies from a maximum at the center to a minimum at the walls. At any point along the tube after t = 0, a cross section contains fluid of different age dependent upon the radius through which each streamline passed. This may be appreciated by examining figure 1 which shows the laminar flow of a thin sheet of indicator and the indicator distribution in cross sections of fluid. In Appendix I the mathematical derivation of the impulse response of the catheter is given for the general case of nonturbulent flow. This degenerates, as shown by Sheppard, into the following indicator concentration with time when the velocity gradient is parabolic with zero wall velocity:

\[ C(t) = \begin{cases} 0 & \text{where } t < \frac{r}{2Q} \\ \frac{r}{2Q} & \text{where } t > \frac{r}{2Q} \end{cases} \]  \tag{1}

where

- \( C(t) \) = indicator concentration (or density) as a function of time
- \( I \) = mass of injectate
- \( V \) = volume of the catheter
- \( t \) = time
- \( Q \) = volumetric flow through the catheter per unit time

The hypothesis of a parabolic gradient may be questioned on several grounds: 1. The parabolic gradient is not achieved in laminar flow for a number of radii beyond the entrance of the catheter. 2. The parabolic gradient may never be achieved in blood flow.

Coulter and Pappenheimer have shown the ranges over which the flow is non-turbulent. The experimental work of Lacy et al., which will be used to confirm the theory here proposed, was done with Reynolds's numbers which are \( \frac{3}{4} \) or less of the critical Reynolds's number found by Coulter and Pappenheimer. These latter authors go further, and suggest that the axial flow is parabolic with a plasma sheath of different parabolic velocity gradient from the core. The model for such two phase flow becomes sufficiently complex as to resist theoretic treatment.

The deviation from parabolic gradient, if it exists in a significant sense, should be detectable by appropriate experiments comparing the theory with reality. The work of Lacy et al., was analyzed and gave experimental support to the theory derived herein. To use data of reference, the impulse response of the catheter was extended (Appendix II) to show the response of a catheter to square steps of indicator used by Lacy. Theoretically, the square step response of the catheter should have, as a percentage of the final re-
Figure 2. The ordinate shows the percentage of final response which is reached at the time indicated in seconds along the abscissa after flow through a catheter of dimension and rate indicated on the graph. The solid curves are experimental data taken by Lacy et al. The dotted curves are theoretical predictions from equation 6.

\[ \frac{C(t)}{C_f} = \frac{T}{T + \frac{1}{QV}} \]

where

- \( C(t) \) = measured concentration as a function of time at the output of a catheter of volume \( V \)
- \( C_f \) = final concentration at the catheter output
- \( V \) = volume of the catheter
- \( Q \) = volumetric flow per unit time
- \( T \) = time measured after the time of initial appearance

Figure 2 shows samples of the comparison between the theoretical predictions of Appendix II and the experimental data of Lacy.
These authors measured the step function response of catheters with varying flow rates and volumes. The original experimental data used in figures 2 and 3 were furnished through the courtesy of Dr. Elliot V. Newman. The comparison of measured and predicted results shows acceptable agreement. The experimental data of their figure 2 (bottom) do show a wide discrepancy in response at low flow rates (0.28 cc./sec. in 1.19 mm. diameter tubing) between blood having packed cell volumes of 25 per cent and 40 per cent which is not predicted by parabolic gradient theory with zero velocity at the walls. At such low rates Coulter and Pappenheimer have pointed out that the Reynolds number will be in the region of rapidly changing and increasing viscosity due to the disorientation of the red blood cells.
Fig. 4. Slope errors in catheter sampling systems. The percentage error of indicated slope is plotted on the ordinate as a function of pump strokes at which the slope is determined, plotted along the abscissa. For $K = 0.25$, the variation of percentage error with catheter parameters (volume $V$ and catheter flow rate $Q$) and with stroke rate is plotted in $A$. Similar curves for $K = 0.5$ and $K = 0.75$ are shown in $B$ and $C$. A composite of curves for $K = 0.25$, 0.5, and 0.75 for a catheter having $f_p V/2Q$ of 1 is shown in $D$.

For a single chamber pump it has been shown by Holt$^2$ that the time concentration of the output is represented by

$$C_r = K^n \text{ for } \frac{n-1}{f_r} < t < \frac{n}{f_r}$$

where

- $C_r$: pump output concentration
- $K$: ratio of pump residual volume to total chamber volume
- $n$: number of pump strokes
- $f_r$: stroke rate

Holt$^2$ has published concentration curves recorded by conductivity measurements in the ascending aorta which approximated such exponential fall-off, and derived equations relating certain cardiac parameters to $K$.

What distortion will a catheter cause to such an idealization as that given in equation 3? Subject only to the law of superposition, which hypothesizes that the effect of a sum of concentrations is equal to the sum of the effects of each concentration alone, the output of a catheter of known impulse response to an arbitrary input can be calculated. Such a computation is made in Appendix III for an input of the form of equation 3. The result of introducing$^*$ a variety of parameters into these equations is shown in figure 3A, B, and C.

The significant general trend demonstrated in figure 3A, B, and C is the increasing loss

$^*$We are indebted to Miss Billy Houghton of the Lincoln Laboratory for the computations in figure 3, and to Mrs. Marion Andrews of the Lincoln Laboratory for the computations summarized in figure 4.
of detail with increased $f_p V/2Q$. This is understandable, since the increased "memory" of the catheter for past events makes it less responsive to recent concentration changes in pump output. Figure 3D was included to show the effect of the catheter on pump output concentration curves representative of three conditions, for pumps having identical chamber volumes; the highest curve has one third of the stroke volume of the lowest curve. It is apparent that the variations in downslope of these three concentration curves do not visually reflect the differences in stroke volume.

While the curves in figure 3 give some qualitative indication of the deviations caused by catheters, a quantitative measure of the distortion may be determined by comparing the actual slope with that indicated by the catheter output. The significant measurement which should be derived from the indicator concentration curve is the normalized slope

$$\frac{C_o \left( \frac{n}{f_o} \right) - C_o \left( \frac{n-1}{f_o} \right)}{C_o \left( \frac{n-1}{f_o} \right)}$$

In Appendix IV the per unit error ($\epsilon$) in normalized slope is derived as a function of the catheter parameters, stroke rate, and actual slope. The per unit error $\epsilon$ in slope is defined as

$$\epsilon = \frac{\text{actual value} - \text{measured value}}{\text{actual value}}$$

The computations for a variety of parameters are shown in figure 4. The following generalizations may be made concerning figure 4A, B and C: 1. The slope error as measured at the catheter output decreases with decreasing $f_p V/2Q$ in a predictable but nonlinear manner. 2. After the sixth stroke, less slope error is to be expected for higher $K$ (less efficient pumps) within the range of values shown. 3. The error in slope as measured after various numbers of strokes may be comparable to the actual differences in the value of $K$.

**Discussion**

In normal clinical usage the catheter sample is taken at some distance from the heart. Thus the actual flow distortion is the result of the concatenated flow in the artery and the catheter. If the artery has laminar flow, then the results of this paper are also applicable to the flow distortion in the artery. Calculations on the concatenation of arterial and catheter flow are complicated by the flow characteristics at the interface of the catheter and the artery. Even if the catheter had ideal flow characteristics, the ventricular output concentration curve would be distorted by arterial flow.

It is possible to take the concentration curves measured at the catheter output, the characteristics of the catheter, and to insert these in the final equations (11, 12, and 13 of Appendix III) to solve for $K$. That is, knowing the characteristics of the catheter, and the output, one can solve for the input. In the general case this is a tedious and difficult calculation. If the special case of a stepped input is valid, the computation is simple. The result will be in error by variations from the parabolic velocity gradient in the catheter and by deviations from the assumed function at the input of the catheter due to arterial flow. One can avoid the distortion due to arterial flow by inserting the catheter to the ascending aorta at the expense of increasing the catheter volume $V$.

What figure of merit can be assigned to various catheter systems? A precise figure of merit can be assigned only if a value judgment is made regarding the acceptable error in a given set of circumstances. As a rough approximation, the impulse response for a parabolic velocity gradient with zero wall velocity falls to one fourth of its initial value $\left( \frac{2t}{V} \right)$ after a time equal to $V/Q$. This "memory" time should be significantly less than the time in which significant changes can occur in the input function—otherwise changes will
be "slurred" by catheter memory of past flow. In the case of slope measurements, this response time of the catheter should be significantly less than the intersystolic interval.

What can be done to improve the fidelity of sampling systems? In order to avoid arterial distortion, the catheter input should be placed as close to the source of change (ascending aorta) as possible; however, this would have the detrimental effect of increasing the volume of the catheter. The internal volume of the catheter should be minimized by decreasing length (which conflicts with the need of avoiding arterial distortion) and by decreasing radius. The flow rate should be maximized, but an upper limit is set by the source and by the other demands, than sampling, for the blood.

A deviation from parabolic velocity gradient with zero wall velocity is indicated for high hematocrit and low flow rates. If this deviation increases the wall velocity substantially, one may improve response by tagging the red blood cells with indicator and using low flow rates. Further confirmation of velocity gradients at low flow rates is needed before this possibility can be exploited.

The most hopeful approach for higher fidelity samplers would abandon systems which require the transport of fluid for sampling and would use rapid response indicators at the catheter tip. Holt has used a device of this type which measures conductivity at the tip of the catheter in the ascending aorta.

SUMMARY

A study has been made of the distortion of indicator dilution time-concentration curves by catheter sampling systems. The time response of a catheter having laminar flow with a parabolic velocity gradient to an impulse of indicator indicates that the catheter has "memory."

For an input of a thin sheet of indicator, the catheter output concentration at a given time (greater than $V/2Q$) is proportional to the mass of indicator ($I$) at the input of the catheter and the volume of the catheter ($V$), but inversely proportional to twice the product of the square of the volumetric flow ($Q$) through the catheter per unit of time and the square of the time ($t$) after injection.

There was acceptable agreement between the experimental data of Lacy et al., who studied the effect of different sizes and lengths of catheter on the shape of indicator dilution curves, and between curves derived on a theoretic basis.

The relations of various pump parameters (stroke volume, residual volume, stroke rate) and catheter parameters (volumetric flow through the catheter per unit of time, catheter volume) on the shape of the indicator concentration curve have been defined for an idealized input. Mathematical corrections of catheter distortion are unsuitable for routine clinical application. In addition, it would be difficult to correct for similar distortions in the peripheral arterial system itself. The most hopeful approach to high fidelity recording would be the use of rapid response recorders on the tip of a needle or catheter located in the ascending aorta.

SUMMARIO IN INTERLINGUA

Esseva facite un studio del distorsion efectuata in le curvas de tempore e concentration in tests a dilution de indicator per varie systemas de obtenere specimen per cathesterisation. Le responsa de tempore evocate per un impulso de indicator in le caso de cathesteres a fluxo laminar con gradiente parabolic de velocitate indica que le catheter ha un "memoria."

Post le introduction de un tenue lamina de indicator, le concentration del rendimento del catheter a un specific tempore (de plus que $V/2Q$) es proportional al massa del indicator ($I$) al introduction del catheter e al volumine del catheter ($V$); illo es inveremento proportional a duo vices le producto del quadrato del fluxo volumetric ($Q$) a transverso le cathester in le unitate de tempore con le quadrato del tempore ($t$) post le injection.

Esseva trovate un grado acceptabile de acordo inter le datos experimental de Lacy et al. (qui studiava le efecto exercite per cathesteres de diferente dimensiones e longores su-
per le conformazione del curvas de dilution de indicator e le curvas establece super un base theorie.

Le effeeto exercite per varie parametres de pumpage (volume per pulso, volume residue, frequentia de pulsos) e per varie parametres de catheterismo (flusto volumetric a transverso le catheter in le unitate de tempore, volume del catheter) super le conformazione del curva del dilution de indicator ha essite definite pro idealisate conditiones de introduction. Correctiones mathematic del distorsion catheteral non es appropriate al routine del practica clinic. In plus, il esserea difficile corriger simile distorsiones in le systema del arterias peripheric mesme.

Le plus promittente methodo de registration a alte grados de fidelitate pare requirer le uso de registratores a responsa rapide attachate al puncta del agulia o catheter locate in le aorta ascendente.

REFERENCES

SHERMAN, SCHLANT, KRAUS, MOORE

APPENDIX I

Impulse Response of a Catheter to Laminar Flow
A mass of indicator I is initially placed in an infinitely thin sheet across the catheter at time t = 0 at a point l = 0 (fig. 1). Since we have postulated a finite mass and an infinitely thin sheet, then the density of this sheet must be infinite. The mathematical impulse function δ(x) is used to describe a density distribution which has the following properties:

\[ \int_{-\infty}^{\infty} \delta(x) \, dx = 1 \]

At any location the argument of the impulse function is

\[ x = t - \frac{l}{v(r)} \]

where

\[ v(r) = \text{velocity of the fluid at radius } r \]

\[ l \]

\[ \text{length of time it takes the impulsive sheet to reach the point } l \text{ along the streamline whose velocity is } v(r). \]

At t = 0, the indicator has been hypothesized to be uniformly (i.e., independent of r) distributed across the catheter. The amount of indicator per unit area is therefore the total mass of indicator divided by the area, I/πR².

The amount of injectate \( i(t) \, dt \) passing a location \( l \) in the time between \( t \) and \( (t + dt) \) is

\[ i(t) \, dt = \frac{I}{\pi R^2} \times 2\pi R \, dr \quad \text{when } t = \frac{l}{v(r)} \quad (\text{i.e., } z = 0) \]

For this relationship between \( t, l \) and \( v(r) \)

\[ v(r) = \frac{l}{t} \]

\[ \frac{\partial v}{\partial r} \times \frac{dr}{dt} = -\frac{l}{v^2} \]

\[ dr = \frac{l \, dt}{v^2(r)} \]

were \( v'(r) = \frac{\partial v}{\partial r} \)

Substituting this into the expression for \( i(t) \)

\[ i(t) \, dt = \frac{I}{\pi R^2} \times \frac{2\pi R}{v^2(r)} \times \frac{l}{v(r)} \, dt \]

The expression \( \frac{r}{v'(r)} \) is evaluated for that radius \( r \) which satisfies the relationship \( t = \frac{l}{v(r)} \).
The concentration or density of injectate at point \( l \) and at time \( t \) is the mass of injectate flowing per unit time divided by the total volume of fluid \( Q \) passing the same point per unit time:

\[
\frac{i(l)}{Q} = C(t) = \frac{-2ll}{Q R^2 \alpha} \frac{r}{\nu(r)} \frac{z}{x} = 0
\]

Apply this general result to a truncated parabolic velocity gradient of the form:

\[
v(r) = v_m \left( \frac{R^2 - r^2}{R^2} \right) + u_w
\]

where:

- \( v_m \) is the maximum velocity which occurs at \( r = 0 \)
- \( u_w \) is the wall velocity
- \( \nu(r) = \frac{-2v_m r}{R^2} \)

The flow \( Q \) associated with any velocity gradient \( v(r) \) is:

\[
Q = \int_0^R 2\pi r v(r) \, dr
\]

Substituting the truncated parabolic velocity gradient and integrating:

\[
Q = A_c \left( \frac{v_m + u_w}{2} \right)
\]

where \( A_c \) is the area of the catheter.

Solving for \( v_m \) and substituting, and indicating the physical epoch over which the concentration exists:

\[
C(t) = \frac{IV}{2Q(A_c + u_w)}; \quad \frac{l}{v_m} > \frac{l}{v_m} > \frac{l}{v_m}
\]

where

\[
V = A_c L = \text{volume of catheter from the point of injection} (l = 0) \text{ to the point of examination} l
\]

Where the velocity at the catheter wall goes to zero, the result reduces to the familiar form:

\[
C(t) = \frac{IV}{2Q'} \quad \frac{l}{v_m} > \frac{l}{v_m} = \frac{V}{2Q'}
\]

\[
= 0 \quad \frac{l}{v_m} < \frac{l}{v_m} = \frac{V}{2Q'}
\]

This result is used throughout this paper. In order to obtain some appreciation of the error accruing from ignoring the wall velocity, Peterson et al. have noted that the maximum velocity for their catheters was 1.6 times the mean velocity, which changes the factor of 2 in the denominator of \( C(t) \) to 1.6. The flow rates mentioned in Peterson's work were low (1/6 cc. per second) and may have made the wall velocity appreciable, due to the increased viscosity at low Reynold's numbers as measured by Coulter and Pappenheimer.

The formulas given permit the study of gradients other than parabolic. A Taylor series approximation to the velocity gradient can be introduced. The parametric quantity which describes the concentration curve is proportional to \( kV/Q' \) where \( Q' \) is the flow corrected for wall velocity and \( k \) is a proportionality constant. The more blunt the nose of the velocity gradient is (i.e., the higher order the Taylor series), the shorter is the response time of the catheter (i.e., \( k \) decreases).

### APPENDIX II

**Step Function Response of a Catheter**

Where the indicator occurs in other form than as an infinitely thin sheet the output of the catheter can be calculated from the impulse response. The output of the catheter \( C(t) \) at time \( t \) is the sum of the impulsive contributions from each prior instant of time weighted by the impulse response or "memory" of the catheter \( C(r) \). Specifically, the following relations hold for the calculation of catheter response to an infinitely long "slug" of indicator. For parabolic velocity gradients with zero wall velocity:

\[
C(r) = 0 \quad \text{when} \quad r < \frac{V}{2Q}
\]

\[
C(r) = \frac{IV}{2Q'} \quad \text{when} \quad r > \frac{V}{2Q}
\]

\[
C(t) = \int_{r/Q}^t C(r) \, dr = \frac{1(t - V/2Q)}{Q'}
\]

Substituting \( T = t - V/2Q \) to correct to the instant of initial appearance:

\[
C(t) = \left( \frac{T}{T + V/2Q} \right) \left( \frac{Q}{Q'} \right)
\]

As \( T \) approaches infinity \( C(t) \) becomes the final value \( C_f \)

\[
C_f = \frac{I}{Q'}
\]

The percent of final response is therefore:

\[
\frac{C(t)}{C_f} = \frac{T}{T + V/2Q}
\]

Where the wall velocity, \( u_w \), is other than zero, and the cross-sectional area of the catheter is \( A_c \),
with total volume \( V \), the step function response can be derived as above, with the appropriate \( C(r) \) to give

\[
\frac{C_{o}(r)}{C_{f}} = \frac{T}{T + \frac{V}{2(Q - A_{w})}}
\]

(7)

The main body of the paper is concerned with zero wall velocities and uses equation 6.

**APPENDIX III**

*Catheter Distortion of Concentration Curves of Form \( K^n \)*

The catheter output to an impulsive input is

\[
C_{c}(r) = \begin{cases} 
\frac{IV}{2Q^2} & t > \frac{V}{2Q} \\
0 & t < \frac{V}{2Q}
\end{cases}
\]

(8)

when the velocity gradient is parabolic with zero wall velocity.

Consider now a pulsatile input function of the form

\[
C_{*} = K^n; \quad \frac{n - 1}{f_{p}} < t < \frac{n}{f_{p}}
\]

(9)

where

- \( n = 1, 2, \ldots \)
- \( n \) = number of pump strokes after injection
- \( f_{p} \) = stroke rate

Consider the interval between the \( n \)th and \((n + 1)\)st strokes. The output at time \( t \) is the sum of the weighted impulsive contributions from all prior inputs. The summation is done by an integral, starting with a delay of \( V/2Q \), due to the finite time to traverse the catheter, taken with a dummy variable \( r \) which is allowed to range from \( V/2Q \) to the total time \( t \) measured from the injection time. This summation follows.

\[
C_{d}(t) = \int_{V/2Q}^{t-\frac{n}{f_{p}}} \frac{K^{n+1}V}{2Q^2} dr + \int_{t-\frac{n - 1}{f_{p}}}^{t-\frac{n - 1}{f_{p}}} \frac{K^{n}V}{2Q^2} dr + \cdots
\]

(10)

This expression holds only if

\[
0 < \left( t - \frac{n}{f_{p}} \right) \leq \frac{V}{2Q}
\]

which states that the catheter traversal time is less than the time since the last stroke.

If the integration is carried out it leads to the following equation.

\[
C_{d}(t) = \frac{K^{n+1}V}{2Q^2} \left[ \frac{f_{p}t - n}{2Q} - \frac{V}{2Q} \right] + \sum_{i=0}^{n-1} \frac{K^{n-i}V}{2Q^2} \left( f_{p}t - \frac{n - i}{f_{p}} \right)
\]

(11)

\[
0 < t - \frac{n}{f_{p}} \leq \frac{V}{2Q}
\]

Equations are given below for cases in which the traversal time \( V/2Q \) of the catheter exceeds the interval between strokes.

\[
C_{d}(t) = \frac{K^{n+1}V}{2Q^2} \left[ f_{p}t - \frac{n - 1}{f_{p}} - \frac{V}{2Q} \right] + \sum_{i=0}^{n-1} \frac{K^{n-i}V}{2Q^2} \left( f_{p}t - \frac{n - i}{f_{p}} \right)
\]

(12)

\[
0 < t - \frac{n}{f_{p}} \leq \frac{V}{2Q}
\]

\[
C_{d}(t) = \frac{K^{n+1}V}{2Q^2} \left[ f_{p}t - \frac{n - 2}{f_{p}} - \frac{V}{2Q} \right] + \sum_{i=0}^{n-1} \frac{K^{n-i}V}{2Q^2} \left( f_{p}t - \frac{n - i}{f_{p}} \right)
\]

(13)

\[
0 < t - \frac{n}{f_{p}} \leq \frac{V}{2Q}
\]

where

- \( f_{p} \) = stroke rate per unit time
- \( n \) = largest integer \( < f_{p} \)
- \( K \) = ratio of pump residual volume to total chamber volume
- \( 1 - K \) = ratio of stroke volume to chamber volume
- \( V \) = volume of the catheter
- \( Q \) = volumetric flow per unit time
- \( I \) = mass of injectate
APPENDIX IV

Error in Normalized Slope of Indicator Concentration Curves due to Catheters

The quantity of special interest is the normalized slope of the concentration curve. In the ideal case in which

$$C(t) = C \left( \frac{n}{f_p} \right) = K^* \text{ for } \frac{n-1}{f_p} < t < \frac{n}{f_p} \quad (14)$$

The normalized slope is here defined as

$$K = \frac{C_n}{C_{n-1}} \quad (15)$$

In Appendix III it was shown that the catheter output

$$C_o(0) = \frac{K^* V}{2Q^2} \left[ \frac{f_p t - n - \frac{f_p V}{2Q}}{2Q (f_p t - n)} \right]^n + \frac{1}{2Q^2} \left( \frac{n-2}{f_p} \right)^n K^{n-i}$$

$$0 < f_p t - n \geq \frac{V}{2Q}$$

The error in normalized slope is derived for that time during the cycle which is $V/2Q$ units of time (the traversal time of the catheter) after the stroke. At this time

$$t = \frac{n}{f_p} = \frac{V}{2Q}$$

and the indicator from the last stroke is just short of traversing the catheter. At this time

$$C_o \left( \frac{n}{f_p} + \frac{V}{2Q} \right) = \frac{-1}{\sum_{j=0}^{n-2} \frac{K^{n-j} V}{2Q} \left( \frac{f_p}{2Q} \right) \left( \frac{K^{n-j} V}{2Q} \right)} \quad (16)$$

Similarly, one stroke earlier

$$C_o \left( \frac{n-1}{f_p} + \frac{V}{2Q} \right) = \frac{-1}{\sum_{j=0}^{n-2} \frac{K^{n-j} V}{2Q} \left( \frac{f_p}{2Q} \right) \left( \frac{K^{n-j} V}{2Q} \right)} \quad (17)$$

The normalized slope can be computed from these two measurements.

$$\frac{C_o(n)}{C_o(n-1)} - \frac{C_o(n-1)}{C_o(n-2)}$$

The per unit error $\epsilon$ is

$$\epsilon = \frac{\text{actual value} - \text{measured value}}{\text{actual value}}$$

$$(K-1) - \left[ \frac{C_o(n)}{C_o(n-1)} - 1 \right] = \frac{\sum_{j=0}^{n-1} \frac{K^{n-j} V}{2Q} \left( \frac{f_p}{2Q} \right) \left( \frac{K^{n-j} V}{2Q} \right)}{(K-1)}$$

$$K = \frac{\sum_{j=0}^{n-1} \frac{K^{n-j} V}{2Q} \left( \frac{f_p}{2Q} \right) \left( \frac{K^{n-j} V}{2Q} \right)}{(K-1)}$$

$$\left( K - 1 \right) - \left[ \frac{C_o(n)}{C_o(n-1)} - 1 \right] = \frac{\sum_{j=0}^{n-1} \frac{K^{n-j} V}{2Q} \left( \frac{f_p}{2Q} \right) \left( \frac{K^{n-j} V}{2Q} \right)}{(K-1)}$$

$$\epsilon = \frac{1}{(K-1)(\beta + n)(\beta + n - 1)} \sum_{j=0}^{n-2} \frac{K^{n-j}(\beta + j + 1)(\beta + j)}{1}$$

$$\beta = \frac{f_p V}{2Q}$$

$$V < \frac{1}{f_p}$$
A Figure of Merit For Catheter Sampling Systems
H. SHERMAN, R. C. SCHLANT, W. L. KRAUS and C. B. MOORE

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