Vibrational Analysis of Bioprosthetic Heart Valve Leaflets Using Numerical Models: Effects of Leaflet Stiffening, Calcification, and Perforation

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The fundamental natural frequency of the closed cusps of porcine bioprosthetic valves, fabricated from the normal leaflets of pig aortic valves, was estimated using a finite element model. Both normal and stiffened leaflets were considered in the vibrational analysis. The effects of conditions that simulated degeneration, such as stiffening, central perforation, a tear, calcium deposits in the commissural attachments, and combinations of these were determined. The primary frequency of vibration of the normal leaflets was within the range of the dominant frequency of the heart sounds determined clinically by spectral analysis of the recorded phonocardiogram. If only one leaflet was stiffened or calcified, there was only a marginal change of frequency. With stiffening and calcification of the commissures of all 3 leaflets, the frequency of vibration increased. Introduction of a tear in a single leaflet of a stiffened and calcified valve markedly reduced the fundamental frequency. In view of the relation between the frequency content of heart sounds and the frequency of valve vibration, this mathematical simulation establishes a possible basis for the observation of a varying dominant frequency of heart sounds in patients with bioprosthetic valves that are in the process of degenerating. (Circulation Research 1987;61:687–694)

Degeneration of porcine bioprosthetic valves (PBV) several years after insertion is a clinical problem, and any noninvasive technique to diagnose degeneration before it becomes clinically obvious would be useful. One such technique is based on the dominant frequency of the heart sounds produced by prosthetic valves. It has been demonstrated that the dominant frequency of bioprosthetic valve sounds increases as the duration of insertion increases, and this change was attributed to stiffening of the leaflets. In vitro tests have shown that the leaflets stiffen when subjected to millions of cyclic stresses. An increased frequency content of the heart sounds produced by degenerated porcine bioprosthetic valves also has been shown to occur if the valves are stiffened or calcified. Whether a tear or perforation of the leaflets would modify the frequency content of the sounds has not been determined.

This problem of whether tears or perforations of the valve modify the frequency content of the heart sounds can be approached by an analysis of valve vibration based on the evidence that the heart sounds are produced by vibration of the valvular cusps immediately after closure and that the frequency of sound relates to the frequency of vibration. The effect on the frequency of valve vibration of a tear or perforation of one of the leaflets, in combination with a stiffened and calcified valve, can be assessed by mathematical modeling of vibration of the valve leaflets.

The purpose of this study was to estimate the fundamental vibrational frequency that occurs following closure of a porcine bioprosthetic valve when one or more leaflets are calcified, stiffened, and torn. The finite element technique was used. The characteristics of valve vibration will lead to further understanding of the factors that affect the frequency content of heart sounds when the valves are abnormally stiffened, calcified, or torn. Hopefully, this information will lead to a more knowledgeable application of the frequency analysis of heart sounds for the detection of bioprosthetic valve degeneration.

Materials and Methods

Assumptions

Several assumptions were made in the development of the numerical model to simplify the solution or because of lack of reliable experimental data. The following assumptions were made in our model: 1) the 3 PBV leaflets were assumed to be equal in size, symmetric, and uniformly thick (0.6 mm); 2) the stent posts were assumed to be rigid; 3) valve annulus deformation was neglected; 4) the leaflet tissue was considered to be isotropic; 5) a Poisson's ratio of 0.499 was used for the tissue to achieve a nearly leaflet incompressibility condition; 6) an “effective” leaflet mass density was used to account for the fluid coupling (blood coupling) during vibration; and 7) the damping was assumed to be very small (<<1).
The geometry of the closed PBV leaflets is complex even in the relaxed state, i.e., under zero pressure difference across the cusps. Based on our observations of various types of PBV, the geometry of a single leaflet approximated that of one half of an elliptic paraboloid. Details of geometric modeling of the closed leaflet have been reported previously. The annular diameter of the valve used in the model was 27 mm, the height of the stent posts 19.0 mm, and the surface area of a single leaflet 5.77 cm².

The geometry of the PBV, viewed from the side of the valve, is shown in Figure 1. The leaflets are attached to the support structure ABC. The assumption was made that the cloth and sutures, in addition to the stent itself, were rigid. Although this is not exactly true, the deformation of these components can reasonably be expected to be small, and this small deformation is neglected in the determination of vibrational frequencies. As the pressure gradient increases, the stresses in the leaflet increase because of an increase in pressure, and hence, the mechanical property (Young's modulus) of the tissue changes. In addition to this change of mechanical properties (Young's modulus), the coaptation surface area also increases. In this analysis, it was assumed that the coaptation surface area and the tissue Young’s modulus were constant during vibration.

Analysis

The finite element technique involves discretizing the structure into small portions called “elements.” These elements are assumed to be interconnected at discrete points called “nodes.” These nodes are located along the boundary of each element. The governing mathematical relations for each element are established in terms of nodal variable parameters (displacements) by invoking the principles of structural mechanics. This technique permits variation of mechanical properties of each element and allows an incorporation of complex geometries in the analysis. Assembly of each element-governing equation represents the overall mathematical model of the complete structure. After appropriate “boundary” conditions are applied, the overall governing equations are solved.

The finite element discretization of all the three leaflets is shown in Figure 2. Triangular thin shell elements are used in the analysis. This discretization resulted in 420 nodal points that connected 828 elements. At each nodal point, three translations and three rotations accounted for the nodal displacements (degrees of freedom).

With the damping effect neglected, the governing equations for free vibration of all 3 leaflets together are expressed in matrix form as follows:

$$[K] \{\delta\} - \omega^2 [M] \{\delta\} = \{0\} \quad (1)$$

\[ \text{FIGURE 1. Three-dimensional view of a Hancock porcine bioprosthetic valve. A, B, and C show the stent post (h, stent height).} \]
FIGURE 2. Top: Finite element discretization of the three leaflets. Triangular shell elements are used. A, B, and C are the commissures. There are 828 elements connected by 420 nodal points. Bottom: Top view of the discretized leaflets. A, B, and C are commissures.

where [K] is the stiffness matrix and [M] is the effective mass matrix of the leaflets. For the assumed closed leaflet geometry, the stiffness matrix is the function of Young's modulus of the leaflet, and the mass matrix is the function of mass density of the leaflet surrounded by blood. Since the “exact” mass density of the vibrating membrane supporting the column of blood is not yet established, an approximate “effective” mass density of 10.4 g/cm³, which is 10 times that of tissue, was used in the analysis. Equation 1 was solved to obtain the frequency \( \omega \) (eigen values) and the nodal displacements \( \delta \), the eigen vectors.\(^{15}\)

The fundamental vibrational frequencies of the leaflets were obtained for normal leaflets, simulated stiffened leaflets, leaflets with simulated focal calcium, and leaflets with a simulated central perforation and tear.

For normal leaflets, the Young’s modulus was assumed to be 0.3 MPa (mega Pascal), and this modulus value corresponds to the modulus at low stress values. For the stiffened valve, the leaflet modulus was assumed to be 2.1 MPa. The Young’s modulus for the calcified portion of the leaflet was assumed to be 21.0 MPa. The density and the Poisson’s ratio used for these various conditions of the leaflet are given in Table 1.

### Results

The results are summarized diagrammatically in Figure 3.

**Normal leaflets.** The fundamental vibrational frequency was calculated at 55 Hz for the normal valve. The modal shape of a single leaflet vibration is shown in Figure 4 because of symmetric conditions.

**Normal valve with perforation of one leaflet.** A central perforation of 1.17 cm² area (20% of the single leaflet area) was simulated (Figure 5). The fundamental frequency was calculated as 52 Hz.

<table>
<thead>
<tr>
<th>No.</th>
<th>Material</th>
<th>Young’s modulus (MPa)</th>
<th>Poisson’s ratio</th>
<th>Density (kN/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Normal leaflet</td>
<td>0.30</td>
<td>0.499</td>
<td>10.4</td>
</tr>
<tr>
<td>2</td>
<td>Stiff leaflet</td>
<td>2.10</td>
<td>0.499</td>
<td>10.4</td>
</tr>
<tr>
<td>3</td>
<td>Calcium</td>
<td>21.0</td>
<td>0.350</td>
<td>27.0</td>
</tr>
</tbody>
</table>
NORMAL VALVE WITH TEAR OF ONE LEAFLET. A tear that involved 35% of the area of a single leaflet was introduced in one leaflet (Figure 6). The frequency of vibration diminished somewhat to 39 Hz.

ONE LEAFLET STIFFENED AND CALCIFIED (TWO LEAFLETS NORMAL). Focal calcium at the commissural attachments of one of the leaflets (17% of a single leaflet area) and stiffening of that leaflet had little effect on the frequency of vibration of the entire valve (Figure 7). The estimated fundamental frequency of the valve for these conditions was 62 Hz.

STIFFENED VALVE (ALL THREE LEAFLETS). The fundamental vibrational frequency of the stiffened valve was calculated as 145 Hz.

STIFFENED VALVE WITH CALCIFICATION (ALL THREE LEAFLETS). All three leaflets were caused to be stiff in the model, and focal calcifications were introduced at the commissural attachments of all the three leaflets (Figure 9). In this configuration, the stiffened and calcified valve showed a frequency of 176 Hz.

STIFFENED AND CALCIFIED VALVE (ALL THREE LEAFLETS) WITH TEAR IN ONE LEAFLET. All three leaflets were modeled as stiffened and calcified at the commissural attachments. In addition, a tear that included 35% of a single leaflet...
area was introduced in one leaflet (Figure 10). The frequency of vibration was 123 Hz.

**Discussion**

Numerical simulations of anatomic structures suffer in accuracy as a result of inherent assumptions necessary to obtain a solution. Some of the assumptions made in this study are readily justified, while others represent approximations. The three leaflets of a PBV are usually of different size and seldom symmetric. Lack of symmetry, however, does not affect the static response of the loaded valves, and this fact can also be extended to the dynamic analysis of the leaflets. The thickness of the leaflets is not uniform, and variability among specimens is possible. Human aortic valve leaflets are not uniformly thick and vary between the relaxed state and pressure-stressed state. Since topographic variation of leaflet thickness of a PBV has not yet been determined, a constant thickness of the leaflets was assumed in our model.

The leaflet material is incompressible, i.e., the volume of the leaflet does not change during deformation. This incompressibility condition was achieved by approximating the Poisson's ratio (ratio of lateral strain to axial strain) as 0.499 in our model.

The leaflet material of a PBV is invariably anisotropic, and the stress-strain properties of PBV leaflets differ in the circumferential and radial directions. Nevertheless, the assumption of isotropy was made to simplify the problem. Also, even though PBV valves have flexible stents, a rigid stent was assumed in this model.

The effective density was assumed to be 10 times that of the leaflet mass density in our model. This approximation reduces the frequency values by a factor of 3.16 times that of the value in vacuum. This estimate of the effective mass density was based on the effects of a liquid in a fluid-filled structure on the vibrational frequency of the empty structure. The natural frequency of vibration of a fluid-filled cylindrical shell is lower than the frequency of the cylinder alone. Also, the natural frequency of vibration of a fluid-filled tank is lower than the empty one. These effects are due to the added mass of the fluid and solid-fluid coupling. The frequency of fluid-filled structures may be reduced to 33 to 50% of the frequency in a vacuum. On the basis of these observations, an effective mass density of 10 times the leaflet mass density was assumed, thereby decreasing the frequency to 32% of the frequency that would occur in a vacuum.

The Young's modulus of the tissue was chosen as 300 kPa at the moment of closure. This value, which was based on test data of others, depends on factors such as the pressure during mounting of the valve leaflets and chemical treatment of the tissue before mounting. The stress-strain curves of gluteraldehyde-preserved porcine leaflet specimens prior to and following accelerated fatigue cycling are given. The initial modulus of the cycled tissue at $648 \times 10^6$ cycles and $808 \times 10^6$ cycles may vary from 1,500 kPa to 3,000 kPa. In our model, a value of 2,100 kPa was arbitrarily chosen for Young's modulus for a stiffened leaflet. This value is approximately 25% of the highest Young's modulus that would occur before the normal tissue ruptures. The Young's modulus of the calcified tissue was assumed to be in the range of Young's modulus of limestone.

Sites of high mechanical stresses in the leaflet were correlated to the sites of calcification and perforation. The calcium sites were introduced in the
commissural attachments at maximally stressed sites. Any real structure dissipates energy as it deforms during vibration. This energy dissipation is called damping. It is difficult, if not impossible, to measure the damping of the vibrating valve because of the environment in which the leaflets vibrate. The damping mechanism may include fluid viscosity, turbulence, acoustic radiation, and material damping. To analyze the mechanical response, one has to consider all of these phenomena. However, in practical cases, one or two mechanisms are generally predominant. The major effects of damping are 1) reduction of vibration amplitude at resonance and 2) more rapid decay of free vibrations. In general, the damped free vibration frequency ($\omega_0$) of any structure can be expressed in terms of the damping ratio ($\zeta$) and the natural undamped frequency ($\omega_n$) as

$$\omega_0 = \omega_n \left(1 - \zeta^2\right)$$

If the damping ratio is as high as 0.2, the frequency reduces by only 2%. In our computation, the effect of damping was neglected.

The natural frequency of the normal valve that was computed in this analysis, 55 Hz, was comparable to the dominant frequency of the heart sounds produced by normal bioprosthetic valves (60 ± 17 Hz). An example of Fourier analysis of the first heart sound in a patient with a normally functioning porcine bioprosthetic valve inserted in the mitral position is shown in Figure 11. In view of the close relation between the dominant frequency of heart sounds and the frequency of vibration of the aortic valve, the computed value is believed to be correct.

The introduction of a central perforation in one of the leaflets had a negligible effect on the frequency, reducing it only from 55 to 52 Hz. Similarly, the introduction of tear that involved 35% of one of the normal leaflets caused a reduction of the frequency only from 59 to 39 Hz.

The introduction in the mathematical model of stiffening and calcification of a single leaflet had a negligible effect on the frequency of vibration since the frequency increased only from 55 to 62 Hz. If the single stiffened and calcified leaflet became torn, the computer modeling also indicated that there would be a negligible effect on total valve vibration (Figure 3). Thus, if the characteristics of only a single leaflet were modified, whether that leaflet was torn, perforated, stiffened, calcified, or calcified, stiffened, and torn, the frequency of the vibration of the valve as a whole changed little.

In contrast to calculations related to modifications of a single leaflet, if all three leaflets were stiffened, then the frequency of vibration increased markedly from 55 to 145 Hz. If all three leaflets were both stiffened and calcified at the commissures, the frequency was even higher at 176 Hz. An example of the frequency spectrum of the first heart sound in a patient with a degenerated porcine bioprosthetic valve in the mitral position is shown in Figure 12. The valve showed stiffening and calcification of all three leaflets, and two of the leaflets were torn at the commissures. The dominant frequency was abnormally elevated at 189 Hz.

In the mathematical model, if one of the stiffened and calcified leaflets was torn at the center of the cusp, then there was a prominent reduction of the frequency to 123 Hz, although the frequency did not fall to normal. This finding may explain occasional observations in patients in whom, over the course of years, the dominant frequency of heart sounds increases and then diminishes strikingly at the time that clinical evidence of degeneration is apparent. It may be that stiffening of the leaflet that is not recognized clinically is followed by a tear that lowers the sound frequency.

Numerical techniques have been applied to valve vibration to correlate the heart sounds with the vibration of the valve cusps. These models assumed the leaflets have the geometry of either a circular membrane or a one-dimensional idealized geometry. The models did not account for the actual geometry and degenerative conditions of the leaflets. A direct comparison of the results with these models is not possible because of the various assumptions and techniques involved in the analyses. However, the frequency of vibration obtained by our model is of the same order of magnitude as predicted by these models.

A number of assumptions were made in our calculations. Mathematical modeling of biologic structures is complex because of a wide variation in the geometry and because of unknown tissue properties. Even

![Figure 11. Frequency spectrum of the first heart sound in a patient with a normal porcine bioprosthetic valve in the mitral position.](http://circres.ahajournals.org/content/61/5/692.f11)
though some of these assumptions represent approximations, it is believed that these calculations are advanced over previous calculations in the literature and represent a significant step toward mathematical modeling of bioprosthetic heart valve vibration. Nevertheless, as with any mathematical model of biologic structures, the conclusions should be interpreted with caution. It should also be kept in mind that the numerical values of the estimated fundamental frequencies reflect the input data, including the valve geometry, material properties of the leaflet tissue, and the coupling of blood hemodynamics with the leaflets.

In conclusion, an estimation of the fundamental frequency of vibration of the closed cusps of a porcine bioprosthetic valve, using a numerical model with the finite element technique, indicates that 1) the dominant frequency of vibration of the valve changes little with tears, calcification, or stiffening of a single leaflet; 2) frequency of vibration of the valve changes little with the coupling of blood hemodynamics with the leaflets.

These observations may be applicable to the evaluation of the frequency content of heart sounds in patients with PBV.

References

**KEY WORDS** • heart valves • heart sounds • frequency analysis • vibrational analysis • finite element modeling • bioprosthetic valves
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