Statistical Properties of Pulsatile Pressure and Flow in the Femoral Artery of the Dog

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Femoral pulsatile pressure and flow in anesthetized dogs were measured as time series. The computed cross products of pressure and flow were also expressed as time series. The statistical variance (\( \sigma^2 \)) of each of these was analyzed into a frequency spectrum. The spectral magnitudes at each frequency were used to compute the pressure-flow coherence, impedance magnitude and phase angle as a function of frequency. The high coherency between pressure and flow at most frequencies indicated that a linear deterministic model may suffice to express the opposition to flow pulsations. The impedance variation with frequency indicated that the model may be frequency dependent. Computed autocorrelation and cross correlation functions measured the statistical nature of the observed time series.

The rhythmic action of the heart muscle imparts large fluctuations to blood pressure and blood flow, the duration of these fluctuations being of the order of the duration of the cardiac cycle. Since both physical and physiological reactions of the cardiovascular system can occur within this time interval, there is an interest in the adequate quantitation of the magnitudes of pressure and flow as mathematical time series.

The analysis of a periodic time series into Fourier harmonic components and the analysis of an aperiodic time series into a continuous spectrum using the Fourier integral have wide application for deterministic physical phenomena. These analyses have been applied to cardiovascular pressure and flow.\(^1\)\(^-\)\(^3\) Considering the spontaneous active vasomotion in the arterioles observed by Nicoll and Webb\(^4\) and considering the variability of biological quantities in general, it would seem to be of interest to observe the temporal statistical nature of the vascular pressure and flow. In fact, the very statistical nature of these 2 time series may constitute quantitative parameters which, when properly interpreted, may be of fundamental importance in the understanding of the vascular opposition to flow.

Statistical time series, or stochastic processes, have been widely studied in communication engineering by the blending of harmonic and correlation analyses, known as autocorrelation and cross correlation. Small et al.\(^5\) found that the time series representing the intestinal motility of the rabbit could be viewed statistically, after autocorrelation analysis, as having 3 separate and distinct components. Autocorrelation of the electrical activity of the heart during ventricular fibrillation indicates a periodic component which is obscured by statistical variations in the conventional time recording.\(^6\) Cross correlation between a photic stimulus and an electroencephalographic potential permits detection of evoked responses not discernible in the electroencephalographic tracing.\(^7\) The present paper is a preliminary report of an exploratory study of the autocorrelation functions of femoral arterial flow and pressure in the anesthetized dog, the cross correlation between these 2 quantities, and their spectra and cospectrum. These computed quantities provide statistical information about these time series and about the opposition to pulsatile blood flow.

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Supported by Research Grant H-2464 from the National Institutes of Health, U. S. Public Health Service.

Received for publication April 10, 1958.
The pressure and flow recordings were quantized to the nearest mm. Hg. and ml./min., respectively, at time intervals of 0.05 seconds. A total of either 500 or 1,000 successive pairs of observations was used for each of the 5 dogs so that 40 or 50 seconds of the pressure and flow recordings from each animal were analyzed.

Much of the theoretical background of the more generalized harmonic analysis which autocorrelation and cross correlation provide has been covered by X. Wiener. The technical aspects of computation have been outlined by James et al. Goodman has indicated the statistical aspects of estimating the time series parameters. A short description of the quantities computed is given in the appendix of this paper in terms of the hydraulic system observed.

The initial correlation and spectra computations were done by desk calculator. Although a complete analysis would require several months of hand calculation, this did permit determination of trends and sampling parameters in a very flexible manner. More complete analyses were performed by the New York University Computation Laboratory, through the kind cooperation of Mr. Raymond G. Stevens and Mr. Leo Tick, with their I.B.M. 650 digital computer. The complete analysis of the data from one animal was computed by them in one hour from punched cards.

RESULTS

The results of computations performed on each of the pressure and flow recordings from 5 animals show features which are common to all. The data of only 1 dog (12 Kg. female) will be reported in detail here since an insufficient variety of experiments has been done to establish structural or functional significance.

A section of the recording of arterial pressure and flow variation with time which was subjected to analysis is redrawn in figure 1. The mean flow over the analyzed time interval of 40 seconds was found to be 37 ml./min. and the mean pressure was 105 mm. Hg. The computed product of these 2 magnitudes, 3,850 mm. Hg X ml./min. (or 8.55 X 10^6 ergs/sec.), represented the hydraulic power which was present as the result of the steady components of pressure and flow. The concept of power is discussed more fully in the appendix. The pressure in the artery varied with a maximum excursion of about 36 mm. Hg and the flow pulse was about 40 ml./min.
The variations in flow and pressure about their means were responsible for an additional amount of hydraulic power, referred to here as pulsatile power. The computed instantaneous pulsatile power is redrawn in the upper tracing of figure 1. Positive or negative values of this curve, ranging from +5,000 to −1,000 mm Hg × ml./min., indicate the amount by which the total hydraulic power deviated from the power which would be expected for steady pressure and flow. Thus the pulsatile power at some parts of the cardiac cycle was considerably greater than the mean hydraulic power. The average pulsatile power over a 40 sec. time interval was found to be 123 mm Hg × ml./min. \((2.76 \times 10^4 \text{ ergs/sec.})\).

During diastole the wave form of the flow tracing had a significant departure from the pressure wave form showing an increase of flow during a time in which the pressure was steadily decreasing. This anomaly is believed to be characteristic of the vascular bed fed by the femoral artery, since it was not observed when the distal end of the exteriorized tubing was disconnected from the artery and then constricted by an adjustable clamp offering equivalent frictional opposition.

The cross correlation function of flow and pressure is shown in figure 2. The function, which varies with \(\tau\), the delay time between pressure and flow, amounts to a statistical condensation of the entire 40 second pulsatile power curve. At the delay time of \(\tau = 0\), the cross correlation function had a value of 123 mm. Hg × ml./min. which was also the mean pulsatile power. The normalized cross correlation function \(\rho_{fp}\) was identical in wave form but with the scale shown at the right in figure 2. This normalized function showed the correlation between flow at time \(t\) and pressure at time \((t + \tau)\). The correlation for simultaneous flow and pressure had the magnitude of 0.9, with a high level of significance \((p < 0.001)\) because of the large number of data pairs (800) used. The amount of this correlation was reduced for greater delay time, although in one experiment in which a more lengthy analysis was performed, this
correlation oscillated with a maximum amplitude of plus and minus 0.1 for as long as 11 seconds. The period of the oscillation in the correlation curve corresponded to the cardiac cycle duration.

The pressure and flow autocorrelation functions and their normalized scales are shown in figure 3. These functions are shown for only positive values of $\tau$ since the function is symmetrical about the ordinate $\tau = 0$. The normalized autocorrelation function is always unity for $\tau = 0$ since each of these time series will have perfect correlation with themselves for zero delay time. Preliminary analysis of the computed autocorrelation functions indicates that they may be represented as having three components: first, an exponential decay; second, a damped oscillation with period equal to the heart rate; and third, a damped oscillation with period equal to twice the heart rate.

The frequency representation of the cross correlation function has spectra corresponding to both the cosine and sine terms in its Fourier expansion. Figure 4A illustrates the pulsatile power spectrum arising as the result of the sinusoidal components of the pressure and flow which were in phase with one another at each frequency. The cospectrum magnitude at each frequency indicates the amount of pulsatile power which was present for a frequency interval of 0.167 c.p.s. The sum total of the power for all frequencies is then equal to the mean pulsatile power, or 123 mm. Hg $\times$ ml./min. in this case. The quadrature power spectrum, figure 4B, represents the hydraulic power at each frequency as the result of pressure and flow components which are 90° out of phase with one another. In the animals observed, the lower frequency spectral magnitudes indicated that the flow
was leading the pressure while at the higher frequencies the pressure was leading the flow.

Figure 5A indicates the manner in which the flow statistical variance of 158 (ml./min.)² was distributed in regard to frequency. Each ordinate of this curve represents the amount of activity which was present within a frequency band or 0.167 c.p.s. The spectral distribution of the pressure statistical variance of 117 (mm. Hg)² is shown in figure 5B. In general the amount of activity within the 1.5 through 3 c.p.s. band is a reflection of the amount of time that the heart was operating at each frequency. The high frequency components are manifestations of the distortion from a pure sine wave at the basic heart rate. The flow harmonics seem out of proportion to the fundamental when compared with the pressure spectrum. Also the pressure had more activity at the low frequencies, probably correlated with respiratory variations in pressure, which was not present in the flow to the same extent. Some animals showed less frequency variation of pressure and flow activities at the fundamental and multiples of the fundamental frequency.

The computed values of power spectral distributions of flow $S_t$, of pressure $S_p$, the co-spectrum $S_c$ and the quadrature spectrum $S_q$ between pressure and flow can be used to estimate several pulsatile pressure-flow relations. One of these, the coherency, is a measure of the linear interdependence of pressure and flow as a function of frequency. A high value of coherency at any one frequency is an indication that a consistent phase angle exists between the pressure and flow components at that frequency, while a low coherency indicates that the phase relationships between the pressure and flow components at that frequency vary at random. Coherency may be computed from the equation

$$\text{Coherency} = \frac{S_c^2 + S_q^2}{S_t \cdot S_p}$$

Figure 6A is a graph of the coherency between pressure and flow as a function of frequency. The coherency was high, of the order of 0.97 to 1.00, in the portions of the spectrum in which there was a significant amount of fluctuation in the original recordings, except at the very low frequencies. In the band of 0 to 0.5 c.p.s. which contained a considerable amount of activity in both the pressure and flow spectra, the coherency was low.

A second pulsatile pressure-flow relationship, the mechanical impedance, can be estimated from the spectral distributions computed. The impedance magnitude, which is the ratio of a sinusoidal pressure fluctuation about the mean pressure to a sinusoidal flow fluctuation at the same frequency, may be given by the equation

$$\text{Impedance magnitude} = \sqrt{\frac{S}{S_t}}$$
The middle curve of figure 6 indicates the computed mechanical impedance to pulsations in flow at the frequencies for which the coherency between pressure and flow was 0.97 or greater. The P. R. U. corresponds to the mm. Hg/ml./min. The impedance computed at 2 times the heart rate was always lower than that at the heart rate. A reduced impedance at any frequency indicates that a given flow pulsation can be produced with a smaller pressure pulsation than would be required at a frequency at which the impedance was greater.

The phase angle between flow and pressure at any frequency may be computed from the relationship between the quadrature and cospectral magnitudes at the frequency using the equation

\[ \text{phase angle} = \tan^{-1} \frac{S_q}{S_e} \]

The computed phase angles for the frequencies at which the coherency between pressure and flow was greater than 0.97 are shown in the lower graph of figure 6. At frequencies below 3.5 c.p.s. the flow pulsations reached their maxima before the pressure, while at higher frequencies the pressure maxima occurred first.

**DISCUSSION**

The autocorrelation functions of pressure and of flow and the cross correlation functions between pressure and flow indicate quite obviously that the pressure, flow and power time series can not be treated as simple periodic phenomena with one repeating cycle. Though one might expect this a priori, the computed functions put the characteristic on a quantitative basis. Within a time of 1.5 to 2 seconds there was a considerable diminution in the correlation between pressure and flow, the correlation remaining small up to at least 11 seconds. The degree of symmetry of the autocorrelation functions about the zero base line is an indication that there was little random noise in the pressure and flow time series. The damped nature of these oscillatory functions indicates that the source of the pulsations, the heart, did not remain "on frequency" for very long. It is hoped that a more detailed analysis of the correlation functions for longer delay times may permit analyzing these functions into separate simple statistical entities in a manner analogous to that which was done for the autocorrelation function of gastric motility.

The spectral information indicates that the major portion of the fluctuations in these time series occurs at the heart rate of the animal and at 2 times the heart rate. The band widths of these spectra indicate the variation in heart rate. The spectral distributions of the band at 2 times the heart rate show a minimum within this band, for the animal reported here. This is a manifestation of a sinus arrhythmia which could be observed in the original time recordings. The dual peak in energy can not be detected in the band of frequencies at the heart rate because of insufficient resolution in the method, the differences were amplified in the process of multiplying the basic frequencies. The power spectra of an animal which did not show sinus arrhythmia were more closely concentrated about singular maxima.

The extremely high coherence between pressure and flow for the frequencies of the heart rate and 2 times this rate, an indication of a fixed phase relation between pressure and flow at these frequencies, suggest that their interrelationships can be treated as deterministic linear systems at these frequencies. The lack of coherency at other frequencies indicates a presence of excessive "noise" or a lack of linear relationship between pressure and flow. Since the major portion of the frequencies at which there is no coherency contained very little pulsatile power in the original time series, it is very probable that there was insufficient signal for an adequate test. However at the lower frequencies, from 0 to 0.5 c.p.s., where there were significant fluctuations in pressure and flow, the lack of coherency is of some interest. No doubt the activity at these frequencies can be explained to a large extent upon the effects of respiration upon blood pressure, though it is difficult to understand why the flow does not undergo
an equivalent linear correlated fluctuation of consistent phase angle. It is possible that there is an incoherent fluctuation in peripheral resistance within the vascular bed which is related to the respiratory cycle.

The suggestion that pulsatile pressure-flow relationships in large vessels are governed by deterministic laws does not necessarily exclude the possibility of the importance of a statistical quantitation of the activity at the small vessels. Most of our gross physical concepts originated as deterministic descriptions which had to be revised to include statistical concepts as the methods of observation permitted quantitation on a more microscopic scale. It is entirely possible that the pressure-flow relationship of a large number of statistically governed blood vessels may appear deterministic.

The frequency characteristics of the impedance magnitude and phase angle suggest that there may be reactive components in the flow opposition of the vascular bed fed by the femoral artery. At the low frequencies where the flow maxima led the pressure maxima, the reactive elements would seem to involve predominantly the process of storing energy in the potential form, as, for example, in distending an elastic structure. At an intermediate frequency, 3.5 c.p.s. in the described animal, the pressure and flow were in phase, indicating a purely dissipative system. At frequencies above this the pressure led the flow, as would occur in a system in which the reactive elements were predominantly storing energy in the kinetic form as in accelerating a mass. The broad nature of the impedance magnitude minimum indicates that the system is highly damped with a considerable amount of dissipative opposition to flow.

These impedance variations with frequency are consistent with those observed by Randall and Stacy, in which they obtained their frequency spectra by taking single cardiac cycle tracings from a large number of animals with different heart rates. The analysis outlined in the present paper offers an advantage in that similar information can be obtained from a single animal with only a short recording of pressure and flow. A limitation is imposed by the spectra of frequencies which are present in the observed pressure and flow time series. Ideally, if these two quantities were pure "white noise" in their power spectral distributions the impedance could be computed for all frequencies. It is possible that a mechanical means of delivering wider frequency spectra of pressure can be devised so that the impedance characteristics of the vascular bed can be observed within a short time under a number of experimental conditions.

**Summary**

Femoral arterial pressure, flow and power were observed in the anesthetized dog. These quantities were subjected to autocorrelation or cross correlation analysis. The computed correlation coefficients revealed the temporal statistical nature of the observed time series. These time series were found to be largely periodic in nature, except that the correlation was not maintained to a very great extent beyond 1.5 to 2 seconds.

The Fourier transforms of the correlation functions were computed as a measure of the power spectra of the recorded time series. Almost all of the power could be attributed to fluctuations at the heart rate and at 2 times the heart rate.

The power spectral magnitudes were used to compute certain pressure-flow relationships as a function of frequency. The coherency between pressure and flow was found to be greater than 0.97 at the heart rate and at 2 times the heart rate. This suggests that the model for pulsatile pressure-flow relationships may be linear and deterministic at these frequencies. The impedance magnitude and phase angle variation with frequency suggest that this model must be frequency-dependent, with flow leading pressure at low frequencies and pressure leading flow at higher frequencies. The methods used herein may be a means of determining the frequency characteristics of the mechanical impedance of a vascular system from a short recording of pressure and flow.
ACKNOWLEDGMENT

The author gratefully acknowledges the technical assistance of Mr. Edward K. Jackson and Mr. Ernest X. Miles.

SUMMARIO IN INTERLINGUA

Pression, fluxo, e forta esseva observate in le arteria femoral de canes anesthesiate. Iste quantitates esseva subjicite a analyses de auto-correlation e de correlation cruciate. Le computate coefficientes de correlation revelava le temporal natura statistic del observate series de tempore. Esseva trovate que iste series de tempore esseva de natura predominantemente periodic, excepte que le correlation non se manteneva a alte grados in ultra de 1,5 o 2 secundas.

Le transformas fourierian del functiones de correlation esseva computate como mesura del spectra de potentia del registrate series de tempore. Quasi omne le potentia poteva esser attribuite a fluctuationes a un vice e a duo vices le frequentia cardiac.

Le magnitudes del spectro de potentia esseva usate pro computar certe relationes inter pression e fluxo como function del frequentia. Esseva trovate que le coherentia inter pression e fluxo excedeva 0,97 a un vice e a duo vices le frequentia cardiac. Isto suggere que le modello pro le relationes inter fluxo e pression pulsatil es possibilemente, a iste frequentias, lineari e deterministico. Le variation del magnitude del impedantia e del angulo de phase con alterationes de frequentia suggera que iste modello depende del frequentia, e que le fluxo precede le pression a basse frequentias durante le pression precede le fluxo a alte frequentias. Le methodos del presente studio representa possibilemente un medio pro determinar le caracteristicas de frequentia in le impedantia mechanic de un sistema vascular super le base de un curte registration del pression e del fluxo.

APPENDIX

Pulsatile Hydraulic Power

The product of pressure (force/area) and flow (volume/time) has the physical dimensions of power (work/time). Such power represents the rate at which work is being done in moving the fluid. The instantaneous hydraulic power is a time series which may be represented by the successive products of instantaneous pressure \( p(t) \) and flow \( f(t) \) as these 2 quantities vary with time \( t \). The average hydraulic power is then the average of a large number of these cross products of pressure and flow or

\[
\text{mean hydraulic power} = \frac{1}{t} \int_{0}^{t} f(t) \cdot p(t) \, dt \quad (1)
\]

For purposes of time series analysis this average power may be considered as resulting from two components, one determined by the product of the mean pressure and flow, the other determined by the pulsations about these means. The magnitude of this pulsatile component of power may be expressed as the difference between the total hydraulic power in equation 1 and the power due to the mean pressure and flow or

\[
\text{mean pulsatile hydraulic power} = \frac{1}{t} \int_{0}^{t} f(t) \cdot p(t) - \left( \frac{1}{t} \int_{0}^{t} f(t) \, dt \right) \cdot \left( \frac{1}{t} \int_{0}^{t} p(t) \, dt \right) \, dt \quad (2)
\]

It can be shown that this mean pulsatile power is also the statistical variance of the instantaneous power time series.

Cross Correlation Function

Since the instantaneous pulsatile power is a time series it may be analyzed into the components which are present at a number of sinusoidal frequencies, thus providing a power spectrum. For power pulsations which are reproducible in their time variations the time presentation synthesized from the power spectrum will have the wave form of the instantaneous power time series. However, when statistical variations of the pulsatile power occur with respect to time, the power spectrum can not be used to synthesize the original power time series. Instead, there will be produced a special time variation known as the cross correlation function between flow and pressure. In mathematical parlance the pulsatile power spectrum and the flow-pressure cross correlation function are Fourier transforms of one another. To obtain a pulsatile power spectrum it is more practical to first compute this cross correlation function and then to compute its Fourier transform.

In addition to its application as a computation-saving intermediary step in determining the power spectrum, the cross correlation function is of some fundamental interest in that it provides a means of separating the random components of the time series from those of a periodic nature. Essentially the cross correlation function is the mean of the cross products of pressure and flow as a function of a delay time between the 2 quantities. The cross correlation function \( R_{xy}(\tau) \) between flow \( f(t) \) and pressure \( p(t) \), each a function of time \( t \), may be calculated from the expression

\[
R_{xy}(\tau) = \frac{1}{t} \int_{0}^{t} f(t) \cdot p(t-\tau) \, dt
\]
PULSATILE PRESSURE AND FLOW

\[ R_p(t) = \bar{f}(t) \cdot \bar{p}(t+r) - \bar{f}(t) \cdot \bar{p}(t) \]  
(3)

This function is computed for each value of time lag from the mean of the cross products of flow taken at time \( t \) and of pressure measured at a time which differs from \( t \) by the amount \( \tau \). Note that this equation reduces to equation 2 for \( \tau = 0 \) and thus \( R_p(0) \) is the statistical variance of the power.

**Autocorrelation Function**

The autocorrelation function of a time series, as flow of pressure, is analogous to the cross correlation function, except that the time series is compared to itself displaced in time by an amount \( \tau \). The autocorrelation functions of flow and pressure, respectively, may be computed as

\[ R_f(\tau) = \frac{\bar{f}(t) \cdot \bar{f}(t+\tau) - \bar{f}(t)^2}{\sqrt{R_f(0) \cdot R_f(0)}} \]  
(4)

and

\[ R_p(\tau) = \frac{\bar{p}(t) \cdot \bar{p}(t+\tau) - \bar{p}(t)^2}{\sqrt{R_p(0) \cdot R_p(0)}} \]  
(5)

When \( \tau = 0 \) the quantity \( R_f(0) \) indicates the statistical variance of the flow about its mean and the quantity \( R_p(0) \) indicates the statistical variance of the pressure about its mean.

**Normalized Correlation Functions**

These correlation functions take on statistical meaning when they are normalized by dividing by a quantity which is a measure of the variance or covariance. The normalized cross correlation function, also called the cross correlation coefficient, is a measure of the correlation between pressure and flow which are separated by a time interval of \( \tau \). It is defined by

\[ \rho_f(\tau) = \frac{R_f(\tau)}{\sqrt{R_f(0) \cdot R_f(0)}} \]  
(6)

Similarly the normalized autocorrelation functions of flow and pressure become

\[ \rho_f(\tau) = \frac{R_f(\tau)}{R_f(0)} \]  
(7)

\[ \rho_p(\tau) = \frac{R_p(\tau)}{R_p(0)} \]  
(8)

These normalized correlation functions are joint probability distributions and may have values within the range of \(+1\) and \(-1\). When the magnitude of flow at time \( t \) is known, the value of \( \rho_f(\tau) \) is the probability of being able to predict the value of pressure at time \( (t+\tau) \). Periodic time series will have maximum correlation at delay times which are multiples of their period. Time series which are random will have no correlation as \( \tau \) is varied. The nature of the correlation function gives an indication of the extent of these two extremes in time series.

**Power Spectra**

The Fourier transforms of the cross correlation and autocorrelation functions indicate the frequency distribution of the statistical variances of the correlated time series. In general the cross correlation function will not be an even function, that is, the magnitude of the function will be different for positive and negative values of the delay time \( \tau \). When this is the case, the cross correlation function must be represented by two spectra. One of these, the power cospectrum, indicates the pulsatile hydraulic power resulting from the components of pressure and flow that are in phase with one another at each frequency. The other, the quadrature spectrum, indicates the pulsatile hydraulic power resulting from the components of flow and pressure that are 90° out of phase with one another at each frequency.

The spectra of the autocorrelation functions have the physical units of (mm. Hg)\(^3\) for pressure and (ml./min.)\(^3\) for flow. These are related to the squares of the sinusoidal components at each frequency which are in the original pressure and flow time series. Such spectral magnitudes are often referred to as power spectra, since under some circumstances the power in a hydraulic system will be directly related to the square of pressure and to the square of flow. Also the variance, \( \sigma^2 \) of the pressure and flow time series may be thought of as having activity at each of the sinusoidal frequencies, the total variance being the sum of the activities for all frequencies.

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Circ Res. 1958;6:689-698
doi: 10.1161/01.RES.6.6.689
Circulation Research is published by the American Heart Association, 7272 Greenville Avenue, Dallas, TX 75231
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Print ISSN: 0009-7330. Online ISSN: 1524-4571

The online version of this article, along with updated information and services, is located on the World Wide Web at:
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