LETTERS TO THE EDITOR

Comments on
"Static Linear and Nonlinear Elastic Properties of Normal and Arterialized Venous Tissue in Dog and Man"
which appeared in
Circ. Res. 37: 509-520, 1975

I have recently read the above-mentioned article in Circulation Research, and would like to relate a few criticisms of the data analysis. My first major point relates to the use of Young's modulus. A modulus is usually defined such that it is proportional to the first derivative of stress to strain:

\[ E \propto \frac{ds}{dy} \]

where \( s = \text{stress and } y = \frac{L - L_0}{L_0} \) (initial length).

Alternatively, this may also be expressed as:

\[ E \propto \frac{ds}{d\lambda}, \lambda = \frac{L}{L_0} \text{ since } y = \lambda - 1 \text{ and } dy = d\lambda. \]

However, in this article Young's modulus is defined as being proportional to the derivative times the stretch ratio:

\[ E \propto (\lambda) \frac{ds}{d\lambda}. \]

Let me give a short description why this is incorrect. Assume a perfectly elastic material, for example, an ideal spring. Plotting stress vs. strain results in a straight line which means the modulus is a constant at all strains. Such a result, if analyzed by this incorrect definition of modulus, would result in a modulus which increases proportionally to the extension.

In a non-ideal system such as a venous segment, the stress vs. strain curve is non-linear. Now the modulus is simply the derivative at any point on the stress vs. strain curve. If one were to multiply that derivative by the extension, \( \lambda \), one is artificially increasing the moduli at large strains. For example, two curves with the same slope at every applied pressure should have the same modulus. I refer to Figure 4 of the data presented, say for j.v. F = 5 and j.v. F = 10, or s.v. F = 5 and s.v. F = 10. These pairs of segments should have the same modulus since they have the same "stiffness," that is to say, they have the same derivative at each applied pressure. However, by this definition of modulus, s.v. F = 5's modulus would be larger than s.v. F = 10's modulus simply because the extension is larger—this being complicated even further by the normalization of the extension ratio since all data should be normalized such that every curve passes through \( \lambda = 1 \) at zero pressure. This last condition must be met by the definition of extension ratio since the initial dimensions should be measured at zero pressure.

From the data presented, the conclusion which should be made is that, initially, all veins tested had the same initial modulus as evidenced by the initial slope of each curve. However, jugular vein segments rapidly increase in modulus and have large but constant moduli at pressures greater than about 40 cm H2O (again referring to Figure 4).

Such an analysis would also explain why arteries are so much better suited for high pressure areas than saphenous veins. The article states, "From a first glance at the data presented in this paper, it might appear that saphenous veins should be a reasonably good grafting material for the arterial system." However, if the data were analyzed keeping these considerations in mind, one would find that the arterial segments have a relatively constant modulus, even up to 200 cm H2O (Figures 4, 5, and 6), whereas saphenous veins' moduli begin to drastically increase at about 120 cm H2O. Such a conclusion was not seen in the data analysis because of the incorrect definition of Young's modulus; multiplying the derivative by the extension artificially increased the artery's moduli even though it is, in fact, constant at all pressures tested. This fact may help to explain why saphenous veins are not as good an arterial substitute as this paper's data analysis seems to suggest.

Ronald J. Weigel
Yale Medical School
1 South Street
New Haven, Connecticut 06510

Reply to the Preceding Letter

The apparent discrepancy observed by Dr. Ronald J. Weigel with regard to our paper is due to his extension of the concepts of the linear, small deformation theory to the mechanics of soft tissue undergoing large deformation. He fails to realize that in large deformation analysis other, alternate, equally valid definitions of stress and strain are possible, and the choice depends on the suitability to a particular application. For example, in the study of the tensile stress-strain behavior of a wire of structural steel, it is customary to define stress as the axial force divided by the initial cross-sectional area. This stress is called the engineering stress. Since the cross-sectional area of the wire changes as it elongates, another stress, the "true" stress, can be defined as the axial force divided by
the cross-sectional area after the deformation has taken place under that load. For steel, in the elastic range of deformations, only negligible area changes occur, and, consequently both of the types of stress are almost equal. However, for a vascular tissue, the two definitions give distinctly different values in the physiological range of deformations. Similarly, various types of strain can be defined also. The conventional strain, \( \gamma \), is, of course, given as the change in length \( (L - L_0) \) divided by the initial length, \( L_0 \), where \( L \) is the current length. Thus, \( \gamma = \lambda - 1 \), where \( \lambda = L/L_0 \) is the extension ratio. For large strains, such as those encountered in soft biological tissue, other strains, for example, the Green strain \( \varepsilon = \frac{1}{2} (\lambda^2 - 1) \), or the natural strain \( \nu = \log \lambda \), are also useful measures of deformation.

Various different types of stress-strain curves can then be drawn depending upon the choice of definitions of stress and strain. For each such curve, two types of elastic modulus can be defined. The stress: strain ratio at each point gives the secant modulus of elasticity. The slope at each point gives the tangent modulus of elasticity. In the linear theory of elasticity, both of these are the same. If we denote the true stress by \( s \), and draw a curve of \( s \) vs. \( \nu \), the slope of the curve would be

\[
E_T = \frac{ds}{d\nu} = \frac{ds}{d\lambda/\lambda} = \lambda \frac{ds}{d\lambda}
\]

which is the definition of modulus we used in our paper. If we plot \( s \) vs. \( \gamma \), the slope of the curve would be

\[
E = \frac{ds}{d\gamma} = \frac{ds}{d\lambda}
\]

This is the definition used in the classical theory of elasticity. Yet another modulus can be obtained as the slope of the curve of the engineering stress vs. \( \gamma \). This is the definition generally used in Strength of Materials. All these definitions give almost equal numerical values in the vicinity of \( \lambda = 1 \), but not further away. Our definition of tangent elastic modulus \( (E_T) \) is thus different from what is conventionally used in the linear theory \( (E_T) \), but it is not incorrect or erroneous as Dr. Weigel suggests. His argument showing why our definition of modulus is incorrect is also improper. If the ideal spring (which recovers its undeformed length when the deforming force is removed) has a linear \( s-\gamma \) response, the modulus \( E \), will be a constant. If it has a linear \( s-\nu \) response, \( E \), will be a constant. For small strains \( (\lambda \approx 1) \), both of these moduli will be equal to the same constant, but for large strains one or the other modulus will vary with strain.

There are some good reasons why we used the definition of modulus we did, although we have nothing to say against using any other definition. Consider the change in length from \( L \) to \( L + dL \). The corresponding \( d\nu \) is given by \( dL/L_0 \) and \( d\gamma \) by \( dL/L \). The initial length \( L_0 \) does not appear in \( d\nu \), but it does in \( d\gamma \). Since in the living state the initial length \( L_0 \) is not available, it is more convenient to use the incremental strain \( d\nu \) than \( d\gamma \). For example, the familiar Bergel's modulus effectively involves the use of the circumferential strain \( dR/R \), where \( R \) is the mid-wall radius at an intravascular pressure \( p \), and \( R + dR \) at \( p + dp \). Another attractive feature of the incremental strains of the type \( d\nu \) is that the vanishing of the sum of the incremental strains of this type in the circumferential, longitudinal, and radial directions is a condition of tissue incompressibility in large as well as small deformations, while the vanishing of the sum of the \( \gamma \) type of strains describes incompressibility only for small strains. Some of these concepts are discussed in our recent textbook, "Basic Hemodynamics and Its Role in Disease Processes," published by University Park Press, Baltimore, Maryland.

We hope that this explanation clarifies the situation.

R. N. Vaishnav
Professor of Engineering
Department of Civil Engineering
The Catholic University of America
Washington, D.C. 20064

R J Weigel

doi: 10.1161/01.RES.49.6.1363

_Circulation Research_ is published by the American Heart Association, 7272 Greenville Avenue, Dallas, TX 75231
Copyright © 1981 American Heart Association, Inc. All rights reserved.
Print ISSN: 0009-7330. Online ISSN: 1524-4571

The online version of this article, along with updated information and services, is located on the World Wide Web at:
http://circres.ahajournals.org/content/49/6/1363.citation