Flow through Collapsible Tubes at Low Reynolds Numbers

Applicability of the Waterfall Model

CAROL K. LYON, JERRY B. SCOTT, AND C.Y. WANG

SUMMARY The applicability of the waterfall model was tested using the Starling resistor and different viscosities of fluids to vary the Reynolds number. The waterfall model proved adequate to describe flow in the Starling resistor model only at very low Reynolds numbers (Reynolds number <1). Blood flow characterized by such low Reynolds numbers occurs only in the microvasculature. Thus, it is inappropriate to apply the waterfall model indiscriminately to flow through large collapsible veins.


VASCULAR collapse occurs in both small and large vessels of the circulation. Examples of blood vessels that are normally subjected to collapse are: blood vessels of the myocardium (Downey and Kirk, 1975), the skeletal muscle (Gray et al., 1967), the lung (Maloney et al., 1968), intra-abdominal veins (Guyton and Adkins, 1954), and the cutaneous veins. Pathophysiological collapse of blood vessels also can occur. For example, cerebral capillaries have been shown to collapse when intracranial pressure is increased suddenly (Hekmatpanah, 1970), and the abdominal vena cava is known to collapse when abdominal pressure is increased by ascites (Vix and Payne, 1972). Occasionally, external interventions are used to induce vascular collapse. For instance, pneumatic pressure cuffs are used to collapse the arteries of the extremities to prevent hemorrhage during surgical procedures and to measure blood pressure by producing the Korotkoff sounds heard with the stethoscope (Brooks and Luckhardt, 1916). Blood flow in the collapsible vessels of the microcirculation of the lung was first modeled in 1962 by Permutt et al. (1962) who offered a simple “waterfall model” to describe the phenomenon. They proposed that blood flow through a vessel when it is partially collapsed by tissue pressure is independent of outflow pressure, just as flow over a waterfall is independent of the height of the falls. However, Permutt et al. (1962) did not set forth the limitations of the waterfall model, thus implying that the waterfall model may be applied to any collapsible vessel. Consequently, this model has been widely quoted to explain the pressure-flow relationships of both small and large collapsible blood vessels (Downey and Kirk, 1975; Green, 1975; Mitzner, 1974; Nakhjavan, 1966). On the other hand, the pressure-flow relationships of large blood vessels have been modeled experimentally by a “Starling resistor” physical model. This physical model takes its name from the fact that the noted physiologist, E.H. Starling
(Knowlton and Starling, 1912), devised a freely collapsible tube passing through a pressure-controlled chamber to model peripheral resistance in his heart-lung preparation. Holt (1941) used the Starling resistor model in an effort to analyze the pressure-flow relationships of large veins. Similar physical models have been used by other experimenters (Brecher, 1952; Rodbard, 1955; Doppman et al., 1966) to model flow through large blood vessels. Although the Starling resistor is a collapsible vessel, its properties do not conform to the waterfall model. Carefully controlled experiments by Conrad (1969), Katz et al. (1969), and Moreno et al. (1969) show the following discrepancies between the pressure-flow relationships of the Starling resistor model and the theoretical predictions of the waterfall model. (1) The waterfall model predicts total collapse of the blood vessels and absence of blood flow whenever tissue pressure (pressure external to the vessel) is greater than inflow pressure. Total collapse and cessation of flow never was observed in the Starling resistor model by these authors. (2) Self-induced oscillations of the vessel, neither predicted by the waterfall model nor observed in the microcirculation, were prominent in the Starling resistor model. These oscillations, or "flutter," were so marked in the experimental model that the pressure-flow relationships were markedly affected at higher flow rates.

It must be noted that, for all of the aforementioned Starling resistor experiments, the flows were at high Reynolds numbers, and these Reynolds numbers are more descriptive of blood flow in large blood vessels than in the microcirculation. The purpose of this research was to propose and test the hypothesis that the pressure-flow relationships of the Starling resistor model tend toward those described by the waterfall model, only for flows with lower Reynolds numbers. This would mean that the waterfall model, although applicable for low Reynolds number flow, does not apply to high Reynolds number flow in collapsible vessels, such as the large collapsible veins.

Methods

Theoretical Model

The concept of "waterfall" flow was first suggested by Duomarco and Rimini (1954). Lopez-Muniz et al. (1968) adopted this idea and set forth the "waterfall model" in explicit terms. According to the waterfall model, if \( P_i, P_o, \) and \( P_e \) are the inflow, exterior and outflow pressures, respectively, then the pressures, flow \( Q \), and resistance \( R \), can be described as follows:

\[
\begin{align*}
Q &= \frac{(P_i - P_o)}{R} \quad \text{if } P_i > P_o > P_e \quad (1) \\
Q &= \frac{(P_i - P_e)}{R} \quad \text{if } P_i > P_e > P_o \quad (2) \\
Q &= 0 \quad \text{if } P_e > P_i > P_o \quad (3)
\end{align*}
\]

where \( R \) is the resistance of the fully open tube. It should be noted that, although \( Q \) depends on three different pressures, \( P_i, P_o, \) and \( P_e \), only two of the three pressure differences \((P_i - P_o, P_i - P_e, P_e - P_o)\) are independent because \((P_i - P_o) = (P_i - P_e) + (P_e - P_o)\). Our experiments plot \( Q \) against both \( P_i - P_o \) and \( P_e - P_o \) for these reasons: \( P_i - P_o \) is traditional for noncollapsible tubes, and \( P_e - P_o \) is a sensitive parameter (more so than \( P_i - P_o \)) at the outflow end of the vessel where the collapse originates. The resulting three-dimensional graph, \( Q = Q(P_i - P_o, P_e - P_o) \), can best be described by cross-sections of this three-dimensional graph taken at various constant values of \( P_e - P_o \) (although \( P_e - P_o \) need not be constant in reality). Figure 1 shows the pressure-flow relationships of the waterfall model. For \( P_e - P_o < 0 \), the tube is fully open and Poiseuille's law applies. The slope of the line is the constant resistance \( R \) (thus the choice of \( Q \) for the abscissa and \( P_i - P_o \) for the ordinate). Let us fix \( P_e - P_o \) at a constant difference, say \( C \). For \( P_e - P_o = C > 0 \), there is total collapse until \( P_i - P_o \) becomes greater than \( C \), i.e., \( P_i > P_o \), when the tube partially opens. The waterfall model shall be compared to our experimental results.

Experiments

The Starling resistor model was used to study the pressure-flow relationships of a collapsible vessel. The physical setup was similar to those used by others over the past 40 years for their study of venous collapse (Holt, 1941; Brecher, 1952; Rodbard, 1955; Conrad, 1969; Katz et al., 1969; Moreno et al., 1969). Figure 2 shows the schematic diagram. A collapsible tube (Penrose tubing, American Hospital Supply), 0.032 cm thick, 1.27 cm in diameter,
A collapsible tube was mounted at both ends on metal tubing which protruded from an air-tight box. Pressure ports, for the measurement of inflow pressure ($P_i'$) and outflow pressure ($P_o'$), were located outside the box on the metal tubing. The pressure in the box was modified by a rubber air pump connected to a port in the box. Fluid was pumped via a compression chamber through the collapsible tube and measured by weight changes during a timed collection period. Pressure measurements were charted by the recorder, and pressure differences were computed and recorded by the X-Y plotter.

A reservoir of fluid, and 7.5 cm long, was mounted at both ends on metal tubes which protrude from an air-tight, transparent Plexiglas box. Pressure ports, for the measurement of inflow pressure ($P_i$) and outflow pressure ($P_o$), were located outside the box on the metal tubing, 8.5 cm from either end of the collapsible tube. The pressure in the box ($P_e$) was modified by the use of a rubber bulb pump connected to a port on the box. The pressures were measured by polyethylene catheters (PE60), filled with distilled water, connected to Statham pressure transducers (P23Gb), and coupled to a direct-writing oscillograph. All pressures were recorded using the average setting of the oscillograph, and pressure differences were recorded by an X-Y plotter which served as an analog computer in series with the oscillograph. The self-induced oscillations of the tube were quantified by use of the electronic high filter setting and expansion of the time scale. Fluid was pumped from a reservoir by a Sigma-motor positive displacement pump (35# torque, 1–400 rpm) and delivered to a compression chamber which effectively extinguished the pulse of the pump. The fluid then passed through the collapsible tubing into a second reservoir atop a Toledo balance. Changes in weight during a timed collecting period were recorded during steady state flow. This collecting period was timed with a hand-held stopwatch. The flow rate $Q$ was obtained from the formula:

$$Q = \frac{W}{\rho T}$$  \hspace{1cm} (4)

where: $W$ = the change in weight; $T$ = the time difference between readings; and $\rho$ = the density of the fluid.

Reliability of timing and reading of weight changes was tested with 15 readings at each of the lowest, medium, and highest constant flow settings of the pump. Recordings varied less than ±5% of the calculated mean. When data were being collected, two readings were taken at each flow rate. If the readings varied by more than 5%, another reading was obtained.

The inflow and outflow pressures ($P_i$ and $P_o$) were calculated from the pressures $P_i'$ and $P_o'$ by application of Poiseuille's Law:

$$P_i' - P_i = P_o - P_o' = 128 \frac{L \mu Q}{\pi d^4}.$$  \hspace{1cm} (5)

Here $L$ is the distance from the collapsible tube (8.5 cm) to the respective pressure port, $d$ is the inner diameter of the metal tube, $\mu$ is the viscosity of the perfusing fluid, and $Q$ is the flow rate. For 0.5-stoke fluid, the theoretical pressure drop across the metal tubing was:

$$(P_i' - P_i) = (P_o - P_o') = 0.134 \text{ mm Hg/ml per sec.}$$  \hspace{1cm} (6)

Experimental measurement of this pressure drop yielded:

$$(P_i' - P_i) = (P_o - P_o') = 0.143 \text{ mm Hg/ml per sec.}$$  \hspace{1cm} (7)

For the 10-stoke fluid, the theoretical pressure drop was:

$$(P_i' - P_i) = (P_o - P_o') = 2.71 \text{ mm Hg/ml per sec.}$$  \hspace{1cm} (8)

The experimental measurement for 10-stoke fluid yielded:

$$(P_i' - P_i) = (P_o - P_o') = 2.67 \text{ mm Hg/ml per sec.}$$  \hspace{1cm} (9)

The Reynolds number ($R$) was calculated by:

$$R = \frac{4\rho Q}{\pi d \mu}.$$  \hspace{1cm} (10)

The method of experimentation was first to initiate a low flow rate through the system by varying an upstream resistance. Pressure in the box was adjusted to a predetermined level of $P_e - P_o$ for each curve. After all transients had disappeared, the pressures, times, and weights were recorded. The flow was then slightly increased, and $P_e$ was adjusted so that a constant $P_e - P_o$ was maintained. A family of curves was obtained in this manner.

Initial pressure-flow relationships were obtained using water (0.01 stoke) for high Reynolds number flow. Dow Corning series 200 silicone fluid (0.5 stoke and 10 stokes) was used for low Reynolds number flow measurements. The pressure-flow curves were highly reproducible in this artificial system.
Results

Figure 3 shows the pressure-flow relationship for the collapsible tube perfused with water (0.01 stoke). For fixed values of \( P_e - P_o \), the perfusion pressure difference, \( P_i - P_o \), rises sharply with flow at low flow rates (\( Q < 6 \text{ ml/sec} \)). In this flow range, the tube appeared flattened, except for two small side channels that offered high resistance to flow. As the flow rate approached 6-8 ml/sec, the flattened area receded from the upstream end, until only the outflow end of the tube appeared to be "pinched" closed. As the pressure-flow curves reached the plateau region, the tube began to open and close intermittently. These self-excited oscillations (flutter) became increasingly vigorous as the flow rate was further increased (\( Q > 8 \text{ ml/sec} \)). The pressure-flow curves in the plateau region were nearly parallel to the flow axis, and \( P_i - P_o \) was approximately equal to \( P_e - P_o \). The changing shape of the tube resulted in the nonlinear pressure-flow relationships. In contrast, the completely open tube yielded the pressure-flow relationship of a straight line which is barely distinguishable from the \( X \) axis. This is compatible with the calculations of Poiseuille's law, which yield:

\[
P_i - P_o = 0.00288 Q. \tag{11}
\]

The Reynolds number for these curves reaches 2000 for \( Q = 20 \text{ ml/sec} \).

Figure 4 shows the pressure-flow relationships obtained when the tube was perfused with fluid of higher viscosity (0.5 stoke). The curves are similar to those in Figure 3, but there is an even sharper rise at low flow rates (\( Q < 3 \text{ ml/sec} \)). Flutter began at higher flow rates, at about \( Q > 10 \text{ ml/sec} \), and with a much lower pressure amplitude than the flutter that occurred during water perfusion. The maximum Reynolds number was about 40. The corresponding curve for Poiseuille flow, 0.5 stoke through the fully open tube, is a straight line from the origin:

\[
P_i - P_o = 0.041 Q. \tag{12}
\]

Figure 5 shows a representative pressure-flow graph obtained during perfusion of the tube with very high viscosity fluid (10 stokes). An initial rising

\[
P_i - P_o = 0.041 Q. \tag{12}
\]

Figure 4 A representative graph of the pressure-flow relationships for the perfusion of the collapsible tube with 0.5-stoke fluid. \( P_e - P_o \) was held constant 60, 30, 15, and 10 mm Hg, respectively. Maximum Reynolds number is 40.

Figure 5 A representative graph of pressure-flow relationships for the perfusion of the collapsible tube with 10-stoke fluid. \( P_e - P_o \) was held constant at 60, 30, and 10 mm Hg. The theoretical pressure-flow relationship for the condition \( P_e - P_o \leq 0 \) (the open tube) is represented by the broken line. Maximum Reynolds number is 2.
Table 1  Effect of Viscosity on Self-Induced Oscillations

<table>
<thead>
<tr>
<th>Viscosity (stokes)</th>
<th>Flow (ml/sec)</th>
<th>Amplitude (mm Hg)</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>16</td>
<td>1</td>
<td>50+</td>
</tr>
<tr>
<td>0.50</td>
<td>14.5</td>
<td>12</td>
<td>6.0</td>
</tr>
<tr>
<td>10.0</td>
<td>14.5</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

phase was not found at low flow rates. When flow was directed through the collapsed tube, $P_e - P_o$ gradually increased from zero to the level of $P_e - P_o$ and the tube gradually opened from the inflow end before any net flow occurred. The outflow end remained slightly constricted when flow did commence. No flutter of the tubing was ever detected throughout the entire flow range. Linear regression on the curves yielded:

For

$$P_e - P_o = 60 \text{ mm Hg},$$

$$P_i - P_o = 60.03 + 1.26 Q \quad (13)$$

$$P_e - P_o = 30 \text{ mm Hg},$$

$$P_i - P_o = 31.42 + 1.16 Q \quad (14)$$

$$P_e - P_o = 10 \text{ mm Hg},$$

$$P_i - P_o = 8.16 + 0.92 Q. \quad (15)$$

These slopes are somewhat higher than those calculated from Poiseuille’s law for perfusion of 10-stoke fluid through the fully open tube:

$$P_i - P_o = 0.829 Q \text{ (broken line in Fig. 5)} \quad (16)$$

The maximum Reynolds number for these experiments with 10-stoke fluid was 2.

Table 1 shows the effect of viscosity on the self-induced oscillations when $P_e - P_o = 10 \text{ mm Hg}$.

**Discussion**

The most important parameter governing viscous flow is the Reynolds number, which is a measure of the relative importance of inertial forces compared to viscous forces. Modeling theory states that all flows with the same Reynolds number demonstrate the same flow characteristics. For instance, the critical Reynolds number for the transition from laminar to turbulent flow in a tube is 2300, whether it is large or small. Therefore, although the diameters of the vessels of the microvasculature are too small to be modeled physically, the character of microvascular blood flow still can be approximated by increasing the viscosity of the perfusing fluid. Physical modeling with high-viscosity fluids has been minimal. Holt (1969) perfused Penrose tubing with 0.125-stoke fluid, achieving a Reynolds number of 130. He observed that increased viscosity caused an increased cross-sectional area of the tubing at any given flow rate. Fung and Sobin (1972) used a 300-stoke fluid and achieved a Reynolds number of 0.02. Photographs of the tube showed a slight constriction at the outflow end of the tube and no evidence of flutter. However, they did not measure pressures or flow rates. The present paper presents for the first time the pressure-flow relationships for a single collapsible tube with low Reynolds number flow. Our pressure-flow curves for water perfusion at high Reynolds numbers confirmed the results of Brower and Noordergraaf (1973) who were the first to plot data in terms of the pressure differences used in this study. The curves of Figure 3 are not compatible with those of the waterfall model of Figure 1. However, as the Reynolds number was gradually decreased (Figs. 4 and 5), the pressure-flow curves approached those of the waterfall model. Except for slight differences in the slope, Figure 5 is indeed the waterfall model of Figure 1. Although the Reynolds number of $10^{-2}$ of the microcirculation was not achieved, we can safely infer that the waterfall model would be even more applicable to flow at such a low Reynolds number. Figure 6 shows the various Reynolds numbers encountered in the vasculature of a dog (constructed from the data of Whitmore, 1968, and Cooney, 1976). From the results of this study, we conclude that the waterfall model is valid for very small blood vessels (probably less than 1 mm in diameter). Our experiments show that the pressure-flow relationships of the waterfall model can be achieved by reduction of the Reynolds number alone. This means that, when the Reynolds number is low enough, the properties of the collapsible tube, tube sizes, and method of mounting seem to be secondary factors. Finally, we refer to a recent controversy about the possibility of self-induced oscillations concomitant with low Reynolds number flow through a collapsed vessel (Conrad, 1973; Fung, 1973). Although we are not in the position to conjecture the cause of such oscillations, some of the questions raised have been answered by the present paper.
Our experiments show that: (1) Oscillations diminish to zero as the Reynolds number decreases; and (2) the pressure-flow relationships for low Reynolds number flow are distinctly different from those of high Reynolds number flow. This implies that the phenomena of pressure-flow relationships of collapsible vessels may be quite different for high vs. low Reynolds number flow. Therefore, we suggest caution in applying experimental results for high Reynolds number flow to low Reynolds number flow and vice versa.

Acknowledgments

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C K Lyon, J B Scott and C Y Wang

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