The Relation between Arterial Viscoelasticity and Wave Propagation in the Canine Femoral Artery in Vivo

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SUMMARY The influence of arterial dimensions and viscoelasticity on pulse wave propagation has been expressed in many theoretical models of blood flow in arteries, but few experimental tests of these theories in vivo have been reported. The measurements required for such tests include not only the arterial viscoelasticity, diameter, and wall thickness, but also the true propagation coefficients and impedances, for comparison with the values "predicted" by solution of the model equations. We made such measurements in 16 experiments on the femoral artery in nine anesthetized dogs. A two-point pressure and flow technique was used to measure wave propagation, and an ultrasonic micrometer was used to measure vessel diameter as a function of time and pressure. Measured attenuation constants ranged from 0.010 at 1.3 Hz to 0.075 at 12.7 Hz, and were more than twice as large as those predicted by two representative linear models. True phase velocity, which increased from 6.71 m/sec at 1.3 Hz to 10.54 m/sec at 12.7 Hz, agreed closely with the values computed by the Cox model but were lower than those given by the Jager model. The resistive, but not the reactive, component of longitudinal impedance was significantly greater than predicted by the models at all frequencies. The experiments do not identify the source of these discrepancies. The use of linear models to calculate pulsatile blood flow from pressure gradients in relatively small vessels, or to calculate attenuation and characteristic impedance from arterial viscoelasticity in vessels of any size, produces significant errors.

THE RELATION between arterial elasticity and wave velocity is one of the oldest and most fundamental principles of hemodynamics. It can be traced from Isaac Newton’s 17th-century analyses of the velocity of sound, through Thomas Young’s work on wave velocity in fluids (1808), to modern theories of pressure-flow relationships in blood vessels.1-4 Numerous mathematical models of this relationship have been devised, though they differ remarkably little in the phenomena they predict for a vessel of given physical properties. In 21 different linearized models compared by Cox,5 those in which longitudinal wall motion was limited by external constraint had higher phase velocities and greater wave attenuation than those lacking such constraint, but the most extreme examples differed by less than 30%.

Almost all theoretical models of blood flow lead to equations that express vascular impedance and wave propagation as functions of what may be called the "system parameters," namely: the dimensions of the vessel, viscoelasticity of the wall, kinetic viscosity of blood, constraint on wall motion, and signal frequency. If these parameters are known, the equations predict the impedance and wave propagation to be expected. For example, model equations are often used to estimate wave velocity from measurements of arterial elasticity, or vice versa.1,6,7 They are also used to compute instantaneous blood flow from the measured pressure gradient.6,9 Hemodynamic theories in the form of mathematical models thus have a wide range of applications, from studies of activity in vascular smooth muscle to clinical estimation of stroke volume.

Considering the large number of proposed models, there have been relatively few experimental studies of the accuracy with which the theories describe normal phenomena in vivo. Discrepancies between theory and actuality are inevitable, because blood vessels never conform exactly to the simplifying assumptions inherent in the equations, but more information on the nature and magnitude of the discrepancies is needed. Several investigators have measured arterial viscoelasticity,6-10 differential pressure and flow,6,8,11,12 or wave propagation,13,14 but rarely have all these variables been measured in the same experiments. Systematic testing of theories of this kind requires simultaneous measurement of: (1) pressure and flow wave transmission, independent of reflections; (2) pulsations of vessel diameter; (3) radius and wall thickness of the vessel. The only previously published experiments that meet these requirements, to our knowledge, are those reported by Cox.15 He showed good agreement between the phase velocities predicted by his theoretical model and those observed in the canine femoral artery in vivo, but did not examine attenuation or longitudinal impedance. Ling and his colleagues16,17 made the relevant measurements in the aorta and coronary arteries of the anesthetized...
dog and compared the results with a nonlinear model, but they did not analyze their data in a form that permits comparison with the propagation coefficients and impedances reported by others. The propagation of artificial waves of 40–200 Hz, well above the physiological range, has been studied by Anliker’s group. The present experiments were designed to provide quantitative comparison of the propagation and impedance actually observed in vivo with those predicted by existing linear theories, given the measured physical properties of the artery. The experiments were carried out on anesthetized dogs, in which we measured true propagation coefficients, vascular impedances, wall viscoelasticity, and arterial dimensions. The dimensions and viscoelasticity were entered in the model equations, which could then be solved to give “predicted” propagation coefficients and impedances, for comparison with the measured values.

**Symbols and Definitions**

### Real Variables

- \( R_o \): Outer radius of vessel.
- \( R_i \): Inner radius of vessel.
- \( h \): Wall thickness.
- \( x \): Axial (longitudinal) coordinate of vessel.
- \( L \): Distance along axial coordinate between the two sites at which pressures and flows are measured.
- \( f \): Frequency in Hz.
- \( \omega \): Frequency in radians/sec.
- \( \sigma \): Poisson’s ratio of wall material; assumed 0.5 unless otherwise specified.
- \( \rho \): Density of blood vessel wall; assumed to be 1.05 g/cm³.
- \( \eta \): Blood viscosity; assumed to be 0.035 poise (dyn sec/cm²).
- \( \alpha \): Womersley parameter, \( \alpha^2 = R_i^2 \omega / \eta \).
- \( c_0 \): Phase velocity in tube filled with nonviscous fluid.
- \( c_1 \): Phase velocity in tube filled with viscous fluid (\( c_1 = \omega / \beta \)).
- \( j \): \((-1)^{1/2}\).
- \( \delta \): \(R_o/h\).

### Complex Variables

- \( \gamma \): True propagation coefficient, \( \gamma = a + jb \), where \( a \) is the attenuation in nepers/cm, and \( b \) is the phase coefficient in radians/cm. For example, if no reflected waves are present, \( P_2 = P_1 e^{-\gamma L} \).
- \( c \): Complex wave “velocity,” \( (\omega/c) = \gamma \).
- \( P \): Complex harmonic component of blood pressure. Subscript “1” indicates the upstream site; “2” downstream.
- \( Q \): Complex harmonic component of blood flow. Subscripts as for \( P \).
- \( \xi \): Complex harmonic component of radial displacement of outer wall.
- \( E_c \): Complex viscoelastic “modulus.” In real and imaginary form \( E_c = |E_c| \cos \psi + j |E_c| \sin \psi \). The real part equals Young’s modulus, \( E \).

### Methods

Experiments were carried out on two mongrel dogs and seven greyhounds. Four of the dogs were anesthetized with sodium pentobarbital (25 mg/kg, iv, initially, and approximately 3 mg/kg hourly thereafter). The other five dogs were given a-chloralose (80 mg/kg, iv, as an 8% solution in polyethylene glycol) to provide lower resting heart rates and hence a wider spectrum of frequencies. Respiration was spontaneous, but a tracheal tube was inserted to maintain a clear airway. The right femoral artery was exposed by an incision about 10 centimeters long, keeping dissection of tissues to the minimum required for placement of transducers. The edges of the skin incision were raised and suspended from a metal support, forming a trough, which was filled with Krebs’ solution. The vessel was completely immersed in this bath during the measurements.

Flows were measured with a two-channel electromagnetic flowmeter (Biotronex BL-610) and flow probes with an inside diameter of 3.5–5.0 mm. The measured dynamic response of the flowmeter was constant in amplitude within ±5% from 0 to 42 Hz, with a linear phase shift of 1.3°/Hz. Zero flow signals were determined by occlusion of the artery peripheral to the distal probe. Static calibrations of both flow probes were carried out simultaneously in situ with measured flows of the dog’s own blood in six of the nine dogs. In the other three dogs, the flow probes had recently been calibrated in other experiments. Two flow probes were placed on the femoral artery, one just distal to the branching of the deep femoral artery, the other just proximal to the first large muscular branch downstream, giving a distance between probes of 3.7–5.3 cm.

Pressures were measured through Teflon tubing (0.75 mm i.d., 8 cm in length) attached to Statham P23Gb strain gauges. Catheters and gauges were filled with boiled physiologic saline solution, and the dynamic response of the pressure systems was measured by imposing a pressure step-function (“balloon pop”) prior to each experiment. Damped natural frequency was greater than 274 Hz in all experiments and averaged 336 Hz. Damping ratio averaged 0.09. The catheters for pressure measurement were inserted through small side branches of the artery so that they measured lateral pressure...
1–3 mm beyond the distal edge of each flow probe. The remaining side branches, which were less than 1 mm in diameter, were tied off near the femoral arterial wall.

Outside diameter of the artery was measured by an ultrasonic transit-time micrometer recently developed by Bertram. This instrument measures the transit-time of ultrasonic (5-MHz) impulses traveling between two piezoelectric transducers attached to the outer wall. The two transducers, each measuring approximately 3 × 1.25 mm and weighing 15 mg, were placed on the arterial wall about 1 cm distal to the proximal flow probe. They were fastened to the wall with a minute amount of Eastman no. 910 adhesive, with particular care taken to align them across the diameter of the vessel. Transit-time measurements were calibrated by displaying the output on an oscilloscope with an accurate high-frequency sweep (Tektronix model 465), and were converted to distance by applying the measured velocity of such signals in blood, 1570 m/sec. The accuracy of absolute measurements of diameter depended on the accuracy with which the oscilloscope display could be read, which we estimated to be about 2%. The resolution of the measurements was less than 1 μm, and their repetition rate 1 kHz. At the end of each experiment, the distance between transducers and the length of the arterial segment in situ were measured. The vessel was then excised and the attached connective tissue removed from the wall by careful dissection. The segment was then weighed in air and in water to determine its density, and the cross-sectional area of the wall was calculated.

The data collected during the experiments were stored on analog magnetic tape (Hewlett-Packard model 3960 four-channel recorder). Proximal flow and both of the pressures were recorded continuously on three of the channels. The fourth channel was used to record distal flow and diameter alternately, by switching from one to the other every 10 seconds (see Fig. 1).

Data Analysis

The stored experimental data were analyzed after each experiment by a digital computer (Digital Equipment Corp. model PDP 11/10, including AR-11 A/D converter). The time interval used in analog-to-digital conversion was either 2 or 4 msec, depending on heart rate, so that at least 160 points per cardiac cycle were converted. From two to four cardiac cycles were analyzed for each experimental determination. Pressures, flows, and diameter were subjected to harmonic analysis so that each variable could be expressed as a Fourier series. Computations were arbitrarily limited to frequencies below 16 Hz, with the result that from five to ten harmonics were computed for each set of data, depending on the fundamental frequency. Appropriate corrections for the known dynamic response of the pressure and flow measurement systems were applied, although most of these adjustments were extremely small. Pressure corrections were less than 0.2% for modulus and less than 0.006 radians for phase, at frequencies of 1–12 Hz. Flow corrections were less than 1% modulus, and 0.03–0.33 radians for phase. No correction was made for the slight pressure lag (<0.02 radians) introduced by the distance between flow and pressure measurement sites.

True (reflection-free) propagation coefficients (γ) were determined by a procedure previously described by Milnor and Nichols. In this method the propagation coefficient is calculated from experimental measurements of pressure and flow wave transmission between two sites in a blood vessel. If, for any one harmonic, $T_p$ denotes the ratio of downstream to upstream pressures, $T_q$ the corresponding ratio for flows, and $L$ the distance between the two measurement sites, then:

$$\gamma_L = \cosh^{-1} \left[ \frac{1 + (T_p T_q)}{T_p + T_q} \right].$$  \hspace{1cm} (1)

Characteristic impedance ($Z_0$) can be calculated from the same experimental data:

$$Z_0 = \frac{Z_1 (\cosh \gamma_L - T_p)}{\sinh \gamma_L}. \hspace{1cm} (2)$$

Details of the technique, which is based on the assumption that the system is analogous to a linear transmission line, have been discussed in an earlier paper. The longitudinal and transverse impedances were computed from $\gamma$ and $Z_0$ by application of the transmission-line equations:21

$$Z_L = \gamma Z_0 \hspace{1cm} (3)$$

$$Z_w = Z_0 / \gamma. \hspace{1cm} (4)$$
The complex viscoelastic modulus \( E_c \) at each harmonic frequency was calculated from the harmonics of pressure and diameter in accordance with Bergel's\(^6\) equation for a thick-walled tube of incompressible material, including the phase shift that represents viscous behavior of the wall:

\[
E_c = \frac{3 R_o R_i^2 P e^{i\delta}}{2 (R_o^2 - R_i^2)} \xi.
\] (5)

Reflection ratios \( \Gamma \), the ratios of reflected to incident pressure waves, were calculated from the equation:\(^9\)

\[
\Gamma = \frac{(Z_n - Z_0)}{(Z_n + Z_0)}.
\]

The experimentally determined values of \( Z_n, Z_0, Z_L, \) and \( Z_w \) were calculated through Equations 1-4, and the results were compared with the values predicted by theoretical models. Statistical significance of differences was calculated by the "\( t \)-test."\(^22\)

Theoretical Models

Two representative models were selected: (1) Jager's model\(^3\) in which the wall is subjected to infinite longitudinal constraint; (2) Cox's model,\(^2\) in which the wall is free to extend longitudinally as well as radially. Both analyses assume that the wall is relatively thick, and viscoelastic. Almost all other linear models reported in the literature give results that lie between the two extremes represented by these examples.\(^5\) The implications of Jager's model are identical to those of Womersley's infinitely constrained tube,\(^4\) provided \( E \) is defined as in Equation 5.

Longitudinal impedance in the Jager model was calculated by:

\[
Z_L = \frac{j \omega p}{\pi R_i^2 (1 - F_{10})}.
\] (6)

where \( (1 - F_{10}) \) is a complex function of \( \alpha \), derived and tabulated by Womersley.\(^4\)\(^9\) Normalized values of longitudinal impedance were calculated by dividing the Poiseuille resistance \( (8\eta/\pi R_i^4) \) into the real part of \( Z_L \), and dividing the theoretical inertance \( (\omega p/\pi R_i^3) \) into the imaginary part.\(^11\) The normalized \( Z_L \) predicted by the Cox model was taken from his tables.\(^2\) In his model, \( Z_L \) is a function of \( R_o/R_i \) as well as \( \alpha \). His tabulated values apply to a ratio of 1.15, but they differ by less than 1.5% from those for the average ratio (of 1.12) in our experiments.

The propagation coefficients \( (\gamma = j\omega/c) \) predicted by the Jager model were computed by way of the complex wave velocity, \( c \):

\[
c^2 = c_o^2 (1 - F_{10})(1 + j \tan \psi).
\] (7)

In this equation, the influence of fluid viscosity and boundary conditions is expressed by \( (1 - F_{10}) \), and the last term on the right describes the effects of wall viscosity. The variable \( c_o \) represents the wave velocity in a nonviscous system, and was calculated in accordance with Bergel's derivation:\(^6\)

\[
c_o^2 = \frac{E(4R_o^2 - R_i^2)}{3 \rho R_i^2}.
\] (8)

The variable \( E \) in Equation 8 is Young's modulus, i.e., the real part of the complex viscoelastic modulus calculated from Equation 5. In the Cox formulation, \( c \) was computed by solution of the quadratic equation:\(^2\)

\[
(1 - F_{10})(\delta^2 - 1)x^2 + \alpha^2[(1 - F_{10})(3 - \delta^2) - (3 + \delta^2)] x - \alpha^2[(\delta^2 + 1)/(\delta^2 - 1)] \cdot [(1 - F_{10}) - \delta^2] = 0,
\] (9)

where

\[
x = \frac{\gamma R_o^2 E_c}{\omega \eta^3}.
\] (10)

Transverse wall impedance, \( Z_w \), was calculated by the equation given by Cox.\(^15\)

\[
Z_w = \frac{E(4R_o^2 - R_i^2)}{3j\omega \pi R_i^3 R_o^2}.
\] (11)

Jager describes the transverse impedance in different terms, but his expression is equivalent to Equation 11.

Results

The average vessel dimensions, mean pressures, and mean flows, shown in Table 1, were similar to those reported in other studies on anesthetized dogs.\(^7\)\(^14\) Animals anesthetized with chloralose had slower heart rates (average, 72 beats/minute; \( \text{SEM} = \pm 8 \)) than the others (157 ± 9 beats/minute), but at the harmonic frequencies common to both (>2.0 Hz) the results obtained in the two groups of dogs were not significantly different \((p > 0.20)\). We assumed, therefore, that there was no difference in the effects of the anesthetics themselves on the variables of interest, and that the two agents simply extended the frequency range of the observations, as was intended.

The measurements of frequency-dependent variables are summarized in Table 2. Data from all experiments were grouped and averaged according to frequency. Bandwidths were selected to give from 7 to 15 observations in each frequency band, with the result that the group boundaries were

**Table 1 Summary of Experimental Data**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body weight (kg)</td>
<td>24.78 ± 1.39</td>
</tr>
<tr>
<td>Heart rate (per min)</td>
<td>105 ± 13.2</td>
</tr>
<tr>
<td>Vessel radius, outer (mm)</td>
<td>2.5580 ± 0.0638</td>
</tr>
<tr>
<td>Ratio wall thickness/outer radius</td>
<td>0.1079 ± 0.0029</td>
</tr>
<tr>
<td>Mean blood pressure (mm Hg)</td>
<td>142.4 ± 4.08</td>
</tr>
<tr>
<td>Mean blood flow (ml/sec)</td>
<td>1.483 ± 0.16</td>
</tr>
</tbody>
</table>

Results are expressed as means ± SE.

* Radius at site of diameter transducers.
### Table 2: Experimental Results, Frequency-Dependent Variables, Mean (SEM)

<table>
<thead>
<tr>
<th>Frequency band</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (Hz)</td>
<td>1.30</td>
<td>2.59</td>
<td>4.96</td>
<td>8.12</td>
<td>12.71</td>
</tr>
<tr>
<td>Number of observations</td>
<td>7</td>
<td>15</td>
<td>14</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>Alpha</td>
<td>3.6 (0.30)</td>
<td>5.0 (0.23)</td>
<td>7.0 (0.23)</td>
<td>8.9 (0.29)</td>
<td>11.2 (0.34)</td>
</tr>
<tr>
<td>True propagation coefficients:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Attenuation (10^-8 nepers/cm)</td>
<td>10.11 (2.18)</td>
<td>12.71 (1.50)</td>
<td>18.79 (2.76)</td>
<td>45.73 (3.73)</td>
<td>75.59 (12.92)</td>
</tr>
<tr>
<td>Phase (10^-2 rad/cm)</td>
<td>14.48 (2.74)</td>
<td>25.67 (4.34)</td>
<td>38.70 (4.54)</td>
<td>62.23 (7.56)</td>
<td>81.12 (6.50)</td>
</tr>
<tr>
<td>Viscoelastic modulus:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Amplitude (10^5 dyn/cm^2)</td>
<td>2.70 (0.56)</td>
<td>2.77 (0.34)</td>
<td>2.89 (0.34)</td>
<td>3.74 (0.71)</td>
<td>3.50 (0.61)</td>
</tr>
<tr>
<td>Phase (rad)</td>
<td>+0.22 (0.02)</td>
<td>+0.09 (0.01)</td>
<td>+0.05 (0.06)</td>
<td>-0.20 (0.13)</td>
<td>-0.07 (0.07)</td>
</tr>
<tr>
<td>Inviscid fluid phase velocity (cm/sec)</td>
<td>1234 (112)</td>
<td>1290 (71)</td>
<td>1333 (60)</td>
<td>1400 (154)</td>
<td>1379 (115)</td>
</tr>
</tbody>
</table>

**Impedances:**

<table>
<thead>
<tr>
<th>Impedance</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>Z_0</td>
<td>(10^8 dyn sec/cm^5)$</td>
<td>10.39 (1.40)</td>
<td>6.71 (0.83)</td>
<td>6.84 (0.36)</td>
</tr>
<tr>
<td>$Z_p$ phase (rad)</td>
<td>-0.42 (0.13)</td>
<td>-0.25 (0.09)</td>
<td>-0.12 (0.14)</td>
<td>+0.10 (0.11)</td>
<td>-0.01 (0.09)</td>
</tr>
<tr>
<td>$Z_t$ (dyn sec/cm^5)</td>
<td>206 (36)</td>
<td>289 (65)</td>
<td>302 (34)</td>
<td>473 (52)</td>
<td>790 (107)</td>
</tr>
<tr>
<td>$Z_e$ phase (rad)</td>
<td>+0.54 (0.20)</td>
<td>+0.75 (0.12)</td>
<td>+0.94 (0.14)</td>
<td>+1.01 (0.11)</td>
<td>+0.93 (0.14)</td>
</tr>
<tr>
<td>$Z_w$ phase (rad)</td>
<td>-1.34 (0.21)</td>
<td>-1.28 (0.10)</td>
<td>-1.19 (0.15)</td>
<td>-0.86 (0.14)</td>
<td>-0.93 (0.10)</td>
</tr>
<tr>
<td>Reflection ratio, amplitude</td>
<td>0.47 (0.06)</td>
<td>0.65 (0.03)</td>
<td>0.58 (0.04)</td>
<td>0.48 (0.07)</td>
<td>0.64 (0.15)</td>
</tr>
<tr>
<td>Reflection ratio, phase (rad)</td>
<td>-0.46 (0.13)</td>
<td>-0.49 (0.08)</td>
<td>-1.00 (0.19)</td>
<td>-1.18 (0.15)</td>
<td>-0.99 (0.34)</td>
</tr>
</tbody>
</table>

Alpha: Womersley's parameter; $Z_0$: characteristic impedance; $Z_t$: longitudinal impedance; $Z_e$: transverse impedance; reflection ratios are those at the distal measurement site.

**0.5–2.0 Hz, 2.0–3.5 Hz, 3.5–6.5 Hz, 6.5–9.5 Hz, and 9.5–15.5 Hz.**

Attenuation and phase coefficients (α and β) increased with frequency as expected and did not differ significantly from those found in our previous study 19 ($P > 0.05$ at 8.1 Hz; $P > 0.40$ in all other frequency groups). The complex viscoelastic moduli were similar in magnitude and phase to those found in the canine femoral artery under similar conditions by other investigators. 7,10> 15 The phase of $E_c$, which is the phase angle between pressure and radial motion, became smaller as frequency increased, and was negative (denoting that pressure lagged diameter) above 8 Hz.

True attenuation coefficients measured in the experiments were significantly higher ($P < 0.025$) than those predicted by either of the models, with the exception of the Cox value for α at 1.3 Hz ($P > 0.05$). These results are expressed in Figure 2 as transmission per 10 cm ($e^{-10h}$). Values in the Jager and Cox models differed by less than 1%. The imaginary part of the propagation coefficient (β) was significantly greater than that of the Jager model ($P < 0.025$), but did not differ significantly from the values predicted by the Cox model ($P > 0.05$). These results are shown in Figure 2 in the form of phase velocities ($c_t = \omega/b$). Observed and theoretical phase velocities increased with frequency between 1.3 and 8 Hz. At 8 Hz, the observed true phase velocity amounted to 75% of the Jager value, and to 70% of the inviscid phase velocity, $c_0$.

The apparent phase velocity of pressure waves was higher than the true velocity at all except the highest frequencies. The ratio of apparent to true velocity was 3.5 at 1.3 Hz, and diminished as frequency increased to a value of 1.00 at 12.7 Hz. This difference, which has been reported previously, 16,14 is attributable to wave reflection. Moduli of the reflection ratio at the distal site ranged from 0.47 to 0.65 (see Table 2). The phase of the reflection ratio ranged from −0.46 to −1.18 radians. Theoretical discussions of reflections often assume a phase angle of either 0° or 180° (so-called "closed" or "open" terminations), 8, but our results demonstrate that intermediate values are common in the circulation.

Complex longitudinal impedances ($Z_t$) are shown in Figure 3. The real or resistive part of $Z_t$ measured experimentally was significantly greater at all frequencies than that of either of the models ($P < 0.01$). This difference increased with frequency, the observed real $Z_t$ being greater than the Jager value by a factor of 2.4 at 1.3 Hz, and by a factor of 4.8 at 12.7 Hz. The imaginary or reactive part of $Z_t$ was not significantly different from the model values ($P > 0.10$). Real and imaginary parts of $Z_t$ were 5–20% smaller in the Cox than in the Jager model. In terms of modulus and phase, the average normalized $Z_t$ modulus was significantly greater ($P < 0.025$) than that in the models at 1.3 Hz, but this was not the case at higher frequencies. The measured phase of $Z_t$ was approximately 0.5 radians smaller than that of the models at all frequencies ($P < 0.01$). This difference was more than three times the probable experimental error of measurement, as comparison...
Figure 2. Phase velocity (above) and transmission per 10 cm (below) as functions of frequency (abscissa). Closed circles represent measurements in the present experiments; bars show ±1 SEM. Open circles represent calculations from arterial dimensions and viscoelasticity by the Cox model; crosses, those from the Jager model. Triangles represent inviscid fluid phase velocity calculated from Equation 8. Transmission in the Cox model (not shown) was almost identical with that in the Jager model (see text).

The measured transverse impedances \( Z_{\text{T}} \), calculated by Equation 4 and summarized in Table 2, were significantly smaller in amplitude than those predicted by the models at all frequencies. The predicted values, which were the same for the Cox as for the Jager model, were 2.1–3.8 times larger than the observed averages. The transverse impedance phase measurements did not differ significantly from those of the models at 1.3 and 2.6 Hz, but at higher frequencies the observed value was less negative than the predicted one.

The characteristic impedance moduli predicted by the models (by way of Equation 3) were significantly larger than those measured experimentally at all frequencies \((P < 0.02)\). Phase angle of characteristic impedance for the models was less negative than the observed values at \( f = 1.3 \) and 2.6 Hz, but not significantly different at higher frequencies.

Discussion

Sources of Error

Substantial errors are possible in measurements of this kind in spite of careful attention to technique; yet it is unlikely that the lack of agreement between our observations and the theoretical predictions can be attributed to inaccuracies of measurement. The differences were systematic, and much greater than the estimated error of the methods. For example, the measured resistive \( Z_{\text{T}} \) at 1.3 Hz exceeded the values predicted by the models by an average of 260%, which is 10 times the probable \((P < 0.05)\) random experimental error. The probable errors in the measurement of vessel radius, or in the values assumed for blood density and viscosity, were not large enough to account for discrepancies of the magnitudes shown in Figures 2 and 3.
Values in the literature for these variables suggest that our data are not in error by more than 10%. The "anomalous viscosity" that develops at very low shear rates presumably played no part in our results, because it becomes significant only when $\alpha < 2.5$.

Nevertheless, we cannot rule out the possibility that the discrepancies reported here may arise in part from the assumptions inherent in our determination of true propagation coefficients. The most critical assumption is that the coefficients of retrograde propagation are the same as those in forward wave motion. Taylor's experiments suggest that this condition is satisfied in rubber tubes, but the question has not been answered experimentally for arteries in vivo. When this assumption is applied in determining true coefficients, the results are consistent with other experimental data that do not depend on the assumption, such as the frequency spectra of local impedance. This finding argues that the assumption is at least approximately correct, although it does not constitute a rigorous test.

The use of harmonic analysis to determine $E_2$ as a function of frequency can be criticized because of the nonlinear elasticity of the arterial wall, but pressure-diameter relations are almost linear for the relatively small strains imposed by normal arterial pulsations. Gow and Taylor concluded from the behavior of the pressure-diameter phase angle, $\psi$, that the higher harmonics could not be determined accurately. They found, as we did (see Table 2), that diameter lagged pressure at low frequencies, but led pressure at higher frequencies, and they demonstrated that this pattern could be explained by nonlinearity. We agree that the calculation of components at frequencies above 5 or 6 Hz is relatively inaccurate, in part because the signal amplitudes in that range are very small, but the discrepancies in phase velocity and attenuation with which we are concerned here were prominent at lower frequencies. Moreover, factors other than nonlinearity may be involved in the behavior of the angle $\psi$, for in theory it is influenced by the degree of external constraint and by the relative elasticity and viscosity of the system, which can be frequency dependent.

**Propagation**

The models of Womersley and Jager imply that the true phase velocity in a viscous system ($c_1$) is the same as the inviscid phase velocity ($c_0$) when $\alpha = 5$, and that $c_1$ should be slightly higher than $c_0$ with larger values of $\alpha$. For a long time there was no reason to doubt that these statements applied to blood vessels in vivo, because the data on pulse wave velocity and those on arterial elasticity are of about the right relative magnitudes. This approximate correspondence is not a conclusive test, however, and the range of values reported in the literature for these variables is very wide because of differing experimental conditions. Dynamic elastic moduli of the femoral artery have been measured in vivo and in vitro, in young and old animals, and at a variety of distending pressures, with results that range from $12 \times 10^6$ to $30 \times 10^6$ dyn/cm$^2$. Values in this range are consistent with the Womersley and Jager models overestimate true phase velocity in a vessel of given viscoelastic properties by 25-40% (Fig. 2).

The observation that true phase velocities were more like those in a freely moving than in a strongly constrained model was unexpected, because experimental evidence and the known anatomic structure of tissues combine to suggest that longitudinal motions of the vessel wall must be very small. In our experiments the vessel lay exposed in situ, surrounded by fluid, and it may have been freer to extend or contract longitudinally than is the unexposed vessel. The artery was tethered at each end to the remainder of the vascular tree, however, and there was no visual indication of longitudinal motion. Moreover, the degree of constraint cannot completely explain the differences between theoretical and observed propagation coefficients, because neither the free nor the constrained model predicted attenuation accurately.

The only previous report on true attenuation of natural pulsations is that of McDonald and Gesnner, who studied the carotid artery of the dog. They found an attenuation coefficient of about 0.011 neper/cm at 3 Hz, which is close to our value of 0.013 nepers/cm at 2.6 Hz (Table 2). Their coefficients, like ours, were larger than those in the models at all frequencies. At the high end of the spectrum, the value of 0.076 nepers/cm observed at 12.7 Hz in our experiment is near the 0.080 nepers/cm reported by Anliker and his colleagues for artificially induced waves of 40 Hz ($\alpha = 50$). Attenuation in vivo is therefore not only much greater than theoretically predicted in the range $\alpha = 3-11$, but apparently remains on a high plateau for larger values of $\alpha$. 
Longitudinal Impedance

Several investigators have reported that the resistance of blood vessels in vivo is higher than would be expected from the resistance term in the Womersley maximally constrained model. Longitudinal impedances in vivo are also greater than this model predicts, as is evident in the studies of Fry and his colleagues on the canine aorta, and in those of O’Rourke and Milnor on the canine pulmonary artery. In both studies, \( \alpha > 12 \) and the resistive (real) component of \( Z_L \) was found to be much higher than the value calculated from the Womersley model, whereas the reactive (imaginary) component was close to the theoretical value. The present experiments show the same kind of results for \( \alpha < 12 \) (see Fig. 3). In spite of this error, the theoretical \( Z_L \) modulus is approximately correct for large values of \( \alpha \), because the resistive component is small in comparison to the reactive component. This fact accounts for the success of pressure-gradient methods of estimating instantaneous blood flow in the aorta. 

Our results indicate, however, that the accurate prediction of \( Z_L \) moduli by linear models fails when \( \alpha \) falls below about 6, i.e., in vessels small enough, or at frequencies low enough, to yield such values. This point is illustrated in Figure 4, which shows \( Z_L \) moduli predicted by the Jager model together with those observed in the present experiments and by other investigators. “Normalized” longitudinal impedance moduli (\( Z_L \) modulus/Poiseuille resistance) have been used to allow comparison of vessels of different diameters. The experimental data for \( \alpha = 0 \) in Figure 4 were reported by Ling and his colleagues, who made very careful measurements of resistance in the canine aorta. For \( \alpha > 0 \), theoretical values in the Cox model (not shown in Fig. 4) were 4-9% smaller than those in the Jager model. It is evident from this figure that the normalized \( Z_L \) modulus in vivo deviates from the theoretical curve below \( \alpha = 6 \). We conclude that the use of linear models to compute flow from measured pressure gradients in situations where \( \alpha \) is less than 6 will overestimate both mean and pulsatile blood flow.

Source of the Discrepancies

The present experiments do not provide an explanation for the differences between observation and theory, but these differences must have their origin in phenomena that are not represented in the models. “Linearized” models are derived by neglecting nonlinear terms in the Navier-Stokes equations, and in blood vessels one or more of these terms may be hemodynamically significant. The largest terms in this category are those that express convective acceleration, as Ling and Atabek have pointed out. The natural anatomic tapering of arteries and changes in diameter with traveling pressure waves would both contribute to such acceleration. In effect, the convection terms must be balanced by a part of the pressure gradient, and the ratio of pressure gradient to velocity of blood flow is consequently not correctly expressed in “linearized” equations. In arteries, where narrowing progresses in the same direction as flow, the pressure gradient would be greater for a given flow than it would be in a cylindrical tube. The longitudinal impedance measured under such conditions would be larger than purely linear models predict, and in a predominantly elastic system this error would appear mainly in the resistive component of \( Z_L \). Arterial tapering could thus account for the differences shown in the upper panel of Figure 3. Unfortunately for such an explanation, there was very little tapering in the arteries we studied. Measurements with a dissecting microscope and eyepiece micrometer revealed an angle of taper less than 1°, which was about the resolution of the method. Very

![Figure 4](http://circres.ahajournals.org/doi/fig/10.1161/01.CIR.62.3.777)
small degrees of taper may be hemodynamically significant, however. Atabek and his co-workers found less than 1° of taper in major coronary arteries of the dog; yet approximately one-third of the pressure-gradient was used to balance the effects of convective acceleration. None of the other nonlinear terms in the Navier-Stokes equations appear to be capable of producing effects of such magnitude. The assumptions used to justify omission of these other terms are probably valid in blood vessels, the principal assumptions being that radius is very small in comparison to wavelength and that phase velocity is much larger than blood velocity.

The hemodynamic effects of certain other arterial properties that are not taken into account in linear models cannot be evaluated at present because a comprehensive theoretical analysis of these properties is not available. Wave reflection is neglected, for example, in the most detailed nonlinear theory that has been tested in vivo, an omission that is warranted for investigation of velocity profiles and wall shear at a single vascular cross-section, but not in the study of wave propagation. Even in short arterial segments, the propagation coefficients are modified by reflected waves. Other examples include the anisotropy, nonuniform elasticity, nonlinear compliance, and curvature. The experiments of Patel and his colleagues show that the arterial wall is not isotropic, but their results suggest that the anisotropy that was observed had little effect on wave velocity. The arterial tree becomes less compliant as it extends toward the periphery, and this nonuniformity accounts in part for amplification of the pressure wave along the length of the aorta, but it was probably not significant in the short arterial segments that we studied. The arterial elastic modulus is unquestionably nonlinear in the sense that it increases as the vessel is distended, but published data on this nonlinearity on wave propagation is probably small in comparison to wavelength and that phase velocity is much larger than blood velocity. The assumptions used to justify omission of these other terms are probably valid in blood vessels, the principal assumptions being that radius is very small in comparison to wavelength and that phase velocity is much larger than blood velocity.

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Conclusions

1. Propagation coefficients calculated by linear model equations from measurements of arterial viscoelasticity are smaller than those observed in vivo. In a blood vessel of given dimensions and viscoelasticity, the attenuation is consequently greater than that predicted by linear models. Phase velocity is correctly predicted by the Cox model, but overestimated by the equations of Womersley and Jager.

2. Linear models tend to underestimate the resistive component of longitudinal impedance, and this error becomes larger as α increases. The modulus of the theoretical longitudinal impedance is affected to the greatest extent at low values of α (<5). As a result, at normal heart rates in arteries less than 6 mm in diameter, the use of linear models to compute blood flow from differential pressure will give inaccurate (false high) results for pulsatile as well as average flow. In larger arteries, only the average flow is seriously in error.

3. Calculations of characteristic impedance based on linear models and measurements of arterial viscoelasticity are significantly larger in modulus than those observed in vivo.

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References

18. Anilker M, Histand MB, Ogden E: Dispersion and attenua-
A Solid Angle Analysis of the Epicardial Ischemic TQ-ST Deflection in the Pig
A Theoretical and Experimental Study

J. Franklin Richeson, Toshio Akiyama, and Eric Schenk

SUMMARY The solid angle theory has long been applied to the interpretation of the ECG; however, quantitative evaluation of its applicability has been minimal. We applied solid angle analysis to the interpretation of the TQ-ST deflection during acute ischemia. Five anesthetized pigs were studied 1-3 hours after coronary occlusion. Multiple unipolar tracings were recorded from precisely determined positions on the epicardium overlying the ischemic and normal zones. The geometry of the hearts and ischemic zones was preserved, the margins of ischemia being defined as the outer border of Thioflavin S nonfluorescence. Wax replicas of the hearts with ischemic zones removed were constructed. Solid angles subtended at electrode positions on the replicas were calculated. A linear relationship ($r = 0.84 - 0.97, P < 0.001$) was shown to exist between the observed TQ-ST deflection and the calculated solid angle. A small but patterned deviation from exact fit of the TQ-ST deflection with the calculated solid angle led us to investigate the possibility that dipole moment strength ($\Phi$) is not confined to the ischemic margins. Computations using idealized heart models with ischemic zones cylindrical and transmural in shape allowed us to distribute $\Phi$ arbitrarily within the ischemic zone, comparing this predicted pattern of TQ-ST deflection with that observed experimentally. The experimental data appear most consistent with the condition in which $\Phi$ is distributed over a 1-cm border region during the first several hours of ischemia. We conclude that the solid angle theory provides a rational basis for interpretation of the ischemic TQ-ST deflection; however, $\Phi$ may be distributed over a marginal zone.

Such interpretations have been challenged by the work of Holland and associates, as well as by data from Cohen and Kirk and Heng et al. Holland applied the solid angle theory to the electrical events resulting from transmembrane potential alterations within the ischemic tissue and suggested that factors other than the extent of the ischemic region (e.g., geometry of the ischemic zone, degree of alteration of action potential morphology, changes in intra- and extracellular conductivity) are influential in determining the extent, polarity, and magnitude of the resulting TQ-ST deflection.

The solid angle theory rests on incontrovertible physical principles and was applied to the interpretation of electric fields by Gauss and Maxwell. Its applicability to the interpretation of the electrocardiogram was proposed over 40 years ago. In this context, the theory is a means of relating the potential created by the electromotive surfaces to that sensed at surrounding points in a uniform...
The relation between arterial viscoelasticity and wave propagation in the canine femoral artery in vivo.

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