The Equivalent Strap Muscle of the Left Ventricle

A Model for Describing Myocardial Force and Length, and Mean Endocardial Fiber-Shortening Rate in Man

COLIN GRANT

SUMMARY The force exerted by cardiac muscle usually has been estimated as wall stress, i.e., force per unit area of cross-section. This calculation fails to allow for the decrease in number of muscle fibers per square centimeter as the fibers shorten and become thicker during systole. To describe better the behavior of actual muscle fibers, a model composed of fibers has been identified and examined. It permits the fibers forming a left ventricle to be uncurled into a straight or strap-shaped muscle equivalent to the actual ventricular muscle mass. This transform allows quantitative conversion of volumetric and pressure data into linear data of force, velocity, and length (more exactly: equivalent myocardial force, mean endocardial velocity of fiber-shortening, and equivalent endocardial fiber-length). Data from 17 subjects (fifteen with coronary disease) show the left ventricles equivalent to strap muscles exerting a maximum force of 2.1 to 4.8 kg and having end-diastolic lengths of 18 to 24.5 cm. Corresponding end-systolic lengths range from 11 to 22 cm and fiber shortening rates from 0.52 to 1.69 lengths/sec. Algebraic analysis, for spherical and elongated (cylindrical) shapes, suggests that different fiber arrangements in the walls may be equivalent to each other. The inward force exerted by each circular loop of fiber is the same, regardless of diameter, so it is possible that the outer and inner fibers of a thick-walled model may make equal contributions to myocardial force (although, of course, the outer fibers will be longer).

THE MECHANICAL behavior of straight or strap-shaped skeletal muscle has been well described in terms of the classical force-velocity analysis. Similar analysis has been applied to cardiac papillary muscle.1-2 A corresponding approach has been applied to the circumferential (equatorial) fiber band of the human left ventricle in vivo.3-5 However, this circumferential muscle belt is not independent of the longitudinal fibers' behavior and there has not previously existed any technique for describing the behavior of a complete ventricle in force-velocity terms. The simplified model analyzed here consists of distinct muscle fibers, rather than sheets or masses of myocardium, and it attempts to develop a format within which the concepts of force-velocity-length analysis may become more directly useful in analyzing the clinical situation.

The main model analyzed is a spherical chamber whose wall consists of fiber loops arranged in all possible great-circle (i.e., geodesic) configurations, to cover it completely. However, in Appendix 2, this model is shown to be equivalent to any network of fibers covering the wall. It will also be shown (Appendix 3) that, for an elongated chamber (at least in the extreme case of an infinitely long cylinder), the arrangement of fibers in the wall does not affect the amount of fiber-material required to generate and maintain internal pressure. Spiral fiber geometry requires the same amount of fiber as does a circular-plus-longitudinal fiber arrangement, or any other more complex and realistic arrangement, or any combination of fiber geometric patterns.

The study considers the forces at the endocardial surface of the ventricle. Wall stress is not calculated, and wall thickness is not measured. Surface tension (grams weight per centimeter of surface length) is not considered at all. The force (tension) in a fiber, in all the fibers, and produced inward by a fiber loop, is expressed in grams weight or kilograms.

This analysis therefore departs substantially from previous more classical engineering-type analyses of cardiac mechanics by Mirsky6 and others. Limitations imposed by the wall-thickness simplification and other factors are considered in the Discussion.

Theory

The total inward force exerted by a tensed circular loop of muscle or other material is equal to 2 \( \pi T \) (Fig. 1A), where \( T \) is the force or tension in the fiber loop (Appendix 1) expressed in grams weight, or kilograms, rather than in dynes. If a large or nearly infinite number of such loops are combined (all having the same size and center, and being randomly or uniformly distributed), they will form the wall of a spherical cavity. With \( N \) such fibers, the total inward force will be 2 \( \pi NT \). (The cavity volume will of course be \( 4 \pi R^3/3 \), where \( R \) is radius of the sphere). This inward force is opposed by the outward pressure force, calculated as the product of pressure and surface area of the spherical cavity, \( P \cdot 4 \pi R^2 \). These outward and inward forces may be equated if the small inertial components (acceleration of myocardium and blood) are neglected.

\[
P \cdot 4 \pi R^2 = 2 \pi NT \quad (1)
\]

\[
2R^2P = NT \quad (2)
\]
Strap muscle of the left ventricle, produced by cutting each great-circle fiber loop once and placing the straightened muscle fibers parallel to each other, so that length and force can be calculated.

If now all the fiber rings are cut once and straightened into a bundle of parallel fibers, a straight muscle will be produced, with length \(2\pi R\); the total force of shortening will be NT, since the muscle consists of N fibers each contributing tension T (Fig. 1B). Thus although neither N nor T is defined, the total force NT of this straight muscle, this “equivalent strap-shaped muscle” has a perfectly definite value which can be calculated from Equation 2. The radius R is an average radius for the left ventricular cavity, calculated as radius of the sphere equal to the actual left ventricular cavity volume. The real left ventricle of course consists of a network of fibers, and it can be shown (Appendix 2) that a branching network of fibers forming a sphere (a more realistic model) is exactly equivalent to the model described above, in which the fiber loops surround the sphere completely, in all the possible great-circle positions.

Analysis of an elongated model, using the extreme case (an endless cylinder, Appendix 3), also shows that simple fiber arrangements are equivalent to all other mathematically possible networks, including those containing complex arrangements of spiral fibers. The possible biological significance of this will be referred to in the Discussion.

The dimensional validity of Equations 1 and 2 depends on the use of T for actual force (mass × length/time squared) rather than surface tension (mass/time squared).

Methods

Angiocardiographic data were examined for a consecutive series of patients undergoing left ventriculography. Seventeen cases were selected with technically adequate angiocardiograms (freedom from premature beats, or a well-outlined beat following a postextrasystolic beat). Left ventricular volumes were calculated by the single-plane area-length method or the single-plane length-width method,6,9 using ventriculograms taken in the 30-degree right anterior oblique projection (RAO).

Magnification was determined by photographing an engraved steel centimeter ruler placed 10 cm above the table top.

DHEW and institutional rules for protection of human subjects were complied with in compiling data for this study.

The ventriculograms were recorded at 30 or 60 frames/sec, and each exposure was recorded (for timing purposes) on the Electronics for Medicine recorder, together with the electrocardiogram. In five cases, the simultaneous radial artery pressure also was recorded. The ventricular pressure corresponding to each measured cineangiogram frame was identified from left ventricular (LV) pressure recorded just before angiography, using the time from R wave peak to the particular frame to identify the pressure at the same time interval on the previously recorded LV pressure. Volumes and pressures were measured from near the onset of mechanical systole (pressure at two-thirds of its peak value) until the peak of tension had clearly been passed, or until peak LV pressure was reached. In addition, maximum and minimum volumes were determined by averaging the two largest and two smallest frames in the study beat, as well as, when possible, maximum and minimum volumes in the previous and subsequent beats. For the equivalent tension determinations, average radius R was calculated (for each frame) by using the sphere volume formula (V = 4/3 \(\pi R^3\)) and the ventricular volume. Equivalent tension (NT) then was calculated from formula 2, \(NT = 2R^2P\), where pressure P was expressed in grams per square centimeter (mm Hg, multiplied by 1.36). The peak of equivalent tension was identified from the plot of this measurement rising and then beginning to fall. The calculated value (grams weight of force) was converted into kilograms for tabulation and recorded as maximum force of the equivalent strap muscle.

Maximum and minimum lengths, L, of the equivalent strap muscle were calculated from the averaged radius, R, using the formula \(L = 2\pi R\), so that the equivalent strap muscle length of this model is the endocardial circumference (for the sphere isovolumic with the actual left ventricle volume). Percent shortening of the average fiber in the sphere was calculated for each patient, and ejection time was measured from either the simultaneous radial pressure trace or the preceding central aortic pressure trace. Velocity of mean fiber shortening was then calculated as percent shortening divided by ejection time, divided by 100 (example: 20% shortening in 0.3 second = 0.67 length per second). The effects of using endocardial surface data and neglecting wall thickness are assessed later, in the Discussion.

Cardiac index was calculated from stroke volume (angiocardiographic) and heart rate. Patients with mitral regurgitation or atrial fibrillation were not studied.
Results

All of the seventeen subjects were male and are described in Table 1. Fifteen had significant coronary artery disease with 1, 2, or 3 vessels showing significantly more than 50% narrowing in lumen diameter; one patient had mitral stenosis (case 2, 11-mm gradient), and one had a normal heart with an undiagnosed chest pain syndrome (case 1).

The hemodynamic data (Table 1) show a range of ejection fractions from 34% to 85%, and no correlation with the extent of the coronary artery disease. Velocity of mean fiber shortening ranged from 0.52 to 1.69 lengths/sec and correlated well with the ejection fraction, a directly related measurement of myocardial performance ($r = 0.95$ for fiber velocity and ejection fraction, using the data of Table 1).

Similarly, there is as expected a high correlation ($r = 0.94$) between equivalent myocardial force and end-diastolic volume, using Table 1 to calculate absolute end-diastolic volume. The effects of elongated ventricular cavity shape and of widely varying ventricular pressure on equivalent myocardial force are discussed later.

The shortest equivalent strap muscle (18 cm, case 11) exerted a maximum force of 2.4 kg (near the lower end of the force range), was operating well up on its force-velocity curve, and was able to shorten by 30%; these numbers are quite similar to those for the first case, the patient with a normal heart. The longest equivalent strap muscle (case 5) measured .4.5 cm (Fig. 2B). It was working at a maximum force of 4.2 kg, among the highest in the series. Fiber shortening during the time-duration of systole amounted to only 13% of initial length. This muscle was clearly operating far down on its force-velocity characteristic curve, with markedly reduced velocity of shortening and near-maximum force generation. Notably, the subject had only one major coronary artery lesion (right coronary occlusion); initially it was surprising to find such severe myocardial impairment. The subject was under treatment for hypertension, and the case illustrates use of the strap muscle approach to make a presumptive diagnosis of cardiomyopathy secondary to hypertension rather than to coronary disease.

![Figure 2](http://circres.ahajournals.org/)

**Figure 2**: A: The time-course of pressure and equivalent myocardial force in a patient with good ventricular contractility (ejection fraction, 71%) and normal ventricular size (78 ml/m$^2$). The rapid fall in equivalent force during ejection (while pressure remains high) is clearly shown. B: The time-course of pressure and equivalent myocardial force in a patient with poor ventricular contractility (ejection fraction, 34%) and an enlarged left ventricle (125 ml/m$^2$). Peak force is both higher and more sustained throughout systole. Symbols as in 2A.
The time-courses of pressure and myocardial force can be seen in the subject with good myocardial function (Fig. 2A) and in the subject with poor myocardial function (Fig. 2B). With good left ventricular emptying, myocardial force is maximal early on and falls rapidly, even while pressure is still rising. By contrast (Fig. 2B), an enlarged and poorly contracting ventricle is exerting 25% more force to produce a very similar pressure in early systole (4.2 kg vs. 3.0 kg). In late systole, the equivalent myocardial force has not fallen significantly (because of poor ejection and slight radius change); the more vigorous ventricle (Fig. 2A) has been able to lower its force by nearly 40% (from 3.75 to 2.3 kg) before pressure starts to fall significantly. It is clear that the tension-time product (or, rather, the force-time product) is increased as a result of the mechanical effect of chamber enlargement and also the effect of diminished ejection fraction (both mechanisms operating through the Laplace relationship between size and wall tension). The timing of maximum force (Table 1) was always in early systole, between 95 and 175 msec after the QRS onset. At this time, as tabulated, pressure was rising close to its peak, and rather slight myocardial shortening had taken place.

Discussion

The methods used represent a transform from volumetric and pressure measurements (whose interrelations can be somewhat elusive, and can be affected substantially by the Laplace relationship when chamber dilation supervenes) to the very elementary dimensions of length, shortening, and force. Transforms do not create new information, but they may markedly affect the accessibility and usefulness of the information. Equivalent force is a function of pressure and chamber volume, equivalent length is a direct function of volume, and velocity of shortening is a function of volume change and ejection time.

Wall stress has been regarded as an important parameter of cardiac mechanical analysis, but wall thickness changes markedly during systole, whereas the number of cardiac fibers remains constant. This difference between the biological situation and the usual engineering calculation of stress (where wall thickness changes only minimally) suggests that this measurement may not give the best information about changes in force development by myocardial fibers. Our earlier calculations of stress incorporating a correction for the change in wall thickness (fiber-corrected stress) have confirmed this expectation. The present model bypasses the problems associated with wall thickness measurement. However, by using the endocardial surface as the study surface, it does calculate shortening velocity and equivalent length for an average of the endocardial fibers. A similar measurement (endocardial shortening velocity of circumferential fibers) has been validated by Bristow et al. and also by Karliner et al. The latter compared this mean velocity with maximum velocity of shortening for midwall fibers. Differences between subjects were highly correlated, although absolute values for the endocardial mean velocities were consistently lower. It is expected that a mean velocity will be lower than a peak velocity; the data imply that this effect is larger than the apparent velocity-raising effect of calculating with endocardial rather than midwall dimensions; i.e., it is suggested that the lack of wall thickness measurements is not a serious limitation. If this be accepted, then the ability to dispense with wall measurements is a major asset of the method.

The effect of a thick wall on wall force calculations is discussed by Mirsky in references 6 and 13. This complex problem will be addressed here briefly, and only in relation to its effect on the strap muscle model. With actual thick walls, the equivalent strap muscle length will be greater for mid-wall and outer-wall shells, and the calculated velocity of shortening will be less. Karliner et al. did calculate endocardial and mid-wall velocity of shortening, and demonstrated the expected differences in absolute numbers but not in ranking between subjects.

However, the additional length of the outer-wall fibers in a strap muscle model (thick-walled) is very large. The outer-wall "strap" may be 30% longer in diastole and twice as long at end-systole, or even more discrepant in pathologically thick-walled ventricles which empty nearly completely. The alternative approaches to be tested in the future include calculating the inner-wall length (as here) while bearing in mind its limitations, calculating a mid-wall length (approximately the average), or constructing a more realistic "strap" muscle with an oblique end, including both short and long fiber-shell components.

The principal model used here is spherical, but the finding in Appendix 3 (that fiber geometry is unimportant in cylinders, as well as in spheres) suggests that the ellipsoid could have the same property and that the present approach may be valid for this shape.

Since total force is calculated as the product of endocardial surface area and internal pressure, the increased area of an ellipsoid (prolate spheroid), compared with the isovolumic sphere, is potentially important. The excess area calculates out to be surprisingly small; an ellipsoid with long axis double the short axis (a common clinical ratio) has only 7.7% more surface area than the isovolumic sphere. To maintain conceptual simplicity, no correction for this has been applied, although the data exist to make such a correction.

The use of pressure recorded just before angiography (rather than simultaneous intraventricular pressure) is virtually standard practice in clinical work and has been reasonably well validated. In principle, the Millar angiographic catheter-tip manometer could avoid this potential error, but limitations on flow rate (and maximum permissible pressure) of the contrast injection make this a suboptimal technique.

The difference between pressure-loading and volume-loading the ventricle is well appreciated by physiologists. Calculations of wall stress have made it clear that the extra force imposed on myocardium as a result of chamber dilation (and the Laplace relationship) can be equivalent to a pressure overload. However, this concept may become both more widely appreciated and more exact if equivalent force and equivalent length become readily available measurements. The extent to which, for example, aortic regurgitation may be a force overload as well as a volume or length overload may help to clarify...
the true amount of extra burden and the point at which compensation becomes inadequate. In Table 1, case 5 illustrates well a subject who has a normal blood pressure but who probably is suffering from an uncompensated force overload, as a result of chamber enlargement.

Another apparent limitation in the model is its generalized nature. Coronary disease of course is a multifocal process, frequently with localized effects on several parts of the left ventricle. Two comments can be made in defense of applying the model to such subjects; the ejection fraction, a measurement directly related to fiber shortening, is open to the same criticism but has, nonetheless, become valuable and widely used in assessing myocardial damage from coronary disease. Second, it would be reasonable to set up a strap muscle model in which some fraction of the “muscle length” is akinetic or non-contractile; the problem of obtaining data for such a model has not yet been addressed, although some promising conceptual approaches exist, based on the quantitation of left ventricular aneurysm size.20-22

The inward force exerted by a small element of the circular loop, which subtends a small angle dL at the center of the circle (Fig. 3), can be calculated: dF = T sin dL (Appendix 1).

\[ dF = T \sin dL \]

Thus the only determinants of inward force are the total tension T and total angle (expressed in radians).

The triangle of forces for this element is drawn in Figure 3, and it shows the inward force dF to be equal to T sin dL. For a small angle, sin dL will equal dL (expressed in radians).

\[ \text{Segment inward force} = dF = T \sin dL \]

\[ = T \cdot dL \]

Summating or integrating for the total inward force from every segment of the circular loop, since there are 2π radians in the circle, the total inward force (product of tension T and total angle) will be 2πT.

\[ \text{Circle inward force} = \int T \cdot dL \]

\[ = T \int dL \]

\[ = T \cdot 2\pi \]

\[ = 2\pi T \]

**Appendix 2**

The Equivalence of the Spherical Great-Circle Fiber Model to All Possible Networks on Nonrigid Fibers Covering the Surface of a Sphere

The simplest possible network is constructed of three semicircles of muscle fiber round the sphere, joined at “north pole” and “south pole,” as in Figure 4. Each of these half-circles will exert half the inward force of a complete loop (πT) so that total inward force will be 3πT, exerted by the equivalent of 1 1/2 great-circle loops. Similarly, a quarter-circle fiber segment will exert one-quarter of the inward force developed by a complete loop, and (by extension) every short segment of a complete network will exert inward force proportional to its length. Thus the only determinants of inward force are the total length of fiber in the wall and the tension T in each fiber. T is defined as equal for every fiber, and real fibers with differing tensions can each be considered equivalent to a number of finer weaker fibers. Thus the great-circle fiber model, with a total fiber length of N 2πR, is equivalent to every possible branching network with the same total length of fibers.
Appendix 3

The Equivalence of Different Fiber Arrangements in an Elongated Structure, a Cylindrical Model

There are two simple ways in which an elongated high pressure cavity (such as the lumen of a fire hose) can be contained by a textile material composed of tense fibers. One fiber arrangement will contain circular and longitudinal fibers (resisting the outward and longitudinal forces, respectively). The other will consist of spirally wound fibers (clockwise and counterclockwise interwoven) so that each fiber resists both the outward and the longitudinal forces.

Surface tension in a distended cylinder is twice as great in the circular direction (which would be cut by incision parallel to the long axis) as it is in the longitudinal direction. The most efficient spiral fibers will therefore be wound, not at 45 degrees to the long axis (which would generate equal components of longitudinal and circular force), but close to the circular fibers, at such an angle (actually 63 degrees from the long axis) that fiber tension can be resolved into a circular component twice as great as the longitudinal component. Figure 5A shows the two fiber arrangements, while figure 5B shows the triangle of forces for one spiral fiber which has been straightened to permit graphic representation on paper. Figure 5C, superficially resembling 5B, shows the actual lengths of longitudinal ($L_1$) and circumferential ($L_2$) fibers required to do the same job as the spiral fiber $L_3$.

The total amount of fiber material required to do a particular job is of course the product of length and tension, $LT$. From the two figures it can be seen that LT for the spiral fiber ($L_3T_3$) is $\sqrt{5} L_1 \times \sqrt{5} T_1$, or $5 L_1 T_1$. Similarly, the LT products for longitudinal and circular fibers are $L_1 T_1$ and $4 L_1 T_1$, totaling $5 L_1 T_1$; so that the single spiral fiber is mechanically equivalent to the combination of circular and longitudinal fibers doing the same job. The spiral fiber, although shorter in total length than the two nonspiral fibers which are equivalent to it, is under greater tension; these two effects being equal, the length-tension product is the same, and the total amount of fiber material needed is the same, whether it be woven spirally or in a longitudinal and circular meshwork.

Similar diagrams and calculations for a spiral mesh of fibers at an angle of 45 degrees (with the necessary additional circular fibers) show the same equivalence in the amount of fiber material needed. This also holds true for other possible angulations of spiral fibers. By applying reasoning similar to that of Appendix 2, it appears that every mathematically possible fiber network or arrangement encompassing an endless cylindrical cavity will require the same amount of fiber material. Just as in the spherical model, the fiber arrangement appears to be unimportant, with spiral fiber segments being resolvable into their longitudinal and circular components.

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