Inverse Calculation of QRS-T Epicardial Potentials from Body Surface Potential Distributions for Normal and Ectopic Beats in the Intact Dog

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SUMMARY Inverse epicardial potential distributions were calculated from potential distributions measured from the body surface in 12 intact dogs, divided into six pairs. The calculation procedure made use of measurements from the initial dog of each pair, giving the geometric location of each epicardial and body surface electrode and the approximate variance of the epicardial potentials and of the body surface noise. The same calculation procedure subsequently was applied to the other dog of the pair. For all dogs, inverse solutions were calculated throughout QRST for several sequences of excitation and repolarization which were produced by stimulating singly and in pairs any of eight ventricular sites. All calculated inverse results were checked by detailed quantitative comparison to the corresponding measured epicardial potential distributions, which were obtained from chronically implanted epicardial electrodes. The root mean square (RMS) numerical differences between the inverse computed and the measured epicardial distributions were a substantial fraction of the RMS measured epicardial voltages, often 0.7 or more, and the correlation coefficients between measured and computed epicardial distributions were in the range 0.6-0.8. Nonetheless, the major epicardial electrical events of excitation and repolarization easily were seen in all of the inverse maps in the initial dogs of each pair, and with only slightly less clarity in the subsequent dogs. Events readily observed included the initial minimum around the stimulus site as excitation began, the movement of the zero contour line across the epicardium as excitation progressed, and the characteristic pattern of repolarization with a maximum near the stimulus site. The results demonstrated that specific physical features of epicardial potential distributions can be determined from body surface measurements and can be verified by comparison with direct epicardial measurements.

THIS STUDY considered the question of whether potential distributions on the epicardium ("epi maps") of intact dogs can be calculated from recordings of potential distributions measured from the body surface ("body surface maps"). Previous inverse calculations of cardiac electrical events from body surface measurements have been based predominantly on representing the heart by means of multipoles or multiple dipole current generators, as extensively reviewed by McFee and Baule. We have used instead a model based on epicardial potential distributions because of their numerous advantages with present day experimental and computational methods. Particular impetus toward studying the characteristics of inverse solutions in terms of epicardial potentials has been provided by recent experimental studies of Spach and coworkers which showed that epi maps were related readily to the underlying intramural extracellular potential distributions and could be computed quantitatively from intracellular potential distributions during the T wave. These studies demonstrated that the epi maps, which at many times were rather complex, contained a considerable amount of information about the underlying intracellular and extracellular intramural cardiac electrophysiological state. This information obviously would be of great value if it could be obtained from body surface measurements. However, comparison of epicardial and body surface maps during normal and abnormal beats in the chimpanzees and in forward simulations in dogs also showed that maps observed from the body surface frequently were much smoother and simpler than the epi maps, a point which had been demonstrated much earlier by Taccardi in tank studies using turtle and dog hearts. The relative simplicity of the body surface maps raised the question of how much significant information had been lost in transit through the volume conductor; at the same time, the relative simplicity of the body surface maps made the value of an inverse calculation procedure for reconstructing the epi maps from body surface measurements more interesting and more valuable, if it could be achieved, since direct interpretation of the electrical events within the heart was not possible from the body surface maps in a fashion similar to that possible from the epi maps.

The potential distributions measured and used for the calculations of this report were those from the intact dog preparation. The excitation and repolarization sequences examined were produced by stimuli applied to either one or two ventricular sites. Both the electrodes used for stimulation and other electrodes used to record the epicardial potential distribution were chronically implanted. The preparation was well suited to providing a good test of the method of inverse calculations since it was possible to measure potentials at a large number of body surface and epicardial electrodes almost simultaneously through QRS-
T under stable conditions, (2) since numerous different sequences of depolarization and repolarization were obtained in the same dog by applying stimuli to a variety of ventricular sites, and (3) since the measured epicardial distributions provided a quantitative check of the computed results.

The method used for calculating the epi maps was based on finding "inverse transfer coefficients," a set of numbers showing the relationships between voltages at each body surface electrode and voltages at each epicardial one. The transfer coefficients were determined from measurements of the geometric coordinates of electrodes on the epicardium and on the body surface and from a priori values for the variance of the epicardial potentials and the variance of the body surface noise. In initial calculations, the geometric measurements and a priori values were determined from the same dog from which the epicardial and body surface potentials were measured. However, in a second set of calculations, inverse epi maps were computed and compared to the measured ones for dogs different from the one used to derive the transfer coefficients, so that the procedure tested was one that could be used with any dog, given only measurements of body surface potentials.

Methods

Experimental Measurements

Data from 12 dogs were used. Potential distributions from all dogs were obtained by the same experimental procedure. In summary, this procedure consisted of implanting surgically about 75 electrodes over the ventricles and atrium, with insulated wires for each one extending subcutaneously to the lower abdomen. About 2 weeks, when the body surface potentials had returned to normal, potential distributions were recorded from the body surface (150 leads) and epicardium (75 leads) in the intact dog, the epicardial leads having been connected via a small abdominal incision. Potentials were recorded in groups of 20 leads plus four references at a time, with a sampling rate of 1000 samples/channel per sec. Measurements were made with respect to Wilson's central terminal. About 20 minutes were required to record waveforms from all 225 epicardial and body surface electrodes for a given pacing site; waveforms from 6 to 12 sequences were recorded in each dog.

In six dogs, postmortem measurements of the geometric coordinates of each body surface and epicardial electrode were obtained, using a procedure developed previously for forward simulations. This procedure consisted of (1) freezing the dog's carcass without changing its position from the time at which the potentials were recorded, (2) slicing the carcass into serial sections about 2 cm thick, (3) photographing the slices, and (4) determining the coordinates of each electrode by combining measurements from the photographs with measurements on the slices themselves. Forward transfer coefficients were computed from the measured coordinates of the epicardial and body surface electrodes, assuming that the intervening volume conductor was homogeneous.

Initial analysis was conducted using data from pairs of dogs. In each pair, for one dog, geometry and potentials were measured and, for the other dog, only potentials were measured. Forward and inverse transfer coefficients were found for the first dog from the geometric coordinates; the performance of these transfer coefficients then was tested both on the potentials from this dog and subsequently on the potentials from the other dog of the pair. It quickly became clear that variations between pairs of dogs were much less significant than from one sequence of excitation and repolarization to another or even between the initial and subsequent dogs within a pair. Thereafter, calculations and comparisons were confined to only two pairs of dogs. Since the results from each pair were quite similar, results from only the first and second dogs of a single pair are presented below.

Mathematical Methods

The mathematical method that we used for inverse calculations was a further development of mathematical methods used previously for forward simulations. These simulations modeled the way in which body potentials were produced in the dog from epicardial potentials, using the equation:

\[ Z_{HH} \Phi_H^M = \Phi_B^S = \Phi_B^M + E_B \]  

In Equation 1, the "forward transfer coefficients," \( Z_{HH} \), multiply heart potentials \( \Phi_H^M \) to produce body surface potentials, \( \Phi_B^S \), which differ from the measured body potentials, \( \Phi_B^M \), by error \( E_B \). Each quantity in Equation 1 is a matrix. Matrix \( Z_{HH} \) has a number of rows equal to the number of body surface electrodes, \( N_{BL} (=150) \), and a number of columns equal to the number of heart (epicardial) electrodes, \( N_{HL} (=75) \). \( \Phi_H^M \) has \( N_{HL} \) rows and a number of columns equal to the number of time instants, \( N_T \). \( \Phi_B^S \), \( \Phi_B^M \), and \( E_B \) have \( N_{BL} \) rows and \( N_T \) columns.

The inverse calculation was represented by Equation 2:

\[ Z_{BH} \Phi_B^M = \Phi_B^S = \Phi_B^M + E_H \]  

In Equation 2, a matrix of measured body potentials, \( \Phi_B^M \), is multiplied by a matrix of "inverse transfer coefficients," \( Z_{BH} \), to compute simulated epicardial potentials, \( \Phi_B^S \). These simulated epicardial potentials are equal by definition to the corresponding measured epicardial potentials, \( \Phi_H^M \), plus the error in the heart potentials, \( E_H \). Matrices \( \Phi_B^S \), \( \Phi_H^M \), and \( E_H \) all have the same number of rows as the number of heart electrodes, \( N_{HL} \), and the same number of columns as the number of time instants used (number of maps), \( N_T \). Matrix \( Z_{BH} \) had \( N_{HL} \) rows and \( N_T \) columns.

The magnitudes of the individual elements of matrix \( Z_{BH} \) determined the weightings given to each body surface potential in computing each epicardial potential. It was approximately true that each epicardial electrode had associated with it \( Z_{BH} \) values that were higher in magnitude for the closer body surface electrodes and lower for more distant body electrodes. A problem in determining specific values for the elements of \( Z_{BH} \) was to take into account correctly the effects of the torso volume conductor.
In which smaller potentials were present on the body surface at further distances from the epicardium, without accentuating the effects of noise in the body surface recordings to an excessive degree. It is important to realize that, in this context, "noise" was any deviation between the potentials that would have been calculated on the body surface in a forward simulation from the correct (measured) epicardial potentials. As such, noise incorporated the effects of any errors in the model relating epicardial to body surface potentials, e.g., differences in geometry between the model and the real volume conductor. Therefore the noise was considerably greater than the relatively small electronic noise of about 30 μV peak-to-peak that was visible on the measured body surface waveforms.

To find values for the elements of matrix $Z_{\text{BH}}$ that would best estimate the epicardial potentials in the presence of noise, we used the mathematical results of Foster and Strand and Westwater that minimized the sum-squared error of the epicardial potentials. Thereby, $Z_{\text{BH}}$ was calculated by the equation,

$$Z_{\text{BH}} = (\Phi_{\text{HC}} + Z^T_{\text{BH}} E_{\text{BC}} Z_{\text{BH}})^{-1} Z_{\text{BH}} E_{\text{BC}}.$$  \hspace{1cm} (3)

In Equation 3, $\Phi_{\text{HC}}$ is the covariance matrix of the epicardial potentials and has $N_{\text{HL}}$ rows and columns. $E_{\text{BC}}$ is the covariance matrix of the body surface noise (errors) and has $N_{\text{BL}}$ rows and columns. Superscripts $T$ and $-1$ signify transpose and inverse, respectively. In obtaining Equation 3, we assumed a mean of zero for the epicardial potentials.

In using the Foster and Strand and Westwater mathematical results, we were mindful of the previous use of these results by Martin, who computed potential distributions over a sphere between the torso and the heart from infinite-medium body surface potentials, and who found the results encouraging. In addition, Martin et al. examined the effects of various errors in inverse calculations with statistical constraints and concluded that, although their feasibility study did not imply that one can actually compute values for $Z_{\text{BH}}$ and execute a series of inverse calculations showed that the various sources of errors will not present insurmountable difficulties.

In order to avoid having to specify all of the a priori information required in Equation 3, namely, the covariance matrices for the epicardial potentials and body noise, we assumed that the potentials at each epicardial electrode had the same RMS magnitude and were uncorrelated with each other as considered over a long series of maps. Similarly, we assumed that over many maps the noise had the same RMS magnitude at all body surface electrodes and was uncorrelated. These assumptions were expressed in equation form as

$$\Phi_{\text{HC}} = \phi_{\text{HC}} I_{\text{H}}$$  \hspace{1cm} (4)

and

$$E_{\text{BC}} = \epsilon_{\text{BC}} I_{\text{B}}$$  \hspace{1cm} (5)

where $\Phi_{\text{HC}}$ and $\epsilon_{\text{BC}}$ are the mean square values assumed for the epicardial potentials and body noise. $\Phi_{\text{HC}}$ and $E_{\text{BC}}$ are scalars and $I_{\text{H}}(I_{\text{B}})$ is an identity matrix with $N_{\text{HL}}(N_{\text{BL}})$ rows. Using Equations 4 and 5 in Equation 3 and rearranging gave the simplified form used for the calculation,

$$Z_{\text{BH}} = \left(\phi_{\text{HC}} I_{\text{H}} + Z^T_{\text{BH}} Z_{\text{BH}}\right)^{-1} Z_{\text{BH}}^T$$  \hspace{1cm} (6)

Note in Equation 6 that only the ratio $\epsilon_{\text{BC}}/\phi_{\text{HC}}$ appears. This has several significant connotations. First, only a single a priori value, the one for this ratio, is required to evaluate Equation 6. Second, the fact that only the ratio appears resolves an otherwise awkward situation that is present when applying Equation 6 to electrocardiographic problems. The difficulty is that the magnitudes of both the epicardial potentials and the body surface noise vary systematically over a wide range during QRS-T, thereby making it impossible to identify a single number for either $\Phi_{\text{HC}}$ or $E_{\text{BC}}$ that is a good approximation for all times. However, since the epicardial potentials and body surface noise tend to rise and fall together, as can readily be seen from forward simulations, the ratio remains roughly constant. Finally, inspection of Equation 6 shows that, as $E_{\text{BC}}$ becomes quite small relative to $\Phi_{\text{HC}}$, Equation 6 reduces to a form of matrix inversion of the forward transfer coefficients, $Z_{\text{BH}}$, the magnitude of the ratio $E_{\text{BC}}/\Phi_{\text{HC}}$ therefore determines the extent to which the use of the a priori values changes the calculation from a straightforward matrix inversion method.

A useful feature of this mathematical method is that, once $\Phi_{\text{HC}}$ and $E_{\text{BC}}$ are specified, the expected error can be computed prior to finding or using the inverse transfer coefficients, $Z_{\text{BH}}$. In particular, for our situation, the ratio of the sum-squared errors in the epicardial potentials divided by the sum-squared values of the epicardial potentials for given values of $\Phi_{\text{HC}}$, $E_{\text{BC}}$, and $Z_{\text{BH}}$, designated $E_{\text{R}}^2$, was determined by the equation

$$<E_R^2> = \text{Trace} \left[ \frac{1}{\Phi_{\text{HC}}} I_{\text{H}} + \frac{1}{E_{\text{BC}}} Z^T_{\text{BH}} Z_{\text{BH}} \right]^{-1} / (N_{\text{HC}} \Phi_{\text{HC}})$$  \hspace{1cm} (7)
homogeneous volume conductor between the epicardium and body surface. Resulting $Z_{HB}$ values were scaled by 0.69 to produce forward simulations of approximately the correct magnitude as well as pattern. $\Phi_{PB}$ and $\Phi_{PB}$ were assigned values of $(2.50 \text{ mV})^2$ and $(0.10 \text{ mV})^2$, values which were chosen after inspecting the measured epicardial potentials and resulting simulation errors in a series of forward simulations.

Execution of a PL/1 program to find the inverse transfer coefficients required 79 seconds on an IBM 370/165 computer system. The algorithm for the required matrix inversion used the Gauss elimination method as described by Ralston.\(^7\) Computation of inverse epicardial distributions from the corresponding measured body surface distributions by Equation 2 required less than 1 second for each map. For each millisecond, the measured body surface and epicardial potential distributions, along with the inverse simulated epicardial distributions, were displayed automatically as contour maps on a Tektronix 4002 graphic display in the format shown in Figure 1.

**Epicardial Grid Transfer**

Once computed, the inverse transfer coefficients found from measurements of geometry on the initial dog were used to compute inverse epicardial distributions both for that dog and for a subsequent dog. Use of the same $Z_{HB}$ with body potentials measured from another dog presented no procedural difficulty, since potential measurements were obtained from the same body surface grid of 150 electrode locations in all dogs. However, in comparing the computed epicardial distributions to the measured ones for the subsequent dog, it was necessary to take into account the fact that the computed epicardial distributions estimated potential values at the electrode positions of the initial dog rather than the subsequent one. This presented no difficulty when comparing the measured and computed results visually, since both grids covered the epicardium. To compare the inverse simulated vs. measured results numerically, however, a means for transferring results from the epicardial grid of one dog to another was required. The means we chose was to transfer the measured epi distributions of the subsequent dog from their original grid pattern onto the epicardial grid pattern of the initial dog. This transfer was achieved by visually examining the location of each electrode in the initial dog's grid and identifying which electrode in the subsequent dog's grid was in the most closely corresponding location, as judged by anatomical landmarks. Then, for each map considered, the voltage at each electrode location on the initial dog's grid was assigned as the voltage from the electrode on the subsequent dog's grid identified as the closest one. Note that this procedure was not required to execute the inverse calculation but only to check the results against the corresponding epicardial measurements. The effect of the grid transfer procedure was to produce numerical comparisons that always indicated somewhat more disagreement than probably was present; i.e., because of the grid transfer the accuracy of the inverse calculation was underestimated in the subsequent dog.

**Results**

The following results include measured and simulated potential distributions from several sequences of depolarization and repolarization for the two dogs of one pair, i.e., first for the initial dog whose geometry formed the basis of the $Z_{HB}$ and then for the subsequent dog for which only body surface potentials were available. More than one sequence is included from each dog since this allows the differences between the simulated and measured epi maps within a particular sequence to be evaluated relative to the differences between sequences. Sequences from both the initial and subsequent dogs are included since comparing the results from the two dogs provides an indication of how much the results are affected by assuming no change in torso and epicardial geometry from dog to dog.

Since only a few maps from a number of sequences of maps are included, these maps have been selected to best compare the simulated and measured epicardial maps; a more complete evaluation of similar measured maps with emphasis on their physiological origin and significance has been reported previously.\(^7\)

**Initial Dog, LV Stimulus Site (Figs. 2 and 3)**

The first sequence of potential distributions to be presented was produced by stimulating the posterior left ventricle (LV) near the base. Characteristics of sequences such as this one that followed stimulation of a posterior site were more difficult to discern from the body surface.

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**Figure 1** Format used to display body surface potential distributions (I) and epicardial potential distributions (II). Each potential distribution was displayed superimposed on the appropriate drawing. In panel I, the encircled letters A and P identify the body surface sites from which an observer would see approximately the same part of the epicardium as drawn in the anterior and posterior views, respectively, of panel II. Atr = atria; RV = right ventricle; LV = left ventricle.
than those for anterior sites, especially in early excitation and early repolarization, since the stimulus site was further from the body surface. As expected, after the stimulus, a region of negative potentials surrounded the stimulus site. This negative region increased in area as depolarization progressed, with positive potentials of increasing magnitude on the right ventricle (RV) where excitation ended. During repolarization, a pattern of reverse polarity was present. The relation of this sequence of epicardial events to the body surface maps was a subtle one, since the major aspects of the body surface pattern remained the same throughout QRS, but visible differences and some movement of the body surface pattern was evident as depolarization progressed.

At 23 msec (Fig. 2), both the inverse and measured epicardial distributions showed negative potentials around the stimulus site. The negative region of the measured epi map was smaller than that of the inverse map, and the measured minimum had a greater magnitude. The correlation coefficient between the measured and inverse epicardial distributions was moderately positive at 0.55, and the RMS-measured epicardial value (M) of 1.76 was about twice that of the inverse map ($S$) of 0.84. Nonetheless, the same major features were present in both the inverse and measured epi maps. On the body surface, a line from the minimum to the maximum went from the lower right side to the left side anteriorly, reflecting the orientation of the heart within the body in the dog.

At 37 msec, both the inverse and measured epi maps showed multiple maxima adjacent to the negative region, which had increased in both size and voltage magnitude on both the inverse and measured epi maps. The inverse epi map showed more maxima, and these were more scattered. On the body surface, two relative minima were detectable with the broad negative region, although only a single minimum was present on the epicardium. The two body minima probably were a consequence of the interaction of the torso shape with the location of the epicardial minimum.

At 77 msec, both the inverse and measured epi maps showed that the negative potentials had expanded significantly onto the anterior LV and posterior RV. The RMS size of the inverse epi map (6.40 mV) was now slightly larger than that of the measured epi map (5.72 mV), and the correlation coefficient had increased to 0.77. Although both epi maps at 77 msec were considerably different from those at 37 msec, the body surface pattern looked about the same, except that the magnitudes of the maxima and minima were much larger.

At 99 msec (Fig. 3), near the end of QRS, both inverse and measured epi maps showed large positive potentials on the anterior RV, with negative potentials of relatively low magnitude over most of the posterior epicardium, except near the apex. The body surface distribution continued to show the same major features, but the positive potential area had shifted toward the head.

At 113 msec, the measured epi map had positive potentials of almost 10 mV on the LV, associated with the end of depolarization, and a smaller relative maximum of 2.2 mV on the RV at the stimulus site due to beginning repolarization. There were two maxima in similar positions on the inverse epi map, even though the body surface pattern remained similar to the pattern at 59 msec with smooth contours and only a single maximum and minimum.

Comparison of the inverse epi maps for the RV stimulus site of Figure 4 with those for the LV site of Figures 2 and 3 showed that the differences between the sequences were far greater than the differences between the inverse and measured maps within either sequence.

Initial Dog, RV Sequence

For comparison with the preceding sequence, Figure 4 shows three time instants from a sequence produced by stimulating the lateral region of the inferior RV surface.

At 41 msec, both the inverse and measured epi maps had a large region of negative potentials surrounding the stimulus site, extending over both the anterior and posterior RV epicardium. Note that at this time both inverse and measured epi maps had an extension of the zero voltage contour onto the LV epicardium also. Although both inverse and measured epi maps showed distinct anterior and posterior maxima, only a single maximum was present on the body surface map.

At 59 msec, both the inverse and measured epi maps showed that the negative potential region was over almost the entire RV epicardium, anterior and posterior, with most of the LV epicardium positive. The considerable shift in the position of the zero contour line on the epicardium from 41 to 59 msec was reflected on the body surface primarily by changes in potential on the superior left chest, where the potentials shifted from positive to negative.

At 113 msec, the measured epi map had positive potentials of almost 10 mV on the LV, associated with the end of depolarization, and a smaller relative maximum of 2.2 mV on the RV at the stimulus site due to beginning repolarization. There were two maxima in similar positions on the inverse epi map, even though the body surface pattern remained similar to the pattern at 59 msec with smooth contours and only a single maximum and minimum.

Initial Dog, RV and LV Simultaneous Stimulus

To provide an examination of whether it was possible to use the inverse epi maps to identify the characteristics of
Figure 2: Inverse simulated and measured potential distributions, LV stimulus site. The potential distributions in this and the following figures all are presented in the same format. Across a given row, the three columns show the measured body surface potential distribution, the inverse epicardial potential distribution, and the measured epicardial potential distribution, as labeled by the heading above the columns. The measured body map was used to compute the inverse epi map; the inverse and measured epi maps were compared to check the accuracy of the inverse calculation. Different rows of the figure show different time instants as specified in the measured body column. The time given was measured from an arbitrary origin 10 msec after the ventricular stimulus; the vertical bar identifies the same time on the reference waveform. On maps in all columns, the small numbers served on the original drawing to give the voltage at each electrode, and these numbers are useful in this figure as identification of the electrode sites. Contour lines were drawn by a computer program automatically; the contour lines in a positive voltage region are solid, and those in a negative potential region are dashed. For purposes of presentation, the zero voltage contour line was superimposed by the wide dashed line, and large plus and minus signs were added near major maxima and minima. The voltage near each plus and minus sign was written in larger numbers at the top of the map. The voltage difference between adjacent contour lines is given beneath each map and identified by the designation G_B, G_S, or G_M. The designation "G_B = log" signifies that, on the body surface, the separation between contour lines was logarithmic with lines at 4, 6, 10, 15, 25, and 40 μV and multiples of 10 thereof, with only the five or six contours of largest absolute magnitude drawn. The root mean square (RMS) voltage of each epicardial map is identified by S (inverse simulated) or M (measured). The correlation coefficient computed between the inverse and corresponding measured epi maps is given by CC, and the ratio of the RMS differences to the RMS measured value is given by ER. The stimulus site is identified by the pulse on the measured epi map.

More complicated excitation and repolarization sequences, Figure 5 shows three maps selected from a sequence produced by stimulating both of the previous RV and LV sites simultaneously.

At 32 msec, both the inverse and measured epi maps had negative potentials surrounding each stimulus site, with several intervening maxima. On the measured epi map, the minima were of greater absolute magnitude, and...
the maxima were located closer to the minima than on the inverse epi map. The body surface showed only a single maximum and mimis as the composite effect of the several epicardial maxima and minima.

At 60 msec, both inverse and measured epi maps showed that the previously separate negative areas had merged to produce a single large negative region across the posterior epicardium. At the same time, both epi maps showed that peak LV epicardial voltages had increased to above 10 mV. On the body surface, the polarity from the top to the bottom of the map had reversed, although the central maximum had moved only slightly.

At 252 msec, during the T wave, the measured epi map showed a maximum at each stimulus site. The inverse epi map also had two maxima, with the RV maximum displaced slightly toward the apex and the LV maximum moved considerably toward the apex. The body surface continued to have only a single maximum.

**Graphical Comparison of Epicardial Maps, Initial Dog**

To provide a comprehensive overview of the degree to which the inverse and measured epi distributions were the same, a series of graphs were constructed comparing the inverse and measured values millisecond by millisecond throughout QRS-T. The three panels of Figure 6 show these graphs for the sequences produced by the RV, LV, and RV + LV stimulus sites. Inspection of these graphs shows that numerical aspects of the relationship between the inverse and measured epi maps remain about the same in all cases.

In particular, in all cases, the RMS measured ($\bar{M}$) and inverse simulated ($\bar{S}$) epi potentials had about the same magnitude as follows:

**Figure 3** Potential distributions following LV stimulus. This figure continues the same sequence of potential distributions begun in Figure 2. The same format as Figure 2 is used.
magnitude, first rising to a peak during QRS, falling during the S-T segment, and rising to another lower peak during T. The RMS error (E) was usually a substantial fraction of the RMS-measured value, frequently about three-fourths as large. The correlation coefficient (CC) was remarkably constant throughout QRS-T in the RV and LV sequences at a value of about 0.75, and was more variable and slightly lower during the RV + LV sequence. As expected, CC declined as voltages decreased to baseline values at the beginning of QRS and end of T. Also, there was a sharp dip in CC near the end of QRS, the time of complicated distributions and low voltage magnitudes, as ending depolarization overlapped with beginning repolarization. Overall, we interpreted the graphs as showing that, although the relationship between the inverse and measured epi maps was noisy, it also was quite consistent from msec to msec and sequence to sequence.

**Subsequent Dog**

After establishing the characteristics of the results of the inverse calculation on the initial dog, for which measurements of geometry provided the basis for the inverse transfer coefficients, Z_{HB}, additional inverse epi maps were calculated for the other dog of the pair, the "subsequent dog," without changing the inverse transfer coefficients. Therefore, the only data from the second dog that were required for the calculation were body surface maps. (Epicardial distributions also were measured to provide a check of the results.) No geometry measurements or other a priori values were available or needed from the subsequent dog since, in effect, these were assumed to be the same as for the initial dog. Since the values for these quantities and therefore the Z_{HB} were poorer approximations for the subsequent dog than the

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**Figure 4** Inverse and measured potential distributions following a RV stimulus. The same format as Figure 2 is used.
initial dog, it was expected that the inverse results would not be as good; however, we considered it to be important to know how much degradation occurred because of the significance of being able to apply a fixed inverse procedure (a fixed matrix $Z_{in}$) to different dogs.

**Subsequent Dog, RV Stimulus Site**

The stimulus site for this sequence was on the RV near the base, more anterior than the RV site presented above for the initial dog.

At 48 msec (Fig. 7), both inverse and measured epicardial maps had negative potentials on the anterior RV around the stimulus site. The negative region extended further onto the RV epicardium in the measured distribution, relative to the inverse distribution, where it was centered more on the atria. Both epi maps also had an extension of negative potentials onto the LV epicardium.

At 54 msec, the size of the negative region was substantially larger and covered most of the anterior and part of the posterior RV in both epi maps. On the body surface map, the maximum and minimum were closer together and the zero contour line was more nearly horizontal.

At 108 msec, late in QRS, both the inverse and the measured epi maps had negative potentials over the RV and partially on the LV, both anteriorly and posteriorly, with a single maximum on the lateral LV. The body surface map remained similar to the one for 54 msec, with some rotation of maximum and minimum.

At 122 msec (Fig. 8), near the end of QRS, the body surface pattern was quite different and had a jumbled appearance. However, the inverse epi map computed from the body surface map had two maxima clearly delineated, which corresponded to two maxima of the measured epi map. The LV maximum was associated with...
ending depolarization, whereas the RV maximum near the stimulus site was associated with beginning repolarization.

By 129 msec, the body surface pattern had taken on the major features of the pattern that it had through most of the T wave. Correspondingly, both the inverse and measured epi maps had a maximum near the stimulus site with negative potentials at the site of the LV maximum previously present at 122 msec.

At 244 msec, during the T wave, all three potential distributions retained the major features of the beginning repolarization maps of 129 msec, with increased voltage magnitudes. Both epi maps had a prominent maximum near the stimulus site.

**Subsequent Dog, LV Sequence**

Maps from three QRS instants from a second sequence from the subsequent dog are shown in Figure 9. The stimulus site for this sequence was on the posterior LV, in a site comparable to the posterior LV site shown for the initial dog.

At 31 msec, both epi maps had negative potentials over much of the posterior LV. Both maps also had anterior and posterior maxima, but these were in different locations. The body surface map for this dog differed from the first body surface maps in the initial dog, especially in the position of the minimum, even though the stimulus site was comparable.

At 108 msec, later in QRS, both inverse and measured epi maps had negative potentials extending over most of the LV with both anterior and posterior minima. In both maps, the single RV maximum had a much higher magnitude than any of the maxima on the earlier maps of 31 msec.

At 128 msec, both epi maps had both anterior and posterior maxima, showing the overlapping effects of terminal repolarization and beginning repolarization. Although CC was only 0.41, the inverse epi map agreed remarkably well with the measured epi map in major features, as judged visually. This was somewhat surprising, since the pattern on the body surface map was much simpler than the pattern measured on the epicardium.

**Graphical Comparison of Epicardial Maps, Subsequent Dog**

A numerical comparison of the inverse and measured epi maps was calculated for the sequences in the second dog and plotted in the same format used for the initial dog. These results are shown in Figure 10.

Inspection of the graphs shows that the quality of the inverse calculation was about the same for the second dog as for the initial dog. The RMS errors were somewhat higher, relative to the RMS voltage on the inverse epi maps, but remained about the same fraction of the RMS-measured epicardial voltage. The correlation coefficient curve was more uneven and lower by about 0.1, usually being in the range of 0.6-0.8 throughout most of QRS and T, where higher voltages were measured.

**Discussion**

Although inverse solutions in terms of current or potential distributions over the epicardium have been advocated for a number of years,² ¹₅ ¹₆ ¹₉-²₁ actual calculations have been restricted to replacing the epicardium by a sphere between the epicardial and body surfaces; results reported for these calculations were not tested against corresponding experimental measurements and bore little superficial
resemblance to the measured potential distributions from the epicardium which have since been reported.

The results presented in this report showed for the first time epicardial potential distributions, estimated from measured body surface potential distributions, that correctly displayed major features of the corresponding measured epicardial maps. Moreover, the inverse epi maps characterized each sequence of excitation and repolarization to an extent not possible from inspection of the body surface maps alone, by providing information that was sufficient not only to make clear the answers to the question of the site of the stimulus, but also to provide a picture of the development and movement of major features of the sequence of excitation and repolarization events. More specifically, the epi maps showed the initial phase of excitation, the movement of the potential distribution as excitation progressed, the overlap of the final events of excitation and the beginning features of repolarization, and finally the development of repolarization to maximum intensity. This step-by-step display of the sequence of excitation and repolarization was extremely difficult to visualize from inspection of the measured body surface maps alone, since many of the features that were distinct on the epicardial surface were merged together in the body surface map.

Comparison of the measured and computed epicardial maps showed immediately, however, that there were many numerical differences in the voltage magnitudes at individual electrodes, as well as some significant pattern differences. The question immediately arises as to whether these differences can be reduced, and some possible approaches to improving the method of calculation to achieve such a reduction are discussed below. In addition, we believe that it is important to weigh the significance of

**Figure 7** Inverse and measured maps from the subsequent dog following a RV stimulus (first part). The same format as Figure 2 is used.
FIGURE 8 Inverse and measured maps from the subsequent dog following RV stimulus (part two). The same format as Figure 2 is used.

A second counterbalancing consideration is the recognition that the question involved in any inverse solution is how to make the best use of available body surface measurements to evaluate cardiac electrical activity. In this regard, the choice available is not whether to select the inverse over the measured epicardial distributions, but whether to use the inverse epi maps in addition to the measured body surface maps to evaluate best the sequence of epicardial events. Since considerable information about the electrophysiological state of intramural and even intracellular events can be inferred directly from the epi maps, but only indirectly from the body surface maps, even approximate epi maps are immediately useful. Visual comparison of the inverse and measured epicardial distributions shows that, even in the presence of substantial amounts of noise, physiologically significant features of the epicardial distributions that are difficult or impossible to recognize from inspection of the body surface maps are
immediately and correctly observed from the inverse epi maps.

**Possible Improvements in Method of Calculating Inverse Maps**

One of the objectives of the experimental design was to use a mathematical and computational approach that was sufficiently powerful to offer a reasonable chance of producing physiologically meaningful inverse solutions, but at the same time incorporated enough simplifying assumptions that its implementation remained straightforward. Our thought was that solutions achieved by such a method would give an idea of the quality of the inverse results, and if these results were promising, then many revisions of the method of calculation could be considered. Since the results demonstrate that important features of the epicardial maps can be computed from body surface maps, it clearly is desirable to consider how to reduce the extent of the numerical discrepancies between the inverse and measured epicardial voltages. What available methods might achieve a significant reduction?

Two approaches are immediately evident for modifying the inverse calculation method. The first and most direct approach is to reduce the errors in the epicardial potentials by reducing the noise so that the RMS noise level of 100 μV now assumed to be present on the body surface can be lowered. Since this amount of noise is far more than the electronic noise that is a part of each tracing, this approach seems plausible. To evaluate its prospects more quantitatively, Equation 7 was used to find the error fraction expected for different amount of noise. The results are plotted in Figure 11. On the graph, the vertical bar through the plot identifies the expected error fraction as 0.86 for the 100-μV noise level used to find the results presented here.

Note from the graph that a 10-fold reduction in the...
noise level results in a decrease in the error fraction to only 0.66, and even a 100-fold decrease in the noise level still leaves the error fraction at 0.28. These values are disappointingly high when considered in light of the fact that even a 10-fold noise reduction probably cannot be achieved in practice, since the sources of noise are not just electronic amplifier noise but also include any discrepancies due to geometric variations or volume conductor effects. In this regard, it is important to consider that if the same transfer coefficients (forward or inverse) are used with more than one volume conductor (as was done here), then even if these transfer coefficients are extremely accurate for any one volume conductor, they will remain only approximate for the others. In this sense, the limiting factor in reducing the "noise" due to transfer coefficient inaccuracy arises from the variation from one volume conductor to another, since this "noise" is still present even if the transfer coefficients as initially determined are exact. For these reasons, although reduction in the noise can only improve the quality of the inverse solution and therefore is certainly desirable, it appears unlikely that sufficient noise reduction can be achieved to produce any dramatic reduction in the error levels of the epicardial inverse maps.

A second possible approach for modifying the inverse calculation is to incorporate into the solution procedure additional information about the characteristics of physiologically real epicardial potential distributions, in addition to using the same variance for the potential magnitude at all electrodes. A number of possibilities are immediately evident. For example, the potentials at atrial electrodes have a smaller variance than those at ventricular electrodes. A more complicated, but possibly more powerful, characteristic that might be used is that potentials at adjacent electrodes are correlated with each other, because of the nature of the underlying intramural potential distribution. Although use of further a priori information about the epicardial potentials must be made judiciously.

**Figure 10** Graphical comparison of two sequences from the subsequent dog. The results of a msec by msec comparison of the sequences following the RV stimulus (A) and LV stimulus (B) are presented in the same form used for the initial dog in Figure 6.

**Figure 11** Graph of expected error as a function of body surface noise. Values along the abscissa give the hypothesized body surface noise (note log scale). The RMS epicardial voltage was considered constant at 2.50 mV. The vertical axis shows the expected value of the mean squared error as a function of the mean squared epicardial voltage, \( \langle E^2 \rangle \), as determined by Equation 7. Thereby, as the curve moves from 1 toward 0 with lower and lower noise, the expected quality of the solution becomes better and better. The vertical bar at the value 0.86 corresponds to the RMS body surface noise of 100 \( \mu \text{V} \) used for the calculations presented in the previous figures.
to avoid inadvertently excluding unanticipated but physiologically valid solutions, we believe that this approach offers considerable possibilities for further evaluation.

**Extension of the Inverse Calculation Procedure to Man**

The present study was conducted using measurements from dogs to allow the measurement of numerous epicardial potential distributions to check the results and also to allow experimental measurements of the position of each of the epicardial electrodes. For the same procedure to be extended to humans, only the geometry information would be required, since the epicardial maps were used only as a check, and therefore were not required for the calculation itself. A possible approach to obtaining human geometry is to have one or several standard sets of measurements obtained and stored in advance, that provide the measurements from prior anatomical studies that can then be used with body surface maps from many subjects. Because of this possibility, we felt it was important to explore the analogue of this procedure in dogs, i.e., the extension of the inverse calculation method from the initial dog on which it was developed to a subsequent dog. Since this extension produced results in the second dog that were of about the same quality and contained most of the same physiological information, this suggests that an inverse procedure can be developed for a standard geometry in man and then used with productive results with potentials from many subjects.

**References**

Inverse calculation of QRS-T epicardial potentials from body surface potential distributions for normal and ectopic beats in the intact dog.

R C Barr and M S Spach

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