Predicted Effect of Chronic Apical Aneurysms on the Passive Stiffness of the Human Left Ventricle

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SUMMARY  A theoretical analysis of the human left ventricle in diastole was performed to evaluate the quantitative effect of an aneurysm on local fiber elongation in the ventricular wall and to establish quantitative relationships between chronic aneurysm size and stiffness and overall ventricular stiffness. The myocardium was assumed to be a homogeneous, isotropic, essentially incompressible material which exhibits large, nonlinear, elastic deformation. A finite element procedure was used which allows explicit representation of aneurysms in the ventricular wall. It was found that, even when the myocardium near the aneurysm has normal elasticity, the restraining influence of the aneurysm results in a substantial reduction in end-diastolic length of normal muscle fibers located in this area. This reduction in length places these fibers at a less favorable position on the Starling curve for developing tension and shortening during systole. In addition, given that the diastolic pressure-volume (P-V) relationship in the normal ventricle is of the form, \(dP/d(V/V_0) = \alpha P + \beta\), the presence of both fibrous and fibrous-muscular, apical, transmural aneurysms encompassing up to 20% of the wall volume, the parameter \(\alpha\) increases linearly with aneurysm size. Finally, it was found that the ventricular secant modulus, \(\Delta P/\Delta V/ESV\) (\(\Delta P = \) difference between end-diastolic pressure and lowest observed diastolic pressure, \(\Delta V = \) angiographic stroke volume, and ESV = end-systolic volume) remains normal with aneurysms encompassing up to approximately 10% of the wall volume, even though the diastolic P-V curve has been shifted substantially to the left relative to the normal P-V curve. However, for larger aneurysms, this modulus increases rapidly with aneurysm size.

The effects of myocardial infarction on the passive stiffness of the left ventricle were reviewed recently by Mirsky and Grossman and McLaurin. In general, it has been found that, subsequent to an early (1-hour) decrease in ventricular stiffness following infarction, this stiffness progressively increases over a period of days to weeks. An elevated end-diastolic pressure following infarction therefore is not necessarily an indication of heart failure, since end-diastolic volume may remain relatively unchanged. The increase in ventricular stiffness following infarction is attributed primarily to a stiffening of the infarcted area which, in turn, is considered to be hemo-dynamically beneficial since it reduces aneurysmal bulging during systole.

Several years ago, Covell and Ross alluded to the difficulty in evaluating, quantitatively, the effects of graded amounts of scar tissue on left ventricular compliance. Consequently, only two attempts have been made to perform this evaluation. Lowe and Love developed a model for the left ventricle for the purpose of computing the extent of aneurysmal bulging immediately preceding the opening of the aortic valves. The infarcted area was modeled as a liquid, and tension in the remainder of the ventricular wall corresponding to the lowest observed diastolic pressure was assumed to be equal to zero. Therefore, the internal pressure that was applied to the model was equivalent to transmural pressure.

The diastolic phase of the cardiac cycle was simulated by applying a hydrostatic pressure to the interior surface of an axisymmetric finite element model for the left ventricle. This pressure was applied in increments of 0.5 mm Hg from 0 to 12 mm Hg. Pleural pressure was assumed to be equal to zero. Since the state of deformation and stress in the ventricular wall corresponding to the lowest observed diastolic pressure is not known, the theoretical simulation actually began at zero transmural pressure. The requirement for deformation and stress data was therefore replaced by the requirement for a description of the undeformed wall geometry. In a previous study, it was found that the ratio of wall volume and cavity volume at zero transmural pressure for the normal rat left ventricle was approximately equal to 3.0. Since a comparable value was ob-
ventricular size and wall volume can be estimated from and volume as follows (Appendix I, Equation A3):

\[ P = \frac{2V_w}{3V_o} F(V/V_o; E, \beta) \]

where \( P \) = cavity pressure; \( V_w \) = wall volume; \( V_o \) = cavity volume at zero pressure; \( V \) = cavity volume at pressure; \( P, E, \beta \) = elastic parameters; and \( F \) = a function of the volume ratio, \( V/V_o \), and the elastic parameters.

It is clear from this formula that, while increases in ventricular size (\( V_o \)) and wall volume (\( V_w \)) influence the relationship between \( P \) and \( V/V_o \), simultaneous increases in these two quantities tend to negate each other. Consequently, while the assumption is made that neither \( V_o \) or \( V_w \) changes subsequent to chronic aneurysm formation, the discussions of diastolic P-V behavior which follow are qualitatively applicable to ventricles in which simultaneous increases occur in both ventricular size and wall volume.

The presence of an apical aneurysm in the ventricular wall was simulated by using an appropriate model for the elastic behavior of those finite elements comprising an aneurysm. Only apical aneurysms were considered, since they can be represented realistically in an axisymmetric model. Aneurysms located elsewhere in the ventricular wall are not symmetric with respect to the long axis. The remaining finite elements were assumed to exhibit normal elastic behavior. Zones of injury or ischemia adjacent to the aneurysm were not taken into account. Also, the aneurysms were assumed to be transmural. The P-V behavior for the aneurysmal ventricles was computed in a manner identical to that used for the normal ventricle. That is, pressure was applied to the inner surface of the finite element model in increments of 0.5 mm Hg up to a maximum value of 12 mm Hg.

Under uniaxial loading conditions (i.e., those conditions which exist in isolated muscle experiments), the stress-stretch relations that were used in this study to characterize the elastic behavior of the chronic aneurysms and normal myocardium reduce to the following form (Appendix II, Equation A8):

\[ \lambda S = \frac{E}{\beta (1 + \nu)} (\lambda^n - \lambda^{-n}) \]  

where \( \lambda = \text{stretch} = \ell/\ell_0; \ell = \text{deformed length}; \ell_0 = \text{length at zero stress}; S = \text{stress (force per unit of undeformed cross-sectional area)}; \) and \( E, \beta, \nu \) = elastic parameters. This form, rather than the usual exponential form, was selected for convenience.

The passive uniaxial stress-strain data of Parmley et al.\textsuperscript{17} were used to obtain values of \( E \) and \( \beta \) for the aneurysms. This involved transforming Parmley's data into values of \( \lambda S \) and \( \lambda \) and fitting Equation 1 to these values in a least squares sense. Values of \( E \) and \( \beta \) for the normal myocardium were not obtained from uniaxial stress-strain data, but from the passive P-V data of Fester and Samet,\textsuperscript{18} using a procedure described in Appendix II. Values of the parameters based on these two sets of data are listed in Table 1. Graphs of Equation 1 which used these values are given in Figure 2. This figure clearly shows that Equation 1 is an accurate mathematical model for the uniaxial elastic behavior of chronic aneurysms. Since passive P-V data were used to obtain the elastic parameters for the normal myocardium, the accuracy of Equation 1 as a model for the uniaxial elastic behavior of...
Table 1  Elastic Parameters

<table>
<thead>
<tr>
<th></th>
<th>$E$ (mm Hg)</th>
<th>$\beta$</th>
<th>$\nu^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal myocardium</td>
<td>13.61</td>
<td>14.69</td>
<td>0.45</td>
</tr>
<tr>
<td>Fibrous-muscular aneurysm</td>
<td>1081.4</td>
<td>34.46</td>
<td>0.45</td>
</tr>
<tr>
<td>Fibrous aneurysm</td>
<td>24120.</td>
<td>135.96</td>
<td>0.45</td>
</tr>
</tbody>
</table>

* Refer to Appendix II for a discussion of $\nu$.

Figure 2  Passive, uniaxial stress-stretch data for fibrous and fibrous-muscular aneurysms. These data were used to determine the elastic parameters, $E$ and $\beta$, for the aneurysms. Normal muscle behavior based on the P-V data of Fester and Sament is included for comparison purposes.

normal muscle could not be determined. However, in a previous study, Blatz et al. found that this equation did provide an excellent fit to the elastic behavior of human papillary muscle. The dashed curve in Figure 2 illustrates the point that, in order to be consistent with observed P-V behavior, the resistance of normal muscle to stretch under a given uniaxial load is much less than that of either the fibrous-muscular or fibrous aneurysms considered in this study.

Results and Discussion

The results of seven theoretical case studies are presented in this section: a "normal" ventricle, three ventricles with fibrous-muscular, apical aneurysms comprising 9.58%, 21.25%, and 41.18% of the ventricular wall volume, and three ventricles with fibrous, apical aneurysms comprising the same three percentages of the wall volume. (The coarseness of the finite element mesh is responsible for the deviations from percentages of 10, 20, and 40—values that are assumed to span the size range of interest.)

Predicted shapes of the free wall in the normal left ventricle and a ventricle with a fibrous-muscular apical aneurysm comprising 9.58% of the wall volume are illustrated in Figure 3A and B at a transmural pressure of 12 mm Hg. (Note that aneurysm size is based on percentage of wall volume, not wall cross-sectional area. The aneurysms illustrated in these figures are a much larger percentage of wall area. Also, the valvular region in the model was not included in the computation of wall volume.) It is apparent from this figure that the outward expansion of the apex in the aneurysmal ventricle is less than in the normal ventricle. Also, the presence of stiff, aneurysmal tissue reduces the change in wall thickness which takes place, not only within the aneurysm itself, but also in the normal myocardium adjacent to the aneurysm. However, at a sufficiently large distance from the aneurysm, deformation of the wall appears to be essentially normal at this pressure, consistent with a previous observation made by Hood et al.

This result is graphically illustrated in Figure 4 which shows the predicted variation of midwall circumferential stretch (a measure of midwall fiber elongation) with position in the ventricular wall at a transmural pressure of 12 mm Hg. Each value of stretch shown in this figure was obtained by averaging the two stretches at the centroids of the specified finite elements. It is apparent from this figure that, while predicted midwall fiber elongations in the aneurysmal and normal ventricles are essentially equal between the equatorial plane and the valves, the aneurysm severely limits fiber elongation in the normal myocardium adjacent to the aneurysm.

As a result of the restraining influence of the aneurysm on deformation, the predicted cavity volumes of the aneurysmal ventricles at a given transmural pressure always are less than the cavity volume of the normal ventricle. The magnitude of the change in the P-V relationship shown in Figure 5 is not unexpected, considering previous experimental studies. This change is due primarily to aneurysm size and the relative stiffness of the aneurysmal tissue and normal muscle. Since this study...
Midwall circumferential stretch (i.e., deformed fiber length relative to the undeformed length) at the positions in the ventricular wall illustrated in Figure 4. The transmural pressure is equal to 12 mm Hg.

FIGURE 4

Comparison of predicted pressure-volume curves for the normal ventricle and ventricles with fibrous aneurysms.

was limited to an analysis of apical aneurysms, the effect of aneurysm location on the P-V relationship could not be determined. However, a recent study indicates that location may not be a primary determinant of ventricular compliance.

The theoretical P-V relationships in Figure 5 indicate that, for example, the end-diastolic pressure in the normal ventricle is equal to 4 mm Hg (end-diastolic normalized volume = 1.98) and a fibrous apical aneurysm encompassing 21.25% of the ventricular wall is created with the same end-diastolic normalized volume, end-diastolic pressure would increase to 10 mm Hg. The effect of this elevation in end-diastolic pressure on midwall fiber elongation is illustrated in Figure 6. As indicated in this figure, near the aneurysm, fiber elongation in the normal myocardium is substantially reduced. However, with increasing distance from the aneurysm, fiber elongation rapidly increases, becoming substantially greater than the elongation at corresponding locations in the normal ventricle above the equatorial plane. These results are consistent with the recent measurements of Theroux et al. They observed a progressive increase in diastolic segment length in normal regions of the canine ventricle up to 4 weeks following permanent circumflex coronary artery occlusion. They also observed a corresponding decrease in segment length near the infarct. While progressive myocardial scarring accompanied by compensatory hypertrophy in normal regions may be responsible for these results, it is of interest to note that similar results are obtained in the present analysis in which the myocardium adjacent to the infarcted area is assumed to be homogeneous with normal elasticity, and the geometry of the ventricular wall is assumed to remain unchanged subsequent to infarction.

The results illustrated in Figure 6 imply that reduced fiber shortening near an aneurysm during systole may be due to the unfavorable position of these fibers on the Starling curve caused by the restraining influence of the aneurysm on end-diastolic fiber length. The beneficial hemodynamic effect during ejection of increased aneurysm stiffness may, therefore, be offset partially by the detrimental effect of this increased stiffness on end-diastolic fiber length in the normal myocardium near the aneurysm. However, this restraining influence is a local effect, and elevated end-diastolic pressures, which frequently accompany chronic aneurysm formation, may result in stretching of normal muscle fibers sufficiently removed from the aneurysm to lengths that are more favorable for the development of active tension.

It is easily shown that, if the P-V relationship during the reduced filling phase of the cardiac cycle has an

FIGURE 5

Midwall circumferential stretch in a normal ventricle at an end-diastolic transmural pressure of 4 mm Hg and in a ventricle with a chronic aneurysm and an elevated transmural pressure. The end-diastolic volume in both ventricles is equal to 92.4 cc (i.e., V/V₀ = 1.98).
exponential form, the slope of this relationship is a linear function of pressure. That is,

$$\frac{dP}{d(V/V_a)} = \alpha P + \gamma$$

(2)

where \(V_a\) denotes cavity volume at zero transmural pressure. It has been shown that the slope of this linear relationship (\(\alpha\)) varies significantly with the stiffness of the myocardium.20 However, if the P-V data are not exponentially related and the parameter \(\alpha\) is determined by fitting Equation 2 to these data, this parameter may also vary significantly with the range of the data.21

The predicted P-V relationships in this study for both the aneurysmal and normal ventricles were found to be essentially exponential. The parameters \(\alpha\) and \(\gamma\) were determined in a least squares sense from these relationships for the same range in pressure in all cases (6 mm Hg \(\leq P \leq 12\) mm Hg). A central difference formula was used for the derivatives (i.e., \(dP/dV = [P(V + \Delta V) - P(V - \Delta V)]/2\Delta V\), where \(\Delta V\) denotes a small volume increment). Values obtained for the parameter, \(\alpha\), are given in Table 2. It is clear from the table that \(\alpha\) increases significantly with aneurysm size for a given stiffness (i.e., fibrous-muscular or fibrous) and with aneurysm stiffness for a given size. The dependence of \(\alpha\) on both aneurysm size and stiffness limits its use as an index of size alone. For aneurysms known to be primarily fibrous, aneurysm stiffness can be estimated, and \(\alpha\) may have utility as an index of aneurysm size. Also, it should be apparent that, even when the composition of the aneurysm is not known, the stiffness of a fibrous aneurysm provides an upper bound for the stiffness of the aneurysm in question. Therefore, in general, \(\alpha\) may at least have utility as an index for a lower bound on aneurysm size. That is, the value of \(\alpha\) exhibited by the ventricle can be attained by a ventricle with a relatively small fibrous aneurysm or a more compliant aneurysm which is at least as large as the fibrous aneurysm.

The \(\alpha\) values in Table 2 are shown plotted as a function of aneurysm size in Figure 7. It is evident from this figure that, for aneurysms encompassing up to 20% of the wall volume, \(\alpha\) varies linearly with size. The shapes of the curves shown in this figure are consistent with the argument presented in Appendix III.

Smith et al.16 have correlated left ventricular compliance with infarct size. Their measure of compliance was the quantity \((AV/ESV)/AP\), referred to as ventricular secant modulus.20 Their results for single anterior infarctions are illustrated in Figure 8. Actually, the reciprocal of the quantity \((AV/ESV)/AP\), referred to as ventricular secant modulus in this study, is shown plotted as a function of % ACS in this figure. Data for those patients with values of lowest observed diastolic pressures less than zero or greater than 10 mm Hg are deleted from the figure, because they exceed the range of applicability of the present analysis. Predicted values of the ventricular secant modulus are also shown in this figure. (The predicted values assume that ESV is equal to cavity volume at lowest observed diastolic pressure.) Theoretical values of % ACS were determined by computing the ratios of the endocardial aneurysmal and ventricular perimeters as represented by the finite element model at a pressure of 12 mm Hg. These perimeter ratios were found to be equal to approximately 0.22, 0.30, and 0.40 for aneurysm sizes (in terms of wall volume) of 0.0958, 0.2125, and 0.4118. Predicted values of the ventricular secant modulus were obtained using observed pressures for patients with comparable values of % ACS. Corresponding cavity volumes were obtained either from the theoretical P-V relationships or from exponential extrapolations of these relationships for values of end-diastolic pressure greater than 12 mm Hg (the maximum pressure considered in this study). Note that the predicted values of the ventricular secant modulus for ventricles with the fibrous and fibrous-muscular, apical aneurysms considered in this study agree very well with the observed values up to an aneurysm size of 30% ACS. (The significance of the discrepancy between predicted and observed values at an aneurysm size of 40% ACS could not be established, since only one data point was available.) The theoretical analysis demonstrates that, even though the aneurysmal ventricles are stiffer than the normal ventricle, the ventricular secant modulus appears to remain normal up to aneurysm sizes of approximately 25% ACS (i.e., approximately 10% in terms of wall volume.)

**Table 2.** Slope (\(\alpha\)) of \(dP/d (V/V_a)\) vs. \(P\) Relation

<table>
<thead>
<tr>
<th>Aneurysm size (% wall volume)</th>
<th>Aneurysm type</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fibrous-muscular</td>
<td>Fibrous</td>
</tr>
<tr>
<td>9.58</td>
<td>1.34*</td>
<td>1.51</td>
</tr>
<tr>
<td>21.25</td>
<td>1.57</td>
<td>1.92</td>
</tr>
<tr>
<td>41.18</td>
<td>2.08</td>
<td>2.95</td>
</tr>
</tbody>
</table>

* Normal value (no aneurysm) = 1.14.
In summary, a theoretical analysis of the elastic behavior of the human left ventricle during the reduced filling phase of the cardiac cycle was performed which indicates that chronic aneurysms may substantially restrain adjacent muscle fibers from attaining normal end-diastolic fiber lengths, even though these fibers have normal compliance. These muscle fibers may therefore develop active tensions that are significantly less than optimal, reducing their effectiveness with respect to the ejection of blood. To what degree this detracts from the beneficial hemodynamic effect of reduced aneurysm compliance was not determined.

The slope (α) of the dP/d(V/V₀) vs. P curve was examined as a possible index of aneurysm size. It was found that α is sensitive, not only to aneurysm size, but to aneurysm stiffness as well. Consequently, the utility of α as an index of size alone is limited.

Appendix I

Laplace's Law in Terms of Pressure and Volume

The usual form of Laplace's law is as follows:

$$\sigma_r = \frac{Pr}{2t}$$

where \(\sigma_r\) and P are meridional stress and pressure; and \(r\) and \(t\) are cavity radius and wall thickness at pressure, P. This equation may be rewritten in the following way:

$$\sigma_r = \frac{3P(4\pi r_0^2/3)}{2(4\pi r^2/3)} = \frac{3P}{2V_w} \frac{V}{V_0}$$  \hspace{1cm} (A1)

where \(V_n\) and V are cavity volumes at pressures 0 and P, respectively, and \(V_w\) is wall volume (independent of pressure).

If the membrane consists of a material with elastic behavior comparable to that of the myocardium, this behavior can be characterized by a relationship between stress and stretch of the following form:

$$\sigma_r = G(\lambda, E, \beta)$$  \hspace{1cm} (A2)

where G is a nonlinear function of meridional stretch, \(\lambda\); and E and \(\beta\) are constant parameters. For the membrane, meridional stretch is defined as follows:

$$\lambda = \frac{r}{r_0} = \frac{4\pi r^2/3}{4\pi r_0^2/3} = \left(\frac{V}{V_0}\right)^{1/3}$$

where \(r_0\) is cavity radius at zero pressure.

Equations A1 and A2 imply that,

$$P = \frac{2V_w}{3V_0} \left\{ G\left(\frac{V}{V_0}\right)^{1/3}; E, \beta \right\}$$

$$= \frac{2V_w}{3V_0} F(\frac{V}{V_0}; E, \beta)$$  \hspace{1cm} (A3)

where, for given values of the constants E and \(\beta\), F is only a function of the volume ratio, \(V/V_0\).

Appendix II

Mathematical Considerations

The left ventricular wall was assumed to consist of passive, elastic materials during the diastolic phase of the cardiac cycle. This assumption allows the development of constitutive relations (i.e., relations between stress and stretch) for these materials from a strain energy density function.$^{23}$

Derivation of Stress-Stretch Relations

The strain energy density function utilized in this study is defined in Equation A4. As indicated in the discussion that follows, this function accounts for both the nonlinear elasticity and essential incompressibility of cardiac muscle.$^{*}$

$$W(\lambda_1, \lambda_2, \lambda_3) = \frac{E}{\beta^2(1 + \nu)} \sum \left(\frac{\lambda_i^2 - 1}{\lambda_i^2 - 1} \right)$$

$$+ \left(\frac{1 - 2\nu}{\nu}\right) \left[ \left(\frac{\lambda_1}{\lambda_2}\right)^{\frac{\beta_2}{\beta_1} - 1} \right]$$  \hspace{1cm} (A4)

where \(\lambda_i\) is ith principal stretch (* = \(\sqrt{1 + 2\xi_i}\) where \(\xi_i\) denotes the ith principal Lagrangian strain); E, \(\beta\), \(\nu\) are elastic parameters; and \(W(\lambda_1, \lambda_2, \lambda_3)\) = strain energy per unit of undeformed volume. The “stretches” in Equation A4 are normalized measures of deformation. They may be interpreted as the lengths of line elements in the deformed body relative to the lengths of these elements in the undeformed body. The quantity, \(\nu\), should be interpreted as follows:$^{24}$

$$\nu = -\frac{\ln \lambda_{tr}}{\ln \lambda}$$  \hspace{1cm} (A5)

where \(\lambda\) = stretch in the direction of uniaxial load and \(\lambda_{tr}\) = stretch transverse to the direction of uniaxial load.

* Since the strain energy density function defined by Equation A4 is invariant under a permutation in the principal stretches, and since the elastic parameters in this equation are independent of position, the myocardium has been tacitly assumed to be isotropic and homogeneous. This assumption greatly simplifies the analysis and should not alter the major conclusions of this study.
It can be shown that Equation A4 has the necessary mathematical properties which characterize a strain energy density function (i.e., it is positive definite and monotonic relative to the unstrained state, and it approaches the form of a strain energy density function for a linear elastic material in the limit of infinitesimal strain).

The stress-stretch relations implied by Equation A4 were obtained by differentiating W with respect to \( \lambda_3 \) that is,

\[
S_i = \frac{\partial W}{\partial \lambda_i} = \frac{E}{\beta(1 + \nu)} \left[ \lambda^\beta - \left( \prod_{i=1}^{3} \lambda_i \right)^{1 - \beta \nu} \right] (A6)
\]

where \( S_i \) = ith principal stress (force per unit of undeformed cross-sectional area).

According to Equation A5, under conditions of uniaxial load,

\[
\prod_{i=1}^{3} \lambda_i = \lambda \lambda_f = \lambda^{1 - 2\nu} (A7)
\]

Therefore, under this loading condition, Equation A6 reduces to the form:

\[
\lambda S = \frac{E}{\beta(1 + \nu)} (\lambda^\beta - \lambda^{-\beta \nu}) (A8)
\]

As indicated by Blatz et al.,\(^4\) Equation A8 characterizes the nonlinear elastic behavior of human cardiac muscle.

It is also clear from Equation A7 that, for \( \nu = 0.5, \prod_{i=1}^{3} \lambda_i = 1.0 \). That is, by appropriate selection of the parameters \( \nu \), the essential incompressibility of the myocardium is accounted for in terms of the exact mathematical condition which characterizes this property, namely, \( \prod_{i=1}^{3} \lambda_i = 1.0 \).

The stress-stretch relations defined by Equation A6 were used to characterize the elastic behavior of both the normal myocardium and the chronic aneurysms. These materials can be distinguished from each other in this study only by the values of the elastic parameters, \( E \) and \( \beta \). These parameters were estimated using the following procedures.

**Determination of Elastic Parameters in Stress-Stretch Relations**

The parameter, \( \nu \), was simply chosen to have a value (0.45) which ensured the essential incompressibility of both the normal myocardium and the aneurysms. This value of \( \nu \), for example, implies via Equation A7 that, under uniaxial load, volume would change by approximately 3.4% for a stretch equal to 1.4 (40% extension).

The parameters \( E \) and \( \beta \) for the normal myocardium were then estimated by fitting the following equation (derived from Janz et al.\(^28\)) in a least squares sense to a mean pressure-volume (P-V) relationship for the normal left ventricle obtained from Reference 13:

\[
P = \frac{2E}{\beta(1 + \nu)} \int \left( \frac{V}{V_0} \right)^{\beta/3} \left( \frac{X - X_0}{X(1 - X)} \right) dX
\]

\[
= \frac{2E}{\beta(1 + \nu)} \left( \frac{V + V_w}{V_0 + V_w} \right)^{\beta/3}
\]

where \( P \) = intraventricular pressure (mm Hg); \( V \) = cavity volume (cc); \( \nu = 0.45 \) (see above); \( V_0 = \) cavity volume when \( P = 0 \) (cc); and \( V_w \) = wall volume (cc). There are actually four unknown parameters in this equation: \( E \), \( \beta \), \( V_0 \), and \( V_w \). One of these parameters was estimated directly from measured data. That is, given mean values at end diastole for wall thickness, ratio of minor and major axes and cavity volume in the normal human left ventricle,\(^13\) wall volume (Vw) was found to be approximately equal to 143.6 cc. The remaining three parameters could not be obtained directly from measured data and were therefore found by the fitting procedure described above. The results of this procedure indicate that \( V_0 \) is approximately equal to 46.68 cc. This implies that the ratio of \( V_w \) and \( V_0 \) is 3.1, which is surprisingly close to the value previously obtained from the dimensions of the free wall of the rat ventricle.\(^10\) Values that were obtained for the elastic parameters \( E \) and \( \beta \) are given in Table 1. The accuracy of the fit is illustrated in Figure 9.

When the normal parameters given in Table 1 were used to represent the elastic behavior of all elements in the finite element model, a diastolic P-V relationship was obtained over the range in transmural pressure from 6 to 12 mm Hg which can be described mathematically by the following equation:

\[
\frac{dP}{dV} = 0.024P + 0.060 \text{ (mm Hg/cc)} (A10)
\]

This equation was considered to be an acceptable representation of normal human diastolic P-V behavior, since both parameters in the equation deviate from mean values measured by Fester and Same\({\text{13}}\) by less than one standard deviation. (Another approach for obtaining normal muscle elastic parameters would have involved fitting the finite element model directly to normal P-V data. However, the computational cost associated with this procedure was considered to be prohibitive.)

The parameters \( E \) and \( \beta \) for fibrous and fibrous-muscular chronic aneurysms were estimated by fitting Equation

![Figure 9](http://circres.ahajournals.org/)

**Figure 9** Diastolic pressure-volume data for the normal human left ventricle. These data were used to determine the elastic parameters, \( E \) and \( \beta \), and the undeformed cavity volume, \( V_0 \), for the normal left ventricle.
A8 in a least squares sense to the uniaxial data of Parmley et al.\textsuperscript{12} Values obtained for these parameters are also given in Table 1. The accuracies of the fits are illustrated in Figure 2.

**Errors in Finite Element Procedure**

There are two main sources of numerical error in the finite element procedure. (1) The displacement field within each finite element was assumed to be a linear function of position. (The accuracy of this assumption increases with decreasing element size.) (2) A tangent-modulus procedure was used to determine the nonlinear dependency of the displacement field on pressure. (The accuracy of this assumption increases with decreasing pressure increments.) An evaluation of these errors was made by applying the finite element procedure to a problem which could also be solved using a modified Runge-Kutta procedure, namely, a thick-wall hollow sphere with the same elastic parameters, cavity volume at zero transmural pressure, and wall volume as the normal human ventricle. The agreement between the P-V relationships predicted by these two procedures was found to be very good. For example, at a transmural pressure of 12 mm Hg, the difference in predicted cavity volumes was approximately 1.3%. The size of the finite elements and the pressure increment were therefore considered to be small enough so that the contribution of any numerical error to the differences that were obtained between predicted cavity volumes and myocardial stretches in the normal and aneurysmal ventricles is negligible.

**Appendix III**

**Explanation for the Nature of the Dependence of \( \alpha \) on Aneurysm Size (Figure 7)**

Consider the spherical model for the undeformed left ventricle shown in Figure 10. Denote the total wall volume by \( V_w \) and aneurysm volume by \( V_a \). It then follows that the undeformed cavity volume, \( V_o \), is given by the following expression:

\[
V_o = V_1 + V_2
\]

where

\[
V_1 = \left( \frac{V_x}{V_w} \right) V_o, \quad V_2 = \left( 1 - \frac{V_a}{V_w} \right) V_o
\]

Applying a hydrostatic pressure to the inner cavity wall, the myocardium far enough away from the aneurysm will deform as though the aneurysm were not present. For stiff enough aneurysms, the inner wall of the aneurysm will exhibit very little deformation. Therefore, the deformed cavity volume, \( V \), will be given approximately by the following expression:

\[
V = \left( 1 - \frac{V_a}{V_w} \right) V_o + \left( \frac{V_a}{V_w} \right) V_o
\]

where \( V_a \) denotes the deformed cavity volume of the normal ventricle. Therefore,

\[
\frac{dP}{d\left( \frac{V}{V_o} \right)} = \frac{1}{1 - \frac{V_a}{V_w}} \frac{dP}{d\left( \frac{V_o}{V_w} \right)}
\]

\[
= \frac{1}{1 - \frac{V_a}{V_w}} (\alpha_0 P + \nu_0), \quad P \geq 6 \text{ mm Hg}
\]

where

\[
\alpha = \frac{\alpha_0}{1 - \frac{V_a}{V_w}} \quad (\text{A12})
\]

A plot of \( \alpha \) vs. \( V_a/V_w \) would then have the shape illustrated in Figure 7. The deformed volume, \( V \), will actually be less than that given by Equation A11 because the relatively stiff aneurysm restrains the deformation of the normal myocardium in the vicinity of the aneurysm. The extent of this restraint will, of course, depend on the stiffness of the aneurysm relative to the normal myocardium. Therefore, the slope of the \( \alpha \) vs. \( V_a/V_w \) curve will actually be steeper than that implied by Equation A12 and will increase with decreasing compliance of the aneurysm.

**References**

CONTRIBUTION OF VAGAL PATHWAYS TO THE RENAL RESPONSES TO HEAD-OUT IMMERSION IN THE NONHUMAN PRIMATE

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SUMMARY Studies were carried out to determine the contribution of cardiopulmonary receptors to the renal responses to head-out water immersion in the nonhuman primate. Immersion to the suprasternal notch was associated with significant increases in central venous pressure, urine flow, and sodium excretion. The increased sodium excretion was due primarily to a significant increase in the percent of the filtered sodium excreted. Deoxycorticosterone acetate (DOCA) and antidiuretic hormone (ADH) had no substantial effects on these responses. The finding of a vasopressin-resistant hypotension is consistent with the natriuresis of immersion being due, at least in part, to a decrease in sodium reabsorption proximal to the diluting segment, possibly the proximal tubule. Bilateral cervical vagotomy had no substantial effect on the renal responses to immersion, demonstrating that cardiopulmonary receptors whose axons traverse the vagus nerves are not necessary for the homeostatic adjustments to central hypervolemia in the primate. Since the renal and cardiovascular responses of the primate to immersion are essentially the same as those seen in man, it is probable that vagal pathways also are not necessary in man. However, it is possible that sympathetic afferents are involved in the natriuresis observed in the primate during immersion.

IT HAS BEEN well documented that immersion of man in water to the neck induces a substantial diuresis and natriuresis. It has been suggested that these responses are secondary, at least in part, to the demonstrated translocation of blood from the periphery to the thorax. Which, in turn, stimulates cardiac receptors presumed to be involved in the control of salt and water excretion. Central to this hypothesis is that section of pathways which have been shown to transmit information concerning volume from the heart should attenuate, if not abolish, the renal responses to immersion. Since most previous experiments concerning the renal responses to immersion have been made on the human, insight into the mechanisms involved have been limited by virtue of the inability to use invasive techniques. To test the hypothesis that cardiopulmonary receptors are involved in the renal response to immersion, we carried out experiments in the...
Predicted effect of chronic apical aneurysms on the passive stiffness of the human left ventricle.
R F Janz and R J Waldron

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