A Theoretical Analysis of Intracavitary Blood Mass Influence on the Heart-Lead Relationship

By DANIEL A. BRODY, M.D.

As a result of theoretic advances made during the past 10 years, it is now feasible to record scalar and vector electrocardiograms in a manner which is independent of body shape and cardiac location. A similar independence of the body's electric inhomogeneities has not yet been achieved. On the contrary, the evidence presented here shows that inhomogeneity phases in the body, especially the intracavitary blood mass, exert a powerful influence on the heart-lead relationship. The particular effect of the intracavitary phase is to augment the manifest strength of normal components of myocardial doublets, and to reduce the manifest strength of tangential components. This augmentation-reduction effect is quantitatively predictable under conditions of simple idealization, and has been confirmed by experiments on electrocardiographic models. The net effect of the intracavitary phase is probably to produce quasi-vectorial registration of the electromotive forces of the heart, at least during the normal depolarization phase.

THE role of the body's electric inhomogeneities in both theoretic and practical electrocardiography has not yet been clearly defined. Most of the recent reports on the heart-lead relationship either neglect inhomogeneity entirely, or else tend to show that the relationship is not materially affected by the presence of inhomogeneities.

The experimental results which fall in the latter category may be summarized as follows: (1) the Burger triangles of a homogeneous two-dimensional model did not differ greatly from those of electrically inhomogeneous models of the same external configuration,1 (2) the scalene tetrahedron determined upon an electrically homogeneous, three-dimensional phantom2 was essentially the same as that determined upon a similar, but electrically inhomogeneous, model,3 (3) the dipole moment determined from the surface potentials of electrolytic tank models was essentially the same whether the models were electrically homogeneous or inhomogeneous,4 (4) the surface isopotential traces of a lead field were essentially the same for both a homogeneous model and the human subject from which the model was patterned,5 (5) the electrocardiographic equipotentials occurring on the body surface of a normal male subject closely resembled those of an electrically homogeneous model patterned from the subject.6

Despite this evidence which suggests that body inhomogeneity does not greatly influence heart-vector registration, it is our view that inhomogeneity does exert a powerful distorting effect, and that the factors producing this distortion warrant careful theoretic and experimental consideration. In this report we have attempted to analyze inhomogeneity effects primarily from a theoretic point of view with the expectation that some of the conclusions may serve as a basis for future experimental studies.

"Short-Circuiting" Effect of the Intracavitary Blood Mass. According to Schwan and associates7 the conductivities of myocardium and lung are almost the same, and the conductivity of the intracavitary blood mass is approximately ten times that of the surrounding tissue. On this basis it might be expected that the intracavitary blood mass would exert some sort of short-circuiting effect upon potentials generated within the myocardium. The simplest method of analyzing this hypothetic effect consists of idealizing the
intracavitary blood mass into a perfectly conducting sphere immersed in an extended, homogeneous medium. Under these conditions the electric image of a point source located at a distance, \( d \), from the center of the sphere consists of a point source and sink of equal strength. The image source is located at the center of the sphere, and the image sink lies on a line joining the center with the object source at a distance \( R \sqrt{d} \) from the center, \( R \) being the radius of the sphere. The strength of the images is \( \pm R / d \) times that of the object source.

Applying this basic principle to electric doublets* located in the vicinity of the sphere, the following relationships may be deduced: (1) electric images reduce the manifest strength of tangentially oriented doublets by a factor of \( R / d \), and (2) augment the manifest strength of radially oriented doublets by a factor of \( 2R / d \).

The analysis becomes somewhat more complicated for spheres of finite conductivity because the eccentric image no longer assumes a simple unipolar form. However, when the conductivity of the sphere is large with respect to the surrounding medium, the eccentric image is very nearly unipolar, although the strength of the image is less than in the case of the infinitely conductive sphere. A detailed analysis for a sphere whose conductivity is ten times that of the surrounding medium is given in appendix 1.

These deductions lead to the paradoxical conclusion that the relatively large conductivity of the intracavitary blood mass tends to short-circuit the tangential components, but augments the radial components of myocardial doublets.

The augmentation of a radially oriented dipole, as calculated for a specific example, is illustrated in figure 1. These effects were also tested in two-dimensional models consisting of a highly conductive circular disc immersed in an extended medium. In a typical experiment the strength of a tangential dipole was reduced by 87 per cent, and the strength of a radial dipole was augmented by 55 per cent. The expected values, calculated from the physical dimensions of the array, were 86 per cent and 60 per cent, respectively.

Distortion of Ideal Lead Field by Spherical and Spherical Shell Phases. Another way of evaluating the effect of the intracavitary blood mass on the heart-lead relationship is to determine and analyze the distortion which the cavity produces in otherwise ideal lead fields.

The simplest situation is that in which the intracavitary blood mass is represented as a sphere immersed in an extended homogeneous medium. The field equations for a spherical mass whose conductivity is ten times that of the surrounding medium have been published previously.8

Figure 2 illustrates the variety of lead vectors existing within the vertical great circle plane at the surface of such a sphere ("endocardial" surface), at a level one fourth of the radius away from the sphere ("epicardial" surface), and in regions remote from the sphere. The
axis of the field is vertical, and the angular values given in the figure refer to the angle between a horizontal line and the radius vector to points at which the lead vectors were determined.

The inner circles of the figure are the loci of the lead vector termini; the outer circles are the radial component loci of the lead vectors. In the left-hand and middle panels of the figure, the ratio of the greater diameter to the length of the 0 degree vector equals the augmentation-reduction ratio for doublets located at each of the two levels. The ratio of these same diameters to the diameter of the circle in the right-hand panel is the augmentation ratio alone.

Because the radial component loci are circular, any array of radially oriented myocardial doublets will be recorded as the projection of a manifest vector quantity upon the vertical (lead) axis of the figure. For arrays which have relatively small tangential components, registration will be quasi-vectorial due to the relative suppression-augmentation properties of the lead field.

In the spherical case the distorting potentials are due to a centric image doublet whose axis is parallel to the lead axis. Essentially the same situation exists in the case of the spherical shell, the moment of the image doublet being dependent upon the conductivities of the shell, the intracavitary contents and the external medium. For a spherical shell with internal and external diameters of D and 1\(\frac{1}{4}\)D, and whose wall and cavity have a conductivity of two and ten times, respectively, that of the surrounding medium, we calculate that the relative moment of the image doublet is only about 40 per cent of what it would be if the conductivity of the shell were the same as that of the surrounding medium. Therefore, the suppression-augmentation effect in this hypothetic case is decidedly less than in the example illustrated in figure 2.

Distortion of Ideal Lead Fields by Ellipsoidal and Ellipsoidal Shell Phases. When a homogeneous ellipsoid of relatively high conductivity is immersed in an ideal lead field, the electric field produced within the ellipsoid is a relatively weak one with plane-parallel equipotential surfaces. In general, the internal field will not be parallel with the original external field except when an axis of the ellipsoid coincides with the axis of the original field.

The distortion of the field may be expressed mathematically as a first-order ellipsoidal harmonic of the second kind. This harmonic bears much the same relation to the ellipsoidal case that the image doublet bears to the spherical case. Therefore the presence of the ellipsoid augments the manifest moment of normally oriented doublets and reduces the manifest moment of tangentially oriented doublets. At the surface of the ellipsoid the augmentation-reduction ratio is the same as that occurring at the surface of a sphere of identical conductivity.

In the general case (the three axes of the ellipsoid unequal), the distortion harmonic cannot be evaluated by conventional methods. However, in the special cases of oblate and prolate ellipsoids of revolution immersed in ideal lead fields, with their axes of revolution
parallel to the direction of the lead axis, such evaluation may be accomplished. The field equations of these two special cases are given in appendix 2.

The lead vector loci of these particular fields are qualitatively the same as those shown in figure 2 for the spherical case. As implied in the equations of appendix 2, the dimensions of the circular loci depend intimately upon the geometric parameters of the situation. Also implicit in the field equations is the fact that any array of normally oriented myocardial doublets will be recorded as the projection of a manifest vector quantity upon the lead axis.

This latter relationship, however, fails in the general case where it applies specifically only to normally oriented doublets which occupy a given ellipsoidal coordinate level. The orientation of the effective lead axis is different for doublets located at various coordinate levels. However, these differences are greatly minimized for ellipsoids which closely approximate the spherical form.

In the case of the ellipsoidal shell, the distortion harmonic is such that the field within the shell is qualitatively the same as, but quantitatively different from the field about a solid ellipsoid. The relation between the two cases is very similar to that between the spherical shell and the solid sphere.

True Electric Moment vs. Effective Electric Moment. The inhomogeneity problem may be approached in a more general way by idealizing the body into a number of phases of irregular shape, with electric sources and sinks randomly distributed throughout the "myocardial" phase.

As previously shown, the electric moment of numerous electric sources and sinks located within a homogeneous volume conductor is equivalent to a surface integral involving conductivity, unit normal vectors and surface potentials. A given component of the moment may be expressed as

\[ M_x = \gamma \int \int_{S_b} i \cdot N_b V \, dS \]

where \( M_x \) is the X axis component of the moment, \( \gamma \) is the conductivity of the volume conductor, \( i \) is the unit positive vector in the direction of the X axis, and \( N \) is the unit normal of the surface.

This equation may be extended (see appendix 3) to situations in which the body is idealized into a number of phases of different conductivity. For three such phases the equation becomes

\[ M_x = \gamma_b \int \int_{S_b} i \cdot N_b V \, dS + (\gamma_b - \gamma_h) \int \int_{S_h} i \cdot N_h V \, dS + (\gamma_c - \gamma_h) \int \int_{S_c} i \cdot N_c V \, dS \]

where the subscripts \( b, h, \) and \( c \) refer to the body external to the heart, the heart wall and its external surface and the cavity of the heart respectively. The skin-subcutaneous fat phase has been omitted from this equation because of the relatively low conductivity of the phase. The phase may be included simply by substituting integration over the external surface of the skin for the first double integral of the equation.

Equation 2 shows that registration of true heart vectors requires integration over each of the phase boundaries. However, the heart-lung surface (middle integral of the equation) may be disregarded because the conductivities of the two phases are approximately equal. Figure 3 illustrates two-dimensionally a summing circuit which approximately performs the necessary integrations.

Clinical application of this method requires that the external surface of the body, as projected on a plane normal to the lead axis, be divided into a large number of equal areas. The endocardial surface, similarly projected, is divided into units whose area is one-ninth that of the external surface divisions. Electrodes located at the center of each of these area units are connected together through averaging networks consisting of equal resistors of relatively large magnitude.

Averaging of body surface potentials (fulfillment of equation 1 and the first integration of equation 2) is performed by the externally located electrodes. Therefore, these components alone would record "effective" heart vectors
Fig. 3. Two dimensional representation of an averaging network which would accurately record the horizontal component of the true electric moment of the heart, assuming the cavitary contents to have a conductivity ten times that of the heart and body tissue. The figure shows that endocardial as well as body surface potentials must be averaged in order to achieve accurate registration. The body surface electrodes are located on the center lines of a number of horizontal strips of equal width. The cavity phase is similarly divided, but into strips whose width is one-ninth that of the body phase strips. The internal electrodes are located at the cavity-heart interface on the center lines of these narrower strips. The resistors are all of equal and relatively large magnitude. A similar electrode-resistor array, not shown here, is applied to the left side of the figure. The networks perform the integrations indicated in equation 2 of the text, including the proper relative weighting of the external body surface and endocardial surface integrals. Conversely, reciprocal energization through these networks would produce an approximately ideal lead field throughout the model shown in the figure. The method by which such networks could theoretically be applied to the human body is described in the text.

with considerable accuracy. Averaging of endocardial surface potentials (cf. last integral of equation 2) is accomplished by the internally located electrodes. Thus the total electrode-resistor array provides a means for accurate registration of the true electric moment of the heart.

DISCUSSION

Our present analytic methods indicate that the intracavitary blood mass exerts a powerful distorting effect on the heart-lead relationship even though previous studies failed to reveal significant inhomogeneity effects. However, such studies were not specifically designed to reveal effects due to the presence of intracavitary blood. What they tend to show, rather, is that relatively small inhomogeneity phases remote from the heart have little influence on the heart-lead relationship. In this respect there is no incompatibility between the results of such studies and our findings here.

Assuming that our conditions of idealization are approximately correct, the presence of the intracavitary blood mass unquestionably produces effective augmentation of the normal components of myocardial doublets and suppression of the tangential components. In the case of spherical or nearly spherical idealization of the heart's cavity, myocardial doublets which have an approximately radial orientation can be recorded in a quasi-vectorial manner. That is, the registration of such electromotive forces by a so-called ideal lead connection will approximate the projection of a manifest vector quantity upon the lead axis. This conclusion is of particular interest since it has been shown that the orientation of myocardial doublets is approximately normal during depolarization. For every array of radially oriented myocardial doublets two sets of images occur within the spherically idealized cavity. Therefore, the net effect of the doublets and their images is more centrally disposed than the effect of the doublets alone. This relationship, together with the effective suppression of tangential components of myocardial doublets, probably has a great deal to do with the demonstrated behavior of the heart as a single fixed-location dipole.

We have previously suggested that mean lead vectors do not generally exist in association with the usual types of electrocardiographic connections. The considerable distortion of lead fields by the intracavitary blood mass and the great variety of local lead vectors resulting from such distortion greatly strengthens the validity of this concept.

Although this concept appears incompatible with the existence of mirror electrocardiographic patterns, we believe that the latter are due essentially to a fortuitous disposition of myocardial doublets, and do not in any respect depend upon the existence of mean lead vectors. For instance, a cancellation condition
which causes almost complete obliteration of the normal QRS complex may be quite inadequate for the cancellation of T waves.11

The conceptual developments of the past 10 years have resulted in methods of heart-vector registration which are essentially independent of body shape and cardiac location. In contradistinction, no such independence of inhomogeneity effects has yet been achieved. Therefore we believe that none of the presently employed vector- and electrocardiographic methods are valid for the registration of true heart vectors. As implied by Bayley,18 what we call a vectorcardiogram is actually a tensorcardiogram. The powerful influence of the intracavitary blood mass on the heart-lead relationship, as illustrated in this report, increases further the validity of Bayley's idea.

The analysis of the inhomogeneity problem presented here leads to a theoretically correct but clinically impossible method of eliminating inhomogeneity effects. For the present, electric inhomogeneity of the body remains an unsolved and intriguing problem in our efforts to obtain accurate vectorcardiographic registration.

**SUMMARY**

The role of the body's electrical inhomogeneities in electrocardiography has been theoretically analyzed from three different points of view with particular emphasis on the intracavitary blood mass. All three methods of study indicate that the intracavitary blood mass exerts a powerful influence on the heart-lead relationship.

If the conditions of idealization employed in this study are approximately correct, the presence of the intracavitary blood mass unquestionably augments the effective strength of normal components of myocardial doublets and reduces the effective strength of tangential components.

Under conditions of simple idealization this augmentation-reduction effect is quantitatively predictable, and has been confirmed in electrocardiographic models.

An analysis of lead field distortion and local lead vectors in the vicinity of spherical or ellipsoidal intracavitary blood masses strengthens the previously proposed concept that a mean lead vector does not exist in association with the usual type of electrocardiographic connection.

The net result of the augmentation-reduction effect is probably to produce quasi-vectorial registration of normally conducted QRS complexes.

Assuming again that the conditions of idealization are approximately correct, there are great differences between true heart vectors and effective heart vectors.

"Vectorcardiograms" recorded by present methods are actually tensorcardiograms. A practical solution to the inhomogeneity problem must be devised before true vectorcardiography can be achieved.

**SUMMARIO IN INTERLINGUA**

Le rolo del nonhomogeneitates electric del corpore in le electrocardiographia esseva analysate theoricamente ab tres differente punctos de vista con attention special al massa de sanguine intracavitari. Omne le tres methodos de studio indica que le massa de sanguine intracavitari exercite un potente influentia super le relation corde-derivation.

Si le conditiones de idealisation usate in le presente studio es approximativemente correcte, le presentia del massa de sanguine intracavitari augmenta sin dubita le fortia effective del normal componentes de duplettos myocardial e reduce le fortia effective de componentes tangential.

Sub conditiones de idealisation simple, iste effecto de augmento e de reduction es quantitativemente predicible e ha essite confirmate in modellos electrocardiographic.

Un analyse del distorsion de campos derivational e del vectores de derivation local in le vicinitate de sphere o ellipsoide massa de sanguine intracavitari reinfortia le previamente proponite conception que un vector de derivation medie non existe in association con le usual typo de connexion electrocardiographic.

Le resultato nette del efecto de augmento e reduction es probablemente le production de un registration quasi-vectorial de complexos QRS de conduction normal.

Supponite, de novo, que le conditiones de
idealisation es approximativemente corecte, il existe grande differentias inter le ver e le effective vectores cardiac.

"Vectocardiogrammas" registrate per medio del currente methodos es de facto tensecardiogrammas. Un solution practic del problema de nonhomogeneitate debe esser trovate ante que un genuin vectocardiographia deveni possibile.

ACKNOWLEDGMENTS

The author is indebted to Mr. William E. Romans for performing the model experiments described in the text. The illustrations were drawn by Mr. Frederick Allen.

APPENDIX I

Electric Image of a Unipole Located in the Vicinity of a Highly Conductive Sphere. Let a point source of electric current whose strength is 4πγ be located in an extended, homogeneous medium of conductivity γ at a distance, d, from the center of a sphere whose conductivity is 10γ and whose radius is R. The potential function due to this object unipole may be expressed as

\[ V = \frac{1}{d} \left[ P_0(\cos \theta) + \frac{r}{d} P_1(\cos \theta) + \cdots \right], \]

where \( r \) and \( \theta \) are conventional polar coordinates, the functions of \( \cos \theta \) are zonal harmonics, and \( r < d \). Solving for boundary conditions we find that the image potential is

\[ V = \frac{3f}{4r} - \frac{3f}{4r} \left[ P_0(\cos \theta) + \frac{r}{d} P_1(\cos \theta) \right] + 1.043 \left( \frac{r}{d} \right)^2 P_2(\cos \theta) + 1.059 \left( \frac{r}{d} \right)^3 P_3(\cos \theta) + \cdots, \]

where \( f = R^2/d \), and \( r > f \).

The first term on the right of the above equation represents a point source located at the origin. The remaining terms represent a good first approximation of a point sink located a distance \( f \) from the origin. The excellence of the approximation is represented by the small differences from unity of the numerical coefficients within the brackets.

The strengths of the image source and sink are essentially equal to each other, and three fourths of what they would be in the case of a perfectly conducting sphere.

APPENDIX II

Distortion of Ideal Lead Fields by Ellipsoids Which Have an Axis of Revolution Parallel to the Lead Axis. When the ellipsoid is oblate, the potential function in the external medium is of the form

\[ V = \left[ 1 + k \left( \cot \alpha - \frac{\pi}{2} \right) \right] \frac{y}{b} \]

where \( 2b \sec \alpha \) and \( 2b \tan \alpha \) are the major and minor axes, respectively, of the ellipsoidal coordinate surfaces. When the conductivity of the ellipsoid is ten times that of the surrounding medium, \( k = -3.91 \). The field within the ellipsoid is parallel to the ideal field, but is only 16.2 per cent as strong.

For a prolate ellipsoid the potential function in the external medium is of the form

\[ V = \left[ 1 + k \left( \frac{1}{2} \log \frac{\lambda + b}{\lambda - b} - \frac{b}{\lambda} \right) \right] \frac{y}{b} \]

where \( 2\lambda \) and \( 2\sqrt{\lambda^2 - b^2} \) are the major and minor axes, respectively, of the ellipsoidal coordinate surfaces. When the conductivity of the ellipsoid is ten times that of the surrounding medium, \( k = -4.39 \). The field within the ellipsoid is parallel to the ideal field, but is only 23.4 per cent as strong.

APPENDIX III

True Electric Moment as the Sum of Phase Boundary Surface Integrals. Let the body be idealized into the three phases shown in figure 3. Let the outer bounding surface of the "body," "heart," and "cavity" phases be designated as \( S_b, S_h, \) and \( S_c \), respectively. In the same order designate the outwardly directed unit normal vectors of each surface as \( N_b, N_h, \) and \( N_c \); and the specific conductivity of each phase as \( \gamma_b, \gamma_h, \) and \( \gamma_c \).

According to Gabor and Nelson1 the X axis component of the electric moment due to numerous sources and sinks located within a homogeneous volume conductor is

\[ M_x = -\gamma \int \int \int \nabla \phi \cdot dV. \]

Applying Green's theorem in the second form to this equation we obtain for the conditions of figure 3

\[ M_x = -\gamma \int \int S_b \left[ i \cdot N_b \right] dS \]

\[ -\gamma_h \int \int S_h \left[ i \cdot N_h \right] dS + \gamma_c \int \int S_c \left[ i \cdot N_c \right] dS = 0 \]
Heart:  \[ M_x = \gamma_x \int_{S_x} \mathbf{N}_x \mathbf{V} \, dS \]
\[- \gamma_x \int_{S_x} \mathbf{N}_x \mathbf{V} \, dS - \gamma_x \int_{S_x} zN_c \mathbf{V} \, dS + \gamma_x \int_{S_x} zN_c \mathbf{V} \, dS \]
\[= 0 \quad (2) \]

Cavity:  \[ M_x = \gamma_c \int_{S_c} \mathbf{N}_c \mathbf{V} \, dS \]
\[- \gamma_c \int_{S_c} zN_c \mathbf{V} \, dS = 0 \quad (3) \]

Comparing the right hand members of the three equations, note that the third terms of equations 1 and 2 are equal in magnitude and opposite in sign. The same is true of the last terms of equations 2 and 3. Adding the three equations together and collecting terms yields equation 2 of the text.

REFERENCES


ERRATA

The following corrections should be made for papers in the July issue of CIRCULATION RESEARCH:

p. 435. Legend to fig. 2, line 2. "bovine plasma" should read "bovine plasmius."

p. 437. Legend to fig. 4 should read "Graphs showing radioactivity (upper ordinate), fibrinogen level and clotting index (lower ordinate) in dog treated daily with 25 mg./Kg. trypsin.

Legend to fig. 5 should read "Graphs showing flows in arbitrary units (ordinates) through canilaized clots after removal of polyethylene rod in control dog (upper graph) and in trypsin treated dog (lower graph)."

N. Back, J. L. Ambrus, S. Goldstein and J. W. E. Harrison: In Vivo Fibrinolytic Activity and Pharmacology of Various Plasmiin (Fibrinolysin) Preparations
p. 441. Legend to fig. 1, line 3: "human plasma" should read "human plasmius."
A Theoretical Analysis of Intracavitary Blood Mass Influence on the Heart-Lead Relationship

DANIEL A. BRODY

Circ Res. 1956;4:731-738
doi: 10.1161/01.RES.4.6.731

Circulation Research is published by the American Heart Association, 7272 Greenville Avenue, Dallas, TX 75231
Copyright © 1956 American Heart Association, Inc. All rights reserved.
Print ISSN: 0009-7330. Online ISSN: 1524-4571

The online version of this article, along with updated information and services, is located on the World Wide Web at:
http://circres.ahajournals.org/content/4/6/731

Permissions: Requests for permissions to reproduce figures, tables, or portions of articles originally published in Circulation Research can be obtained via RightsLink, a service of the Copyright Clearance Center, not the Editorial Office. Once the online version of the published article for which permission is being requested is located, click Request Permissions in the middle column of the Web page under Services. Further information about this process is available in the Permissions and Rights Question and Answer document.

Reprints: Information about reprints can be found online at:
http://www.lww.com/reprints

Subscriptions: Information about subscribing to Circulation Research is online at:
http://circres.ahajournals.org/subscriptions/