Vessel Caliber and Branch-Angle of Human Coronary Artery Branch-Points

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SUMMARY Measurements were made of parent and branch vessel diameters and of the included angles of branch-points from postmortem human coronary arteriograms to determine the usefulness of theoretical equations relating the relationships between parent and branch vessel calibers and between arterial caliber and branch-angle. The formulas were based on the concept that blood vessel size and arrangement provided for blood flow with minimum energy loss. Size relationships between parent vessel and its branches were determined for 42 left main and 53 other epicardial coronary artery branch-points in hearts with angiographically normal arteries. Left main coronary artery branch-points were studied in 68 hearts with various degrees of angiographically defined coronary artery disease.

The concept of the principle of minimum work as applied to the function of the circulatory system has been well stated by D'Arcy Wentworth Thompson in his classic work, On Growth and Form:

"That this mechanism is the best possible under all the circumstances of the case, that its work is done with a maximum of efficiency and at a minimum of cost, may not always lie within our range of quantitative demonstration, but to believe it to be so is part of our common faith in the perfection of Nature's handiwork...."

The minimum work concept has been applied to a theoretical analysis of the problem of blood flow through arterial branch-points relative to the size of parent and branch vessels and to branch-angles. By assuming the circulation to have the steady laminar flow of a Newtonian fluid which obeys the Poiseuille equation, the relationship has been derived that the cube of the radius of a vessel should equal the sum of the cubes of the radii of the vessels into which it branches. With a similar consideration the angle of vascular branching has been predicted to be a function of the relative sizes of the parent trunk and its offspring. Although it is well known that blood flow does not follow the assumptions made, the possible usefulness of these principles to a description of deviations from normal vascular geometry could then be related to vessel wall disease led us to examine their validity for the coronary arterial tree. Postmortem coronary arteriograms from hearts with various degrees of vascular disease were studied to determine the relationship of parent to branch vessel size and the branch-angle.

Methods

We reviewed the hearts from 738 patients in the autopsy files of The Johns Hopkins Hospital, studied by a standard method employing coronary arteriography and fixation in postmortem human coronary arteriograms to determine the relationship of parent to branch vessel size and the branch-angle.

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they were orthographic projections at right angles to each other. To determine the diameters of the parent and branch vessels, we used an optical measuring device with a scale calibrated to 0.1 mm. A sheet of tracing paper was overlaid and the branch-point area was traced from both radiographs. The center line of the parent and each branch vessel was determined on each tracing and drawn. The point of intersection of the axes of the vessels was taken as the origin of a three-dimensional Cartesian coordinate system. With T-square and triangles, lines were drawn so that the coordinates of an arbitrarily chosen point on the axis line of each vessel, which represented the end of the vector, could be measured on the construction. The angle \( \theta \) between two vessels was calculated by considering the coordinates of the points to represent scalar values for vectors, \( A \) and \( B \), representing the axes of the vessels and using the formula:

\[
\cos \theta = \frac{A_x B_y + A_y B_x + A_z B_z}{|A| |B|}
\]

Repeat determinations on the same vessel showed reproducibility in the range of 1 or 2 degrees difference. Independent determinations of all three angles around a branch-point produced a sum in the range of 330°-362°. That the total of the angles is usually less than 360° is because the majority of epicardial coronary artery branch-points studied lay on a slightly curved surface rather than in a plane.

**Results**

In the first review of the 738 coronary angiograms each usable heart was assigned to a category of severity of angiographically detectable coronary artery disease on a scale of 0 to 4+. In 145 hearts with grade 0 there was no vascular abnormality seen on the angiograms and at most only trivial atherosclerosis recorded from examination of the multiple transsections of the coronary arteries made at 2- to 3-mm intervals. The 108 hearts in grade 1 had only mild irregularities detectable on the coronary arteriogram, with no lesion exceeding 20% occlusion of the luminal area as confirmed by gross study. Grade 2, with 99 hearts, had either moderately widespread obstructions of up to 50% of lumen area or a single severe or complete occlusion with an otherwise normal angiogram, the angiographic findings again being confirmed by the results of direct gross examination. In grade 3, with 115 hearts, and grade 4 with 13 hearts, widespread arterial disease was combined with severe stenoses or complete vascular occlusion. A total of 258 hearts was excluded from the study because the injections or angiograms were incomplete or inadequate.

**THE RELATIONSHIP OF THE CALIBERS OF PARENT AND BRANCH VESSELS**

The diameter of the left main coronary artery and its branches could be determined in 110 hearts. Measurements of parent and branch vessel diameters were made at 53 other branch-points of the epicardial coronary arteries from hearts with grade 0 coronary disease. The Wang computer was programmed to solve the equation, 

\[
(D_{\text{Parent}})^n = (D_{\text{Branch 1}})^n + (D_{\text{Branch 2}})^n + (D_{\text{Branch 3}})^n + \ldots \ldots \text{for } n \text{ for each branch to the nearest thousandth using the measured values of } D \text{. The mean, standard deviation, and range of } n \text{ for each group are shown in Table 1.}
\]

The values of \( n \) do not show significant differences between the groups of successive increases in coronary artery disease. The larger mean exponent for the left main coronary artery branch-points with no angiographic vessel disease is partly explained by two unusual cases with large \( n \) values where the diameters of parent and both branch vessels were nearly equal. With increasing degrees of vessel disease the exponent decreased but the mean diameter of the left main coronary artery and the mean heart weight were similar in these groups.
The area ratio of parent and branch vessels

Area ratios for branch-points were calculated from the formula:

\[
\text{Area ratio} = \frac{(D_{\text{Branch}})^3}{(D_{\text{Parent}})^3} = \frac{(D_{\text{Branch} 1})^3 + (D_{\text{Branch} 2})^3 + (D_{\text{Branch} 3})^3 + \ldots}{(D_{\text{Parent}})^3}
\]

The mean and standard deviation of the values for each group are shown in Table 1. There is a smaller mean area ratio in the atherosclerotic left main coronary artery branches than in the normals. The similarity of parent vessel diameters for these groups suggest that the lower area ratios observed with increasing vascular disease are caused by greater involvement of the branches than of the parent.

The relationship of vessel caliber and branch-angle

The angle between parent and branch vessels and/or between branch vessels was determined as described above for left main and other epicardial coronary artery branches. The result of this study shows that there is a relationship between the size of the parent trunk and its branches in normal coronary arteries. Theoretical considerations based on the concept of minimum work had suggested that the cube of the diameter of the parent vessel should equal the sum of the cubes of the branch vessel diameters. Derivation of the exponent \( n \) for this relationship from direct measurements of coronary artery diameters shows close correspondence to the theoretical exponent (Table 1). When the caliber of the parent vessel, predicted from the measured diameters of its branches by the formula, \((D_{\text{Parent}})^3 = (D_{\text{Branch} 1})^3 + (D_{\text{Branch} 2})^3 + (D_{\text{Branch} 3})^3 + \ldots\), is compared to the measured values in hearts with grade 0 atherosclerosis. By inspection of radiographs it was evident that the angle between the parent vessel and the larger branch was usually greater than that between the parent and smaller branch. The results in the 12 instances in which these angles were determined corresponded to the impression gained from inspection. The range of values obtained was large. For example, five angles between the parent vessel and a branch that was 90% or greater in cross-sectional area than the parent, had a range of 130° to 169°.

Most attention was paid to the angle between branches since it could be more readily and accurately determined. Graphic comparisons of ratios of vessel caliber and the included angle between branches failed to reveal any discernible consistent relationship. A wide range of included angles (32° to 124°) was found between branch vessels. The angle did not appear to be a function of the relative sizes of the vessels (Fig. 2). These branch-angle determinations included a wide range of parent vessel diameters (1.0 to 5.0 mm). In three hearts it was possible to determine, respectively, five, seven, and eight branch-angles that showed a wide range for that heart. No indication of an angle peculiar to a particular heart was obtained. Branch-angles in coronary arteries with atherosclerosis were not studied, because inspection of their radiographs showed no evident differences from the angiographically normal group that had failed to show any correlation between vessel caliber and branch-angle.

Discussion

The results of this study show that there is a relationship between the size of the parent trunk and its branches in normal coronary arteries. Theoretical considerations based on the concept of minimum work had suggested that the cube of the diameter of the parent vessel should equal the sum of the cubes of the branch vessel diameters. Derivation of the exponent \( n \) for this relationship from direct measurements of coronary artery diameters shows close correspondence to the theoretical exponent (Table 1). When the caliber of the parent vessel, predicted from the measured diameters of its branches by the formula, \((D_{\text{Parent}})^3 = (D_{\text{Branch} 1})^3 + (D_{\text{Branch} 2})^3 + (D_{\text{Branch} 3})^3 + \ldots\), is compared to the mea-

### Table 1 Exponent n and Area Ratios for Coronary Artery Branch-Points

<table>
<thead>
<tr>
<th>Group</th>
<th>No. of observations</th>
<th>Exponent n</th>
<th>Area ratio</th>
<th>( D_{\text{Parent}} ) (mm)</th>
<th>Heart wt (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left main coronary arteries</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grade 0 AS</td>
<td>42</td>
<td>3.2 ± 1.6</td>
<td>1.28 ± 0.18</td>
<td>2.5 ± 0.5</td>
<td>538 ± 217</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.5 - 14.2)</td>
<td>(0.93 - 1.4)</td>
<td>(1.0 - 3.3)</td>
<td>(168 - 1040)</td>
</tr>
<tr>
<td>Grade 1 AS</td>
<td>26</td>
<td>2.8 ± 1.3</td>
<td>1.18 ± 0.16</td>
<td>2.4 ± 0.4</td>
<td>561 ± 165</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.9 - 6.2)</td>
<td>(0.79 - 1.49)</td>
<td>(1.9 - 3.5)</td>
<td>(280 - 900)</td>
</tr>
<tr>
<td>Grade 2 AS</td>
<td>25</td>
<td>2.6 ± 1.5</td>
<td>1.18 ± 0.15</td>
<td>2.3 ± 0.4</td>
<td>515 ± 142</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.3 - 4.7)</td>
<td>(0.93 - 1.44)</td>
<td>(1.9 - 3.3)</td>
<td>(280 - 822)</td>
</tr>
<tr>
<td>Grade 3 and 4 AS</td>
<td>17</td>
<td>2.2 ± 2.1</td>
<td>1.10 ± 0.39</td>
<td>2.4 ± 0.4</td>
<td>545 ± 95</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.8 - 10.8)</td>
<td>(0.42 - 1.71)</td>
<td>(1.9 - 3.3)</td>
<td>(425 - 710)</td>
</tr>
<tr>
<td>Other epicardial coronary</td>
<td>53*</td>
<td>2.7 ± 1.3</td>
<td>1.13 ± 0.14</td>
<td>2.8 ± 0.9</td>
<td>389 ± 147</td>
</tr>
<tr>
<td>arteries: Grade 0 AS</td>
<td></td>
<td>(1.7 - 11.2)</td>
<td>(0.88 - 1.77)</td>
<td>(1.0 - 5.0)</td>
<td>(220 - 730)</td>
</tr>
</tbody>
</table>

Results are given as mean ± sd. Ranges are shown in parentheses.

AS = atherosclerosis.

* Made on 17 hearts.

![Figure 2](http://circres.ahajournals.org/)

The ratio of the diameters of larger to smaller branch vessels compared to the included angle for that branch-point. There appears to be no correlation between these values.
measured $D_{\text{parent}}$, the correspondence, for angiographically normal coronary arteries, is excellent (Fig. 3).

With abnormal vessels the measured $D_{\text{parent}}$ tends to be greater than the predicted $D_{\text{parent}}$ (Fig. 4). The same trend is seen with the area ratio, the more severe degree of vascular disease having larger parent vessels relative to branches. The parent trunks are of similar caliber in the diseased vessels and the normals. The decreased area ratio in the left main coronary artery branch for more severe vessel disease is probably a reflection of the common observation that vessel lumen narrowing is usually more severe in the proximal left anterior descending and circumflex coronary arteries than in the left main coronary artery. The mean heart weight for the groups are nearly the same.

The area ratio is not particularly useful for describing the characteristics of branch-points since it could be expected to vary according to the relative sizes of the branch vessels. As shown above, the ratio of the cube of the diameter of the parent vessel to the sum of the cubes of the branch vessels is 1. The same relationship between diameters squared, i.e., the area ratio, will vary according to the sizes of the branch vessels relative to each other. In theory, the area ratio would be 1.26 when the branch vessels are equal and would approach 1.0 if one branch is very small and the other nearly the size of the parent trunk. In a recent review of the subject, Stehbens\(^6\) also has expressed caution in correlating area ratio determinations with abnormalities of blood flow or atherosclerosis.

No relationship could be discovered between the sizes of branch vessels and the angles between them. Our impression, based on measurements and radiographs, was that the larger of two branches tended to pursue a course closer to the path of continuation of the parent trunk than did the smaller branch. Measurements showed a wide variation and considerable overlap in this feature. It appears that relative branch vessel diameter has little to do with determining coronary artery branch-angle. Upon reexamining the radiographs in the light of this negative finding, we have been left with the impression that the destination of the blood vessel may help determine the angles made with its parent and fellows. For example, the artery to the sinoatrial node has a rather constant destination but its point of origin shows considerable variation and the angle it makes with the parent trunk seems determined, in part at least, by the course it must pursue to reach its termination. A similar consideration may apply to the bifurcation of the left main coronary artery into left anterior descending and circumflex branches. It is possible that the branch-angle of the epicardial coronary arteries is determined by relative size when these structures first develop embryologically\(^a\) but that with

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**Figure 3** Comparison of $D_{\text{parent}}$ ($D_p$) predicted from the formula: $(D_{\text{parent}})^3 = (D_{\text{Branch}1})^3 + (D_{\text{Branch}2})^3 + (D_{\text{Branch}3})^3 + (D_{\text{Branch}4})^3$... with $D_{\text{parent}}$ measured from the arteriogram. There is close correspondence for left main coronary artery (left) and other epicardial coronary artery branch-points (right). 0 AS is the grade and means no arteriosclerotic change was seen.

**Figure 4** Comparison of $D_{\text{parent}}$ ($D_p$) predicted and $D_{\text{parent}}$ measured for the left main coronary artery branch-point in hearts with various degrees of arteriographic coronary arteriosclerosis (AS). There is greater scatter and a greater tendency for the measured value to exceed the predicted with increasing vascular disease. The diagonal lines show the relationship for $n = 3$: 1+, 2+, 3 and 4+ denote grades of AS.
subsequent growth and changes of myocardial form\textsuperscript{8} the vascular branch-angles may become altered. The fixed unmodifiable nature of the branch-point location, once the vessel wall forms, is well shown by the abnormal branch-points that may occur in the aorta and give rise to coarctations.\textsuperscript{9}

The approach taken in this study has been to start with theoretical relationships between caliber of parent and branch vessel\textsuperscript{2} and between vessel caliber and branch-angle.\textsuperscript{3} The equations were derived from the principle that vascular geometry will develop in such a manner that the total energy expended in making the blood flow and in the maintenance of blood volume and vessel wall will be a minimum; that is, between large vessels with low resistance to flow but a high cost of maintenance of blood volume and vessel wall, on one hand, and small vessels with high flow resistance but low maintenance energy, on the other, there is an ideal intermediate state of least energy expenditure to which the vascular geometry will approximate during growth. In deriving the equations the assumption was made that the coronary circulation is a steady laminar flow of a Newtonian fluid in a cylindrical tube. The elasticity of the vessel walls, pulsatile nature of flow, wave reflections, shear effects, vascular tortuosity, and effects of branch-points were not considered. Thus, the correspondence between the predicted and observed relationship of calibers does not necessarily support the validity of the minimum work principle as the prime determinant of vascular geometry. Other factors, such as reduction of wave reflection\textsuperscript{11} or maintenance of a uniform wall shear stress, as has been suggested for small blood vessels,\textsuperscript{12} may be important in producing the observed relationship of epicardial coronary artery calibers, especially in view of the lack of correspondence between branch vessel caliber and branch-angle.

In conclusion, measurements show the relationship that the cube of the diameter of the parent coronary artery equals the sum of the cubes of its branch vessel diameters. From this consideration area ratios should be interpreted in terms of the relative sizes of the branch vessels. No consistent relationship could be discerned between coronary artery branch vessel diameters and the included branch-angle. It is suggested that the point at which a coronary artery branches is determined by some unknown factor and that the branch-angle is determined by the destinations of the branch arteries.

References
Vessel caliber and branch-angle of human coronary artery branch-points.
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