Pressure Drop across Artificially Induced Stenoses in the Femoral Arteries of Dogs

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ABSTRACT

Stenoses were artificially induced in 13 large mongrel dogs by implanting small hollow cylindrical plugs in their femoral arteries. The instantaneous pressure drop across the stenosis and the flow rate were measured for a series of stenoses varying in severity from 52.3 to 92.2%. Mean pressure drops ranged from approximately 2 to 30 mm Hg with peak pressure drops ranging from 9 to 53 mm Hg. The pressure drop could be estimated from a relatively simple equation that was originally developed for flow through model stenoses. With this equation, the effects of several factors that contribute to the pressure drop, including stenosis size and shape, artery lumen diameter, blood density, blood viscosity, and velocity and acceleration of flow, could be clearly delineated. For severe stenoses, unsteady flow effects were small, and flow could be treated as quasi-steady. Calculations based on data obtained from the dog experiments revealed that the mean pressure drop across a stenosis increased nonlinearly with percent stenosis and showed quantitatively that the value of critical stenosis decreased with increasing demand for blood flow.

KEY WORDS
flow measurement
blood flow
pulsatile flow
critical stenosis
turbulence
flow resistance

Hemodynamic factors associated with arterial stenoses have been considered by numerous investigators (1–8). Of prime concern has been the relationship between the flow reduction in a stenosed artery and the severity of the stenosis as indicated by the percent reduction in lumen area. From a consideration of simple hydraulic principles, it can be readily deduced that flow to a particular vascular bed is a function not only of stenosis resistance but also of collateral and peripheral resistance. Unless the stenosis is severe, the flow through a vascular bed is controlled primarily by the bed resistance for a given arterial blood pressure. However, as the stenosis becomes severe, its resistance or impedance to flow as evidenced by the pressure drop across the stenosis becomes highly significant and, in fact, can ultimately limit the flow to the peripheral bed. Thus, a knowledge of the relationship between the flow and the pressure drop across a stenosis is a prerequisite to the understanding of the effect of stenotic obstructions on the distribution of blood flow to peripheral vascular beds.

May et al. (3) have suggested that the factors influencing the pressure drop across a stenosis can be studied by considering the stenosis to be a combination of a sudden contraction, Poiseuille flow through the narrowed lumen, and a sudden expansion. Through the use of a combination of simple, steady-flow hydraulics relationships, they have developed an equation for the pressure drop. However, this approach does not take into account the fact that arterial blood flow is nonsteady or pulsatile. Moreover, different stenosis configurations (axisymmetric, nonsymmetric, etc) cannot be incorporated into the equation.

In two recent studies, Young and Tsai (9, 10) have investigated the flow characteristics in arterial stenoses through an extensive series of model experiments. On the basis of both steady- and unsteady-flow tests, they have found that the major factors controlling the pressure drop, $\Delta p$, across a stenosis can be estimated from the equation

$$\Delta p = \frac{K_o \mu}{D} U + \frac{K_i}{2} \left( \frac{A_o}{A_i} - 1 \right)^2 \rho |U| U + K_u \rho L \frac{dU}{dt}, \quad (1)$$

where $A_o = \text{area of the unobstructed tube}$, $A_i = \text{minimum cross-sectional area of the stenosis}$, $D = \text{diameter of the unobstructed tube}$, $K_o$, $K_i$, and $K_u$ = experimentally determined coefficients, $L = \text{length over which the pressure drop is measured}$, $t = \text{time}$, $U = \text{instantaneous velocity in the unobstructed tube (average over the cross section)}$, $\rho = \text{blood density}$.
fluid density, and $\mu = \text{fluid viscosity}$. The average velocity is obtained by dividing the instantaneous value of the volume flow rate by the area $A_o$. The first term on the right of Eq. 1 represents the pressure drop due to viscous effects, the second term is the pressure drop due to nonlinear effects associated with the convergence and divergence of the flow in the stenosis and with turbulence, and the last term accounts for the pressure differential required to accelerate the fluid. $K_v$ and $K_t$ are dependent on stenosis geometry ($K_v$ is strongly dependent on geometry) but can be approximated from steady-flow tests. The value of $K_u$ (obtained empirically) that gives the best fit of the data is 1.2. For a variety of flow conditions and stenosis geometries, including both symmetric and nonsymmetric stenoses, Young and Tsai (9, 10) have been able to predict pressure drops within about 20% of the corresponding experimentally determined values.

The purpose of the present study was to measure in vivo pressure drops in artificially constricted arteries and to investigate the applicability of Eq. 1 for predicting pressure losses due to stenoses in actual arteries.

**Methods**

Thirteen large mongrel dogs (50-70 lb) were anesthetized with sodium pentobarbital and allowed to breathe room air spontaneously through an endotracheal tube. After heparinization, an artificial stenosis was induced in the left femoral artery of each dog by placing a cylindrical hollow plug in the artery through a small incision. Although external banding is a commonly used technique for producing a localized constriction, the geometry cannot be readily determined; therefore, the use of a plug having a well-defined geometry seemed preferable for the present study. The additional trauma due to plug insertion was not an important consideration, since our main interest was in establishing the relationship between the pressure drop and the flow for a variety of flow conditions. The plug was located in the section of femoral artery proximal to the branching of the vessel into the caudal femoral and popliteal arteries.

Two different types of plugs were used (Fig. 1). The plug of Figure 1a has a streamlined entrance, whereas the plug of Figure 1b is blunt at both ends. A series of polycarbonate plugs was fabricated so that a large number of plugs with different outside diameters, $D$, and inner diameters, $d$, was available. For each dog a plug was selected to give a good fit within the artery, and for all tests the outside diameter of the plug was within 0.25 mm of the measured lumen diameter. To obtain the lumen diameter, the outside diameter of the artery was determined from a circumference measurement, and twice the arterial wall thickness was subtracted from this value. The circumference was obtained by encircling the vessel four times with a fine thread and measuring the resulting length (circumference $\times$ 4). This technique was more reproducible than a direct diameter measurement using a device such as a vernier caliper. The ratio of the wall thickness to the outer radius of the artery was taken as 13%; measurements of the double-wall thickness obtained with a micrometer supported this figure. All diameter measurements were made when blood pressure was normal. The length-diameter ratio, $L_s/D$, for all plugs was 2. To determine the coefficients $K_u$ and $K_v$ of Eq. 1, in vitro steady-flow tests were run using water-glycerol mixtures. The results for $K_u$ are shown in Figure 1 (see Appendix for additional details). This curve reveals the strong dependence of $K_u$ on percent stenosis for a given plug shape. Included on this plot are data for both the blunt and streamlined plugs. The coefficient $K_v$ was not as strongly dependent on percent stenosis and fell in the range of 1.57 to 2.31 when the percent stenosis was in the range of 60 to 90%.

For each experiment the left femoral artery was exposed, and two side branches, one proximal and the other distal to the site of the stenosis, were cannulated for pressure measurements. The distance between the two side branches ranged from 27 to 56 mm, with an average value of 43 mm. Pressures were recorded using two Statham P23Db transducers, and the pressure drop was obtained by electronically subtracting the two pres-
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sure signals. The pressure transducers were connected to the artery with relatively stiff catheters, and the pressure transducer-catheter system had a natural frequency in excess of 35 Hz. Prior to each test the pressure-measuring system was calibrated statically, and, subsequently, the catheters were connected to a common dynamic pressure source to ensure that both pressure transducers were properly balanced. For this purpose the right femoral artery was cannulated, and both pressure catheters were connected to this common pressure source. Both pressure-measuring systems were, therefore, exposed to the same dynamic pressure; if they were properly matched, no differential pressure was observed. Small air bubbles in either system caused an unbalance; thus, prior to use the system was flushed repeatedly until no significant differential pressure was detected.

The instantaneous flow rate was obtained using a Biotronix BL-610 electromagnetic flowmeter with non-cannulating flow probes. The zero flow baseline was obtained by occluding the femoral artery distal to the flow probe, and the baseline was checked several times during the course of the experiment. Mean flow rate calibration was obtained in situ at the conclusion of each experiment by cannulating the femoral artery and measuring the time required for various volumes of blood to flow into a graduated cylinder. The dynamic response of the flowmeter was checked electronically and found to be flat to approximately 30 Hz with a linear phase shift of 3.6°/Hz.

For each experiment, the mean and the instantaneous flow rate and the pressure drop were recorded prior to the insertion of the plug. After the plug had been inserted, the measurements were repeated. Flow and pressure data were recorded on both a Grass strip-chart recorder and an Ampex model FR-1300 magnetic tape recorder. The limiting apparent viscosity of the blood, as determined from a Wells-Brookfield cone and plate viscometer, varied between 0.034 and 0.045 dynes-sec/cm².

Results

Figure 2 shows a typical recording of the instantaneous flow and the pressure drop obtained in the femoral artery of one dog. The top set of traces represents normal conditions, i.e., no stenosis present; the bottom set of traces is for the same artery with a 78.1% stenosis (78.1% reduction in lumen area). In general, the presence of a stenosis caused a decrease in the flow through the artery and an increase in the pressure drop. For the experiments performed, the percent stenosis varied from 52.3% to 92.2%. The pressure drop recordings represented the instantaneous pressure drop that developed across the stenosis. Two values were of particular interest: (1) the peak pressure drop occurring during the cycle and (2) the mean pressure drop, \( \Delta p \), defined as the time average of the pressure drop over a cycle, i.e.,

\[
\Delta p = \frac{1}{T} \int_{t_0}^{t_0 + T} \Delta p dt,
\]

where \( t_0 \) is some reference time and \( T \) is the period of one flow cycle. Both mean and peak pressure drops were determined for each experiment; a summary of the results is given in Table 1.

Since the Reynolds number, \( Re = \rho DU/\mu \), is an important dimensionless flow characteristic, values for \( Re \) are also tabulated in Table 1. The mean Reynolds number was obtained by letting \( U = U \), where \( U \) is the mean velocity taken over a flow cycle, i.e.,

\[
U = \frac{1}{T} \int_{t_0}^{t_0 + T} U dt.
\]

The peak Reynolds number was obtained by letting \( U = U_p \), where \( U_p \) is the peak velocity occurring during a flow cycle. The mean Reynolds numbers for unobstructed femoral blood flow averaged 188 ± 120 (SD) for the 13 dogs, the peak Reynolds numbers averaged 648 ± 269, the mean pressure drops averaged 2.0 ± 0.9 mm Hg, and the peak pressure drops averaged 9.9 ± 3.3 mm Hg.

Although the pressure drop generally increased with increasing severity of stenosis (Table 1), the velocity or flow rate was also important. Thus, as the stenosis increased in severity, the flow rate

![Figure 2](image-url)

Typical flow and pressure drop recordings. Top: Unobstructed artery. Bottom: Same artery with a 78.1% stenosis.
tended to decrease, and it was possible to have a smaller pressure drop for a more severe stenosis. The percent reduction in both mean and peak flow varied considerably from dog to dog for a given percent stenosis. We think this variation is due to the important effect of collateral and peripheral resistance on flow through the stenosis. To more clearly reveal in graphical form the nature of the variation in flow rate with percent stenosis, the data, including both mean and peak flow rates, were divided into four groups, and the average percent flow reduction was plotted as a function of percent stenosis (Fig. 3) (individual values are given in Table 1). The resulting curve shows the general decrease in flow rate with increasing severity of stenosis and also the possible wide variation in the change in flow rate at a given percent stenosis.

For each experiment the predicted value of the pressure drop was calculated from Eq. 1 using the values of $K_s$ and $K_t$ obtained from in vitro steady-flow tests in a rigid-walled model system and the values of $U$ and $dU/dt$ measured with the electromagnetic flowmeter. As was the case in the model experiments (10), the value of $K_s$ for the animal tests was found to be approximately unity, and a value of 1.2 gave the best overall fit of the data. Since the instantaneous average velocity, $U$, was related to the flow rate, $Q$, through the equation $U = Q/A_o$, the value of $U$ was obtained from the flowmeter recordings. For purposes of analysis, a typical flowmeter recording, taken over one period, was selected and digitized, and a Fourier series expansion for the velocity $U$ (containing ten harmonics) was obtained. The Fourier series expansion was subsequently differentiated and used as a

TABLE 1
Summary of Data from Artificially Constricted Femoral Arteries in 13 Dogs

| Dog | D (mm) | L (mm) | Plug geometry | % Stenosis | % Flow reduction Mean | Peak | Velocity (cm/sec) Mean | Peak | Reynolds number Mean | Peak | Pressure drop (mm Hg) Experimental Predicted Experimental Predicted |
|-----|--------|--------|---------------|------------|-----------------------|------|------------------------|------|----------------------|------|----------------------|------|----------------------|------|
| 1   | 3.20   | 44     | S             | 52.3       | 25 16                 | 14.7 | 73.6                   | 119 596 | 3.5 2.0              | 13.5 15.7 |                      |      |
| 2   | 3.25   | 41     | S             | 54.0       | 0 0                  | 11.0 | 49.4                   | 90 416  | 2.5 1.4              | 9.5 9.5   |                      |      |
| 3   | 3.75   | 48     | S             | 65.3       | 12 28                | 9.6  | 42.0                   | 91 401  | 3.5 1.7              | 13.0 11.3 |                      |      |
| 4   | 3.75   | 49     | S             | 70.1       | 25 14                | 4.4  | 23.5                   | 36 190  | 4.4 1.0              | 16.2 6.8  |                      |      |
| 5   | 4.10   | 27     | S             | 76.0       | 33 46                | 35.6 | 68.2                   | 324 622 | 17.8 17.9            | 53.3 48.8 |                      |      |
| 6   | 3.60   | 52     | B             | 76.4       | 45 45                | 16.2 | 48.5                   | 145 436 | 4.5 5.4              | 27.8 25.0 |                      |      |
| 7   | 4.25   | 28     | S             | 78.0       | 9 32                 | 19.0 | 43.8                   | 246 566 | 8.7 7.1              | 28.7 23.5 |                      |      |
| 8   | 3.29   | 50     | B             | 78.1       | 22 39                | 16.9 | 46.0                   | 143 387 | 8.2 7.6              | 30.0 27.9 |                      |      |
| 9   | 4.45   | 65     | B             | 79.8       | 27 41                | 16.9 | 52.8                   | 188 587 | 10.6 9.3             | 37.5 39.7 |                      |      |
| 10  | 3.20   | 36     | S             | 87.8       | 8 48                 | 12.3 | 33.5                   | 96 266  | 12.5 12.6            | 53.0 51.6 |                      |      |
| 11  | 3.45   | 45     | S             | 89.7       | 50 36                | 3.4  | 20.4                   | 30 179  | 8.1 4.5              | 25.0 30.8 |                      |      |
| 12  | 4.35   | 46     | S             | 91.0       | 29 56                | 9.6  | 25.5                   | 128 342 | 10.0 14.7            | 33.0 57.2$^*$ |                  |      |
| 13  | 3.95   | 30     | S             | 92.2       | 45 61                | 12.8 | 19.9                   | 126 196 | 30.0 21.8            | 50.0 43.5 |                      |      |

$D$ = internal lumen diameter proximal to stenosis, $L$ = distance between side branches cannulated for pressure measurements, $B$ = blunt plug, and $S$ = streamlined plug (see Fig. 1). Reynolds number = $\nu DU/\mu$ with $U = \overline{U}$ (mean) and $U = U_p$ (peak).

* This unusually high value fell outside the 95% confidence limits for future point estimation and was not used for the linear regression shown in Figure 5.
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convenient means for obtaining the instantaneous value of $dU/dt$ in Eq. 1. This term could also be determined by numerical differentiation. It should be noted that since Eq. 1 is nonlinear, the pressure drop cannot be obtained as the sum of pressure drops caused by individual harmonics in the series expansion for the velocity. Figure 4 shows the variation in $\Delta p$ with time for one dog with a 78.1% stenosis. Both the experimental wave form and the predicted wave form are shown. For all tests in which there was good agreement between the experimental and the predicted mean and peak pressure drops, the complete wave form was also satisfactorily predicted, as shown in Figure 4. Both the mean and peak values of $\Delta p$ were predicted for all experiments, and these values are compared with the experimental values of $\Delta p$ in Table 1. Values of $\Delta p$ given in Table 1 represent the actual measured pressure drop across the stenosis, i.e., the corresponding values in the unobstructed artery have not been subtracted. A graphical presentation of the data is shown in Figure 5. It is apparent that a good correlation exists between the predicted pressure drop, $\Delta p_p$, and the experimentally measured values of pressure drop, $\Delta p_e$. The data were considered in three groups: mean pressure drop data, peak pressure drop data, and the combined data including both mean and peak pressure drop values. For each of these sets of data, two regression lines were obtained; one did not pass through the origin, but the other was forced to pass through the origin. Specific regression equations are given in the legend for Figure 5, along with the corresponding correlation coefficients. The value of the intercept was small in all cases (less than 1 mm Hg). The correlation coefficient was smaller for the mean pressure drops than it was for the peak values. This difference is not surprising, since values of mean pressure drop are small and thus more sensitive to experimental errors in the measurement of the pressure drop.

For the combined data the regression line through the origin is

$$\Delta p_p = 0.93\Delta p_e,$$

with a correlation coefficient of 0.97. The upper and lower confidence bounds on the slope at the 95% level are 0.98 and 0.87, respectively. Since the slope of the regression line is less than unity, the predicted values of $\Delta p$ are generally lower than the measured values by about 7% (for the mean pressure drop data alone the difference is 16%).

**Discussion**

The experimental results reported in this paper support the applicability of Eq. 1 for the estimation of pressure drop across arterial stenoses. As graphically illustrated in Figure 5, there is scatter in the data, but a high degree of correlation exists between the experimental and predicted values. The random scatter is due to measurement errors, whereas the slightly lower values of predicted pressure drops as compared with experimental pressure drops may reflect the more fundamental problem of attempting to describe a very complex flow situation with a relatively simple equation. Since the pressure drop is a function of several variables, all of which are subject to measurement

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**Figure 4**

Comparison of experimental and measured pressure drops at 78.1% stenosis for one complete cycle.

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small errors in each of the measurements can combine to yield significant variations in the predicted pressure drop. The pressure drop is strongly dependent on the velocity, \( U \), and thus an accurate determination of this variable is required. Although the flowmeters used in the experiments were calibrated in situ at the conclusion of each experiment, small errors in the measured flow rate are virtually inevitable for in vivo experiments and may account for some of the scatter in the data. In addition, it is difficult to assess errors involved in dynamic pressure drop measurements. However, as discussed previously, the pressure-measuring system was calibrated statically at the beginning and the end of each experiment, and a dynamic balance check was made during each experiment. Pressure drops were recorded to 0.1 mm Hg, but we estimate that base-line drift and other errors associated with differential pressure measurements give an uncertainty in these measurements on the order of 0.5–1.0 mm Hg.

On the basis of all data, the predicted values of pressure drop appear to be slightly lower than the experimental values. Assuming that the general form of the pressure drop equation is correct, this result indicates that the coefficients \( K_v \) and \( K_i \) are too low. Since these two coefficients were obtained from steady-flow tests in rigid tubes, the apparent difference may be due to the fact that arterial blood flow is unsteady, the fact that the arterial wall is flexible, or both. As discussed previously (10), unsteady flow through a stenosis in a rigid tube gives peak pressure drops that are slightly lower than the corresponding values for steady flow. Thus, we suspect that the increased pressure drop for stenoses located in arteries over that for stenoses in rigid tubes may be due to additional energy losses associated with the distensible arterial walls. In most tests in which the percent stenosis was greater than about 70%, turbulence was induced by the stenosis, and the vibrations of the arterial wall distal to the stenosis could be readily detected. Thus, energy was being transferred from the fluid to the wall, thereby increasing the energy loss in the fluid as evidenced by the increased pressure drop.

The results of the experimental determination of the pressure drop along with Eq. 1 make it possible to ascertain the role of various factors in the determination of pressure loss induced by a stenosis. Since the viscosity and density of blood do not usually vary significantly in the circulation, these two factors can be assumed to be relatively constant. However, the geometry of the stenosis, including the shape, length, and percent stenosis, plays a very important role along with the instantaneous velocity. The time rate of change of the velocity also affects the instantaneous value of the pressure drop. As indicated by Eq. 1, the pressure drop arises from three sources. The first source is the viscous loss given by the first term on the right of Eq. 1, where \( K_v \) is strongly dependent on geometry. As discussed previously (10), the relative importance of this term is characterized by the dimensionless ratio \( K_v/Re_p \), where \( Re_p \) is the peak Reynolds number. The second source of the pressure drop is the nonlinear loss due to the convergence and divergence of the fluid as it enters and leaves the stenosis. This loss is given by the second term on the right of Eq. 1. As the fluid jet leaves the throat of the stenosis, it diverges to fill the lumen. Expanding jets are known to be unstable, and highly disturbed flow patterns and turbulence frequently accompany converging-diverging flows. The relative importance of this term is determined by the parameter \( \frac{1}{2}(A_j/A_s - 1)^2 \), since \( K_i \) is on the order of unity. Due to this term, the pressure drop varies nonlinearly with velocity. Moreover, this nonlinear term is important in the range of Reynolds numbers found in the larger vessels such as the femoral, carotid, or coronary arteries. The third source of the pressure drop is the inertia of the fluid; it is given by the third term on the right of Eq. 1. The relative influence of this factor is determined by the parameter \( La_j/U_p^2 \), where \( U_p \) is the peak acceleration of the mean flow and \( U_p \) is the peak velocity. The presence of the inertial term not only affects the magnitude of the instantaneous pressure drop but also causes a phase lag between the pressure drop and the velocity. For example, unobstructed flow through the femoral arteries shows a significant phase lag between the instantaneous pressure drop and the velocity. This phase lag can be observed in Figure 2. With the addition of a stenosis, the phase lag is decreased; for a severe stenosis of about 85–90%, the pressure drop and the velocity are essentially in phase due to the dominance of the first two terms in the pressure drop equation.

Values for the dimensionless ratios just discussed were determined for the present study, and the results are shown in Figure 6. To obtain the curve for \( K_v/Re_p \), a normal peak Reynolds number of 600 was assumed (this value is typical of those found in the experiments), and the Reynolds number was decreased with percent stenosis in accordance with the flow reduction curve of Figure 3. The parameter \( La_j/U_p^2 \) was found to be relatively constant and independent of percent stenosis (Fig. 6). An important conclusion to be drawn from this figure is that...
above approximately 85% stenosis the pressure drop is clearly dominated by the viscous and nonlinear terms and inertial effects are negligible. Thus, the flow can be treated as quasi-steady under these conditions. For less severe stenoses, inertial effects must be included, particularly if the shape of the pressure drop wave form and the phase relationships are to be considered. Although the results shown in Figure 6 apply specifically to the present study involving canine femoral arteries, it seems reasonable to assume that the general conclusions will be similar for other arteries of approximately the same size.

Since the mean rate at which blood is delivered to a particular vascular bed per heart beat is frequently of prime concern, the relationship between the mean pressure drop, \( \Delta \overline{p} \), and the mean velocity \( \overline{U} \), is of interest. Eq. 1 integrated over one flow cycle yields

\[
\overline{\Delta p} = \frac{K_v \mu}{D} \overline{U} + \frac{K_v}{2} \left( \frac{A_0}{A_1} - 1 \right)^2 \rho \overline{U}^2 \overline{U}^2.
\]  

where the bar over a symbol indicates the time-averaged value over a cycle. Note that the inertial term, when it is integrated over a cycle, drops out of the equation if the flow pulse is periodic. One complicating feature of Eq. 5 is the fact that the time-averaged value of the product \( |U| U \) is not equal to \( \overline{|U| U} \). It is well known that the time average of the square of a time-dependent variable, \( y(t) \), is not equal to the square of the time average of \( y(t) \), i.e., if

\[
\overline{y^2} = \frac{1}{T} \int_{t_0}^{t_0 + T} y^2 dt
\]

and

\[
\overline{y} = \frac{1}{T} \int_{t_0}^{t_0 + T} y dt,
\]

then

\[
\overline{y^2} \neq (\overline{y})^2.
\]

The difference between \( \overline{y^2} \) and \( (\overline{y})^2 \) will depend on the specific form of the function \( y(t) \). Thus, Eq. 5 is not a simple quadratic equation in terms of the mean velocity \( \overline{U} \); rather, the mean pressure drop depends on both the mean velocity and also the flow wave form through the term \( |U| U \). However, Eq. 5 can be written as

\[
\overline{\Delta p} = \frac{K_v \mu}{D} \overline{U} + \frac{K_v}{2} \left( \frac{A_0}{A_1} - 1 \right)^2 \rho |U| U^2 \overline{U}^2,
\]

where the time average of the nonlinear velocity term has been replaced by \( |U| U \), i.e.,

\[
|U| U = \beta \overline{|U| U},
\]

\( \beta \) depends on the velocity wave form. Values of \( \beta \) were determined for the 13 experiments performed; they varied between 1.0 and 4.8, with an average value of 2.6 ± 1.5 (SD). Thus, the time average of the velocity squared was significantly different from the square of the average velocity for the blood flow wave forms studied, and this difference should not be overlooked in predicting pressure drops.

The manner in which the mean pressure drop varies with percent stenosis can be estimated with the aid of Eq. 9. If we use typical values for the artery diameter, blood density, and blood viscosity (as indicated in the legend of Fig. 7) and set \( K_v = 2 \) and \( \beta = 2.5 \), the mean pressure drop can be expressed as a function of percent stenosis and mean velocity for a given arterial geometry. Values of \( K_v \) were determined from Figure 1. For the purposes of this example, the normal mean Rey-
Series of curves showing the effect of flow rate or Reynolds number (Re) on the mean pressure drop across stenoses of different degrees of severity. For these calculations \( \mu = 0.04 \text{ dynes} \cdot \text{sec/cm}^2, \rho = 1.05 \text{ g/cm}^3, \) and \( D = 3.5 \text{ mm} \).

nolds number was taken as 200 and allowed to decrease in accordance with Figure 3. With these assumed conditions, the predicted variation in mean pressure drop is shown by the solid line in Figure 7. A modest increase in pressure drop is noted until the artery is 75-80% constricted. Beyond this critical range, the pressure drop increases rapidly. The pressure distal to the stenosis is the driving pressure for flow through the peripheral vessels supplied by the artery. As this pressure decreases, due to the pressure drop across the stenosis, the flow through the stenosed artery will decrease unless the peripheral resistance is significantly reduced. The percent stenosis at which a precipitous increase in pressure drop occurs corresponds to the critical stenosis, i.e., the percent stenosis at which a small change in the obstructed lumen area causes significant alterations in flow and pressure drop (3, 4).

If the Reynolds number is allowed to remain constant at a value of 200 as the percent stenosis is increased, the results are shown by the lowest broken curve in Figure 7. There is little change for moderate stenoses, but as in the previous case the pressure drop starts to increase rapidly beyond a certain critical range. The value of the critical stenosis decreases with increasing Reynolds number as indicated by the three broken curves in Figure 7. These results show how the effect of a stenosis is strongly influenced by the rate of flow through the stenosis, a result noted by several investigators (3-8). For example, at a 90% stenosis under resting conditions \( (Re \approx 100) \), the mean pressure drop would be approximately 29 mm Hg. However, with increased demand for blood flow, as would occur with exercise or vasodilator drugs, the pressure drop would increase to 88 mm Hg for a mean Reynolds number of 200. It is apparent that the stenosis can easily become the limiting resistance to blood flow in a severely stenosed artery with increased demand, whereas it may not significantly affect the flow under more normal or resting conditions. The results shown in Figure 7 are based on a typical set of parameters and are included to focus attention on significant trends which are not as clearly evident for in vivo experiments in which important factors such as artery diameter and flow rate vary from dog to dog.

In summary, we conclude that the pressure drop across an arterial stenosis can be satisfactorily estimated through the use of Eq. 1. The coefficients \( K_c \) and \( K_r \) must be determined experimentally, but for a specified geometry they can be obtained from a simple steady-flow test. At the present time, there are no suitable analytical methods for the prediction of these coefficients for the essentially infinite variety of stenosis geometries that can be encountered. However, these coefficients may be primarily dependent on a limited number of basic geometrical characteristics, such as stenosis length and percent stenosis. For example, in the present study these coefficients were not strongly dependent on the shape of the stenosis (as shown in Fig. 1 in which blunt vs. streamlined plugs were compared). Thus, it may be possible to estimate \( K_c \) and \( K_r \) from a relatively small number of geometrical parameters which could be obtained in clinical situations using arteriography. Additional in vitro studies using a variety of model stenosis geometries are required before the relationship between \( K_c \) and \( K_r \) and specific geometrical characteristics can be more generally delineated. Although Eq. 1 represents an approximate description of a complex flow situation, it is believed to have an accuracy consistent with the accuracy of the required input data, such as stenosis geometry and flow rate, normally available for the cardiovascular system. With a better understanding of the de-
tailed flow characteristics and pressure losses associated with a stenosis, it should be possible to more accurately assess the influence of stenotic obstructions on regional blood flow.

Appendix

As discussed previously (9), the pressure drop across a constriction under steady-flow conditions can be approximated with an equation of the form

$$\frac{\Delta P}{\rho U^2} = \frac{K_v}{Re} + \frac{K_t}{2} \left( \frac{\Delta \theta}{A_1} - 1 \right)^2,$$

(11)

where the coefficients $K_v$ and $K_t$ depend on the dimensionless geometric ratios $L/D$, $LJ/D$, and $d/D$. The form of this equation can be deduced from a dimensional analysis, if it is recognized that (1) at very low Reynolds numbers the pressure drop must be linearly related to the velocity, i.e., viscous effects are dominant, and (2) at high Reynolds numbers the pressure drop depends on the square of the velocity, i.e., turbulence losses dominate. Thus, at least as a first approximation, a linear combination of low and high Reynolds number regimes seems reasonable.

To obtain the coefficients $K_v$ and $K_t$ for the plugs used in the present investigation, steady-flow tests were run in which the pressure drop was measured as a function of Reynolds number for a given geometry. The results of these tests for the streamlined plugs (Fig. 1a) are shown in Figure 8. The solid lines in Figure 8 represent Eq. 11 with the two coefficients $K_v$ and $K_t$ obtained simultaneously to give the best fit of the data in a least-squares sense. Values of $K_v$ for both the streamlined and blunt plugs are given in Figure 1, and the strong dependence of $K_v$ on percent stenosis is shown in this figure. Values of $K_t$ for the three streamlined plugs are given in the legend of Figure 8 and are much less sensitive to geometry. For the blunt plugs, $K_t$ varied between 1.81 and 2.06. In general, it appears that the pressure drop data can be fit satisfactorily with an equation of the form of Eq. 11. A better correlation with the data could be obtained with a more complicated form of equation, but this additional degree of accuracy is not thought to be justified. For comparative purposes, the corresponding dimensionless pressure drop for a straight tube (Poiseuille flow) is also shown in Figure 8. The large increase in pressure drop with increased severity of stenosis is clearly evident.

For all model tests, the ratio $L/D$ was 16. However, the measured pressure distribution along a tube containing a stenosis (9) reveals that the major pressure loss and the recovery take place over a length-diameter ratio of approximately 10, so the actual value of $L/D$ is not critical if it is not significantly lower than 10 or greater than 16. In the experiments with the femoral artery, $L/D$ varied with each dog and had an average value of 12. Only for three of these experiments was $L/D$ less than 10. For these three cases, the experimental pressure drops were higher for all three peak pressure drops, as would be expected for the low values of $L/D$.

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