Nonlinear Analysis of Aortic Flow in Living Dogs

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ABSTRACT

A nonlinear theory which considered the convective accelerations of blood and the nonlinear elastic behavior and taper angle of the vascular wall was used to study the nature of blood flow in the descending thoracic aorta of living dogs under a wide range of pressures and flows. Velocity profiles, wall friction, and discharge waves were predicted from locally measured input data about the pressure-gradient wave and arterial distention. Precision pressure-gradient waves free of static errors were obtained through forward and backward pressure-gradient measurements, and the arterial pressure–radius relation was obtained through cinematography using a telephoto lens. The results indicated that a major part of the mean pressure gradient was balanced by convective accelerations; the theory, which took this factor into account, predicted the correct velocity distributions and flow waves. The results also showed that for high flow rates the magnitude of the peak wall-shear stress became comparable to the yield stress of the endothelial surface and that radial flows of significant magnitude existed with respect to the arterial wall.

KEY WORDS  pressure-gradient measurement technique  vascular mechanics  blood velocity fields  mass transfer in aorta  shearing stress in vascular wall  mechanical factors in arteriosclerosis

Studies by Fry and his associates (1–3) have shown that the intimal surface of an artery is sensitive to adjacent hydrodynamic events. Therefore, it is desirable to devise a technique with which the flow of blood near the wall can be determined and correlated with atherosclerotic lesions. Recently Ling and Atabek (4) have developed a nonlinear theory describing the pulsatile flow in arteries. This theory takes into account the nonlinear terms of the Navier-Stokes equations as well as the nonlinear behavior and the large distention of the arterial wall. Using the locally measured values of pressure and pressure-gradient waves and the pressure-radius function and taper angle of the arterial wall, the theory predicts velocity distribution, wall shear, and discharge waves. This method was applied in the present investigation to study the nature of pulsatile flow in the descending aorta of dogs under a wide range of systemic pressures and flows.

Since a detailed account of the theory has been given previously (4), we will only present a short summary of it emphasizing especially the physical background of the analysis. For successful application of the present theory, the measured input information about the pressure-gradient wave and the pressure-radius function has to be obtained with precision by special techniques; these techniques will also be described in detail.

Theory

To determine the flow field we must solve a system of simultaneous equations which govern both the motion of the blood and the motion of the arterial wall. The analysis of this system is difficult. However, by introducing certain simplifying assumptions, the problem can be reduced to a manageable form. First of all, it can be shown that the inertial force due to the effective mass of the arterial wall is negligible in comparison with both the pressure and the elastic forces (5). Therefore, the radial motion of the arterial wall can be obtained directly from the pressure wave and the pressure-radius function of the artery. Let \( p \) be the pressure on the arterial wall and \( R \) denote the inner radius of the artery. Then, the pressure-radius function can be expressed as

\[
R = R(p).
\]

Hereafter we will assume that this functional relation is determined experimentally and available for calculations. Second, it has been shown experimentally that the longitudinal motion of the
arterial wall is very small (5). This condition is in part due to strong vascular tethering (6, 7) and in part due to the predominantly circumferential orientation of the elastic and collagen fibers (8) which minimizes the coupling of the circumferential strain to the longitudinal strain. Therefore, we can neglect the longitudinal motion of the arterial wall. Thus, if we measure both the pressure wave and the pressure-radius function, the problem of determining the flow field will reduce to obtaining periodic solutions to the fluid equations satisfying the prescribed wall motion in the radial direction and additional boundary conditions in the longitudinal direction.

Previous experimental studies (4) have shown that the effect of the convective accelerations on blood flow is as important as that of the viscous forces. The convective accelerations are in part generated locally by the arterial taper and the radial motion of the vessel wall and in part due to convection of momentum defect from upstream. This latter part is generally known as the entrance effect and is most pronounced near arterial branches and bifurcations. Locally generated accelerations due to distensibility and taper of arteries tend to cancel the convected momentum defect and prevent it from being carried far downstream. As a result, unlike the case of rigid cylindrical tubes, the entrance effect in distensible arteries is confined to a length less than ten diameters distal from branches (9). Therefore, flows which are not close to major branches, such as those in the descending thoracic aorta, can be considered to be fully developed.

Now, let us consider the equations governing the fluid motion. We will assume that blood can be treated like a Newtonian fluid and that at the location of interest the flow is axially symmetric. Then, the equations expressed in the cylindrical coordinates \( r, \theta, \) and \( z \), with \( z \) along the axis of the vessel, are

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} \right),
\]

\[
\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left( \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2} \right),
\]

and

\[
\frac{\partial u}{\partial r} + \frac{u}{r} \frac{\partial u}{\partial z} = 0.
\]

Here \( t \) denotes time, \( u \) and \( w \) denote the components of the fluid velocity along the \( r \) and \( z \) directions, respectively, \( p \) is the pressure, \( \rho \) is the density, and \( \nu \) is the kinematic viscosity of blood.

Since both the radial velocity and acceleration are small, from Eq. 1 we infer that the radial variation of pressure, \( \partial p/\partial r \), is negligible; thus, within the artery the pressure field is essentially a function of \( z \) and \( t \). Therefore, if we measure \( \partial p/\partial z \) along the arterial wall, the pressure will cease to be an unknown variable in the equation system, and the remaining unknown variables \( u \) and \( w \) can be determined using Eqs. 2 and 3. By a series of mathematical manipulations these equations can be transformed into the following forms, which are suitable for numerical integrations:

\[
\frac{\partial w}{\partial t} = F(z, t) + \left( \frac{\eta \partial R}{R \partial t} - \frac{u}{R} \right) \frac{\partial w}{\partial \eta} + \frac{w}{R} \left( \frac{\partial u}{\partial \eta} + \frac{u}{\eta} \right) + \nu \left( \frac{\partial^2 w}{\partial \eta^2} + \frac{1}{\eta} \frac{\partial w}{\partial \eta} \right),
\]

and

\[
\frac{1}{\eta} \frac{\partial R}{\partial t} \int_0^\infty \frac{\eta w}{\partial \eta} \, d\eta - \int_0^\infty \eta w \, d\eta = 0.
\]

Here \( F(z, t) = -\rho^{-1} \partial p/\partial z \), \( R = R(z, t) \) denotes inner radius of the artery, and \( \eta = r/R \) denotes the
normalized radius of the artery. In Eq. 5, \( \tan \Psi \) represents the natural taper of the artery and the product \( p \partial R / \partial p \) expresses the arterial taper generated by the pressure wave. The factor \( \partial R / \partial p \) is the derivative of the pressure-radius function with respect to \( p \). The remaining terms are related to the axial velocity distribution and the velocity of the arterial wall, \( \partial R / \partial t \), which can be obtained from the pressure wave and the pressure-radius function. Since all explicit dependence on \( z \) has been eliminated from Eqs. 4 and 5, we do not have to specify the boundary conditions in the \( z \) direction. However, these equations still have to satisfy the boundary conditions

\[
\begin{align*}
  u \bigg|_{z=1} &= 0 \quad \text{and} \quad \frac{\partial u}{\partial \eta} \bigg|_{z=0} = 0.
\end{align*}
\]

Furthermore, Eq. 5 reduces to \( u = \partial R / \partial t \) at \( \eta = 1 \), thus satisfying the condition that on the wall the radial velocity of the fluid must be equal to the velocity of the wall.

This set of equations, after being written in the form of a finite difference, can be integrated numerically at a fixed \( z \). The integration process starts at \( t = 0 \) by setting the axial velocity distribution \( u(\eta, z, 0) = 0 \). Then, the values of \( R(z, 0) \) and \( \partial R / \partial t \) are determined from the experimental data. Next, the radial velocity distribution \( u(\eta, z, 0) \) is computed from Eq. 5. This result together with the measured pressure gradient \( F(z, 0) \) is used in Eq. 4 to determine the axial velocity distribution \( w(\eta, z, \delta t) \), where \( \delta t \) represents the finite difference increment in time. After this procedure, the computation cycle is repeated to obtain \( u(\eta, z, 2\delta t) \) and \( w(\eta, z, 2\delta t) \), and then \( u(\eta, z, 2\delta t) \) and \( w(\eta, z, 3\delta t) \), etc. It will take the solution a few cardiac cycles to settle into a steady periodic flow. For further details see reference 4.

**Methods**

**EXPERIMENTAL PROTOCOL**

Eight dogs weighing 23.6—31.8 kg (average 27.8 kg) were studied under sodium pentobarbital anesthesia (about 30 mg/kg, iv). The dog’s chest was opened, and adequate ventilation was maintained with a positive-pressure respiratory pump. The middle descending thoracic aorta was exposed and two intercostal arteries about 3–5 cm apart were cannulated for measurement of proximal, \( p_p \), and distal, \( p_d \), pressures as shown in Figure 1. In every experiment, an electromagnetic flowmeter was placed on the aorta distal to the two pressure taps for monitoring flow rate. All other intercostal arteries between the proximal pressure tap and the electromagnetic flowmeter were ligated. Also in some experiments, a hot-film probe (10) was introduced into the aorta midway between the two pressure taps to obtain the center-line velocity wave. Unfortunately, it was not possible to place the electromagnetic flowmeter close to the point of pressure measurements or to measure the center-line velocity simultaneously with the pressure-gradient waves without introducing error due to the interference of the probes. Nevertheless, these secondary measurements were used as an independent check for the present technique.

In general, six basic signals were recorded simultaneously on a precision FM magnetic tape. They were the event-code signal, the electrocardiograph signal, the pressure-difference signal, the proximal pressure signal, the distal pressure signal, and the electromagnetic flowmeter signal. The event-code signal was created by an event-code generator that provided the identification numbers for the beginning and the ending of a file of recorded data. The electrocardiograph signal provided the means for separating the serially recorded signals into individual heart beats. The pressure-difference signal was the difference of the first pressure transducer with respect to the second transducer. This signal was obtained through a precision differential operational amplifier. Because the pressure-difference signal was a small fraction of the pressure, it was magnified by a factor of two or five through the operational amplifier to utilize the maximum dynamic range of the tape recorder. Two separate data files were required to record the forward and the backward pressure-difference signals, with each file containing a minimum of 30 heart beats. To minimize any change in heart rate during the recording period, the two files were normally recorded immediately one after the other. Also to avoid possible loss of data due to irregularity of heart beats during any one recording, one or two extra sets of recordings were usually made. Since the hot-film probe...
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velocity measurement could not be performed simultane-ously with the recording of pressure signals, it was generally recorded immediately after the last pressure-gradient measurement as a separate file with the electrocardiograph signal.

After the pressure-gradient and flow data were recorded, the taper angle of the arterial segment was determined from the measurement of the radii of the pulsating aorta. To obtain this information, we employed an 8-mm cinecamera equipped with a high-power telephoto lens so that the segment of artery under study could occupy the whole picture frame when the camera was at a relatively large distance (1.2 m) from the object. This technique was essential for minimizing error due to parallax. Motion pictures of the pulsating aorta were taken at 40 frames/sec. Using a microscope equipped with an achromatic dark-field condenser, flat-field objectives, and a 10x micrometer eyepiece calibrated by the scale in the film, the dimensions of the arterial segment were measured at various sites to within three significant figures. From these data the taper angle was calculated.

Throughout the experiment the aorta was kept moist and the blood temperature remained around 36-37°C. The viscosity of the blood was measured three or four times during each experiment, using a cone-and-plate viscometer. The values of viscosity varied between 0.04 and 0.05 poises.

At the end of the in vivo experiment the pressure-radius data were obtained in vitro. The method for obtaining the pressure-radius relation and the instrumentation used to determine the pressure-gradient data will be described in detail.

DETERMINATION OF THE PRESSURE-RADIUS FUNCTION

For the aortic flow studies, it is desirable to obtain the pressure-radius characteristic of arteries covering the full pressure range so that the nature of blood flow under a wide range of pulse pressures can be studied. Therefore, at the end of each in vivo experiment the segment of aorta under study was marked off with two suture lines 5.0 cm apart. Then the intercostal and other small arteries within a length of 10 cm were tied off and the segment checked for leaks. The segment was then excised, and the unstressed length \( L_0 \) between the two suture marks was measured. This procedure provided the necessary information for normalizing all longitudinal extensions \( \lambda_1 = l/l_0 \). In most cases, the in vivo \( \lambda_1 \) had a value close to 1.4. The segment of the aorta was then mounted on a jig. It was stretched longitudinally until the distance between the two suture marks was back to 5.0 cm. One end of the mounted segment was connected to a large syringe through a standard 5-mm drip tube, and the other end was connected to a calibrated pressure gauge. For this setup it was important that all tube connections have an internal diameter of at least 3 mm to prevent phase error in the pressure signals. The whole system was then filled with dextran and mounted next to a chart recorder which displayed the pressure in the test specimen. A centimeter scale was also mounted on the jig for calibration purposes. The same cinecamera was then used to take pictures at 40 frames/sec of the whole system as the artery was being inflated sinusoidally at approximately 1 cycle/sec by pumping on the syringe in two pressure ranges from 10 to 100 mm Hg and 70 to 180 mm Hg. From these pictures the full range of the pressure-radius function was obtained. Minimum hysteresis was observed in the pressure-radius function of all eight dogs we studied. Inflation cycles up to 3 cycles/sec gave essentially the same result. However, at extremely slow inflation cycles the artery tended to develop a set, which resulted in a larger vessel diameter at low pressures. For the present work, data based on extremely slow inflation cycles were avoided.

The data for the outer diameters must be converted into inner radii data through the condition of incompressibility. However, before the inner radii could be calculated it was necessary to first establish the outer radius at zero pressure by extrapolating data taken at higher pressures, because arteries at pressures below 10 mm Hg no longer maintain a circular cross section. For a fixed value of \( \lambda_1 \), the pressure-radius function of arteries in the low pressure range \( (p < 60 \text{ mm Hg}) \) has been shown to be expressible approximately by the following equation (4):

\[
p = \frac{B h_0}{R_0} \left( 1 - \frac{1}{\lambda_2^2} \right)
\]

(7)

Here \( h_0 \) and \( R_0 \) are the unstrained thickness and the inner radius of the artery, respectively, \( \lambda_2 = R/R_0 \) is the circumferential extension ratio, and \( B \) is a dimensional constant which depends on \( \lambda_1 \) and the physical properties of the artery. When \( p = 0 \), Eq. 7 reduces to \( \lambda_2 = \lambda_1^{-1} \). For \( \lambda_1 = 1.4 \), the circumferential extension ratio \( \lambda_2 \) is 0.84 (Fig. 2). We can assume that the extension ratio of the outer radius \( \lambda_2' \) is close to \( \lambda_2 \) at low pressures. Hence the unstressed outer radius can be taken to be equal to the outside radius at \( p = 0 \) divided by 0.84. After the value of the unstressed outside radius is obtained, we can convert all the experimentally measured outer radii into \( \lambda_2' \) values. Now under the condition of incompressibility the wall thickness at any strain level can be expressed approximately as:

\[
h = \frac{h_0}{\lambda_1 \lambda_2'}
\]

(8)

The value of \( h_0 \) was measured by averaging the wall thickness of a transverse slice of the artery under a microscope. For a known \( h_0 \) and a fixed \( \lambda_1 \), \( h \) can be calculated for any given \( \lambda_2' \). Thus by subtracting from the outside radii the corresponding \( h \) values one finds the corresponding inside radii \( R \). Since the unstressed inside radius \( R_0 \) is equal to the unstressed outside radius minus \( h_0 \), we can finally calculate all the corresponding inner extension ratios, \( \lambda_2 = R/R_0 \). Values of \( \lambda_1 \) and \( h_0 \) vs. \( p \) for one experiment, identified as dog \( \alpha \), are shown in Figure 2. It is important to note that the extension ratios for the inner radii at high pressures were significantly larger than those for the outer radii. The calculated nonlinear pressure-radius function for the inner radii was expressed as a seventh-order polynomial in \( p \) and read into the computer as a
subroutine program for subsequent calculation of the flow profiles.

**INSTRUMENTATION FOR PRESSURE-GRADIENT MEASUREMENT**

Although the net flow of blood in the circulatory system is dictated by both the mean systemic pressure and the vascular bed loads, the present theory predicts this flow through local balancing of forces. Thus in the theory, the mean flow of blood is essentially determined by the small mean pressure gradient which balances the mean wall friction and the convective accelerations. To predict the mean flow within an error of a few percent, it is necessary to measure the mean pressure gradient with an accuracy of ±0.001 mm Hg/cm, or less than 0.1% of the peak dynamic pressure gradient. To obtain this precision, a method which involved the measurement of both the forward and the backward pressure gradients was employed. This concept is similar to those commonly used to obtain precision leveling by rotating the spirit level 180° or to obtain precision weighing by alternating the weights between the two weighing pans of an analytical balance. These procedures can be shown to effectively eliminate the error associated with the instrument. To obtain the pressure-gradient information at a point along the descending thoracic aorta, two arterial taps were made through a pair of intercostal arteries, one distal and one proximal to the point of interest. These taps were connected to two Statham P23GD pressure transducers through two 1.2-mm, i.d., nylon catheters and a special duplex-three-way valve (Figs. 1 and 3). The duplex-three-way valve provided the means for either connecting the first transducer to the proximal tap and the second transducer to the distal tap or reversing the connections. At the first valve position the output voltages from the two pressure transducers can be expressed as

\[ E_1 = A_1p_d + e_1 \quad \text{and} \quad E_2 = A_2p_d + e_2, \quad (9) \]

and at the second valve position the output voltages are

\[ E_1' = A_1p_d + e_1 \quad \text{and} \quad E_2' = A_2p_d + e_2, \quad (10) \]

where \( A \) is the calibration constant for the pressure transducer, \( p \) is the arterial pressure, \( e \) is the overall static-error voltage of the pressure-measuring system, subscripts 1 and 2 represent the first and the second transducer, and subscripts \( p \) and \( d \) represent the proximal and the distal tap, respectively. Since the pressure transducers had excellent linear response characteristics, the consideration for nonlinear effect was unnecessary. We defined the forward pressure-gradient signal as

\[ \Delta E_f/\Delta z = (E_2 - E_1)/\Delta z = (A_2p_d + e_2 - A_1p_d - e_1)/\Delta z, \quad (11) \]

and the backward pressure-gradient signal as

\[ \Delta E_b/\Delta z = (E_2' - E_1')/\Delta z = (A_2p_d + e_2 - A_1p_d - e_1)/\Delta z, \quad (12) \]

where \( \Delta z \) is the distance between the proximal and distal pressure taps. By taking the difference of the two pressure-gradient signals we have from Eqs. 11 and 12

\[ (\Delta E_f - \Delta E_b)/\Delta z = (A_1 + A_2)(p_d - p_p)/\Delta z. \]

Hence, the pressure gradient can be expressed as

\[ dp/\Delta z = (p_d - p_p)/\Delta z = (\Delta E_f - \Delta E_b)/(A_1 + A_2)\Delta z. \]

Hence, the pressure gradient can be expressed as
Since both the static instrument errors, \( e_1 \) and \( e_2 \), are invariant to the switching, they are automatically eliminated. In practice, the calibration constants \( A_1 \) and \( A_2 \) can be set to equal one another within 0.1%; hence

\[
\frac{\Delta p}{\Delta z} = \frac{(A_1 - A_2)}{2A_1}.
\]

(14)

We found that the above relation also holds true for the mean pressure gradient under dynamic conditions if the catheters connecting the pressure gauges and the valves are made of identical lengths of stiff nylon tubings and the system is free of all microleaks and air bubbles. All standard Luer connections and valves are notorious for harboring air bubbles and having microleaks. Therefore, at all critical junctures direct tube-to-needle connections were used.

Both the duplex-three-way valve and other control valves in the system were specially designed to eliminate microleaks and air bubbles. A detailed description of the valve is shown in Figure 3. Instead of using a tapered construction to obtain close fitting of the valve body, the valve stem was cylindrical in shape, and the sealing action for the valve was provided by micro-O-rings. The entire valve body was made of Lucite, so that any air bubbles trapped in the system could readily be detected. All the necessary internal pressure connections at different valve positions were cast into the rotating valve stem. In addition to the micro-O-rings which sealed the pressure connections, there were two additional O-rings which sealed both ends of the valve stem. This second set of seals was necessary to prevent air from leaking into the body of the valve. The duplex-three-way valve had two inlet ports for connection to the proximal and distal pressure taps and two outlet ports for connection to the first and second pressure transducers. These connections were arranged in the form of a diamond with the outlets occupying the vertexes and the inlets occupying the side corners. The valve had three control positions, forward gradient, backward gradient, and test. In the forward position the proximal and distal taps were connected to the first and second pressure transducers, respectively. In the backward position the tap connections were transposed to the first and second transducers, respectively. To maintain the dynamic symmetry of the pressure system, the geometry for all pressure connections was made similar. In the test position, both the first and the second transducer were connected to the distal tap. This position was used to check the dynamic symmetry of the pressure system. Since both pressure transducers were connected to the same pressure source the output of the two transducers should be identical. Thus, by observing the magnified difference of the two signals through a precision differential operational amplifier, the operating condition of the system could be determined. For example, if the output showed an oscillatory signal, it would indicate the existence of either air bubbles in the system or a vibrating catheter. If the signal showed a pulsatile wave form similar to the pressure wave, it would mean either that the gains between the two pressure channels were not set precisely to the same value or that there was a leakage of blood into one pressure channel. If the mean value of the output was not exactly zero, it would mean that the zero bias between the two pressure gauges had not been set equal. It is important to note that, although the present technique requires no precise setting of the zero bias, it is absolutely necessary that there are free from drifts during the measuring period. Thus, in the third valve position one can observe the stability of the zero bias for a period of time prior to the measurement of the pressure gradients. Since the switching technique cannot detect any error occurring between the pressure taps and the valve, the catheters connecting these two points must be made short and identical. Connections at the arterial taps must be made with extreme care to prevent any possible occlusion under pulsating and breathing motions.

Flushing of the pressure system was accomplished by connecting the second inlet of each pressure transducer to a special three-way valve whose basic construction is similar to that of the duplex-three-way valve (Fig. 1). One inlet of the valve was connected to a 5-ml syringe which was used for flushing the system, and the second inlet was connected to a drip bottle containing deaerated saline solution. To minimize the formation of blood clots in the catheters, 5% heparin was added to the solution. The three-way valve enabled us to flush the system through either the drip bottle or the syringes.

The pressure gauges were calibrated by first setting the duplex valve to any intermediate position, where the pressure transducers were not connected to any pressure outlets or taps, and by setting the flushing valves to the drip bottle. Then, by raising or lowering the bottle to known heights the pressure gauges could be calibrated. The drip bottle should be equipped with an air tube to ensure that air space in the bottle is under atmospheric pressure. In general, one gauge was first adjusted to an output level which was desirable for maximum utilization of the dynamic range of the FM magnetic tape, and the second gauge was then adjusted to track the first one. Fine adjustment of both zero bias and gain can be performed by observing the magnified difference of the output of the gauges through a precision differential amplifier as mentioned in the preceding paragraph.

Because the precision required for the processing of the pressure-gradient data is beyond the capability of the ordinary analog technique, it was processed by the digital method as described in the following section.

**DATA PROCESSING**

The set of recorded data was converted into digital data by the computer. The digital sampling rate was selected to provide approximately 150–200 data samples per heart beat. The digital files containing the calibration information were averaged to obtain the required calibration constants, and the digital files containing the serially recorded pressure signals were first broken down into records of individual heart beats with each record starting at the beginning of systole. There is usually a few percent variation in the cardiac period from beat to beat. To obtain a precise mean pressure-gradient wave, we selected the beats having equal periods and averaged the data over many heart
beats covering at least one respiratory cycle. Using this technique, the computer was programed to calculate the average pressure-gradient wave based on Eq. 14 and also the average pressure wave based on the sum of the proximal and distal pressure waves divided by two. These processed data were then recorded on a third digital tape and used to compute various flow parameters.1

**Results and Discussion**

Having obtained the local information about the pressure-radius function, pressure-gradient wave, pressure wave, wall taper, and viscosity of blood we can calculate the detailed flow field within the middle descending aorta. Selective data from three of the eight dogs studied, covering a wide range of pressures and flows, are used here for illustrative purposes. These three dogs henceforth will be identified as dogs A, B, and C.

**PRESSURE-RADIUS FUNCTION**

The measured pressure-radius functions of these three dogs are shown in Figure 2. Of the eight dogs studied, dog A had the most distensible and dog C the least distensible aorta. For all cases, as pressure rose from 0 towards 60 mm Hg, the slopes of the pressure-radius curves decreased rapidly. This nonlinear behavior, which is known as the "ballooning effect," (11) was responsible for the large pressure range of 70 to 130 mm Hg varied approximately by a factor of two. As will be seen later, this nonlinear characteristic plays an important role in the dynamics of the flow. The actual pressure range (pulse pressure) for each dog is marked by large open circles in Figure 2. Dog A had the largest pulse pressure which, as will be seen later, was associated with a large cardiac output. Dog C had a relatively small pulse pressure associated with a small cardiac output. The values of $R_0$, $h_0$, $\lambda_1$, and the arterial taper angle $\Psi$ for dogs A, B, and C are given in Figure 2; average values for all eight dogs are also listed. The average pressure-radius function for the eight dogs is shown by the solid curve.

Unfortunately, in the motion pictures of the aorta the edges of the vessel were not always sharp enough to permit accurate in vivo determination of the pressure-radius function during a cardiac cycle. Therefore the in vitro data shown in Figure 2 were used for flow computations. To the extent that such in vitro data differed from comparable in vivo data, we had an error in the computation of the flow field. However, we do not think this error was very large in our experiments for the following reasons. (1) The pressure-radius function is mainly governed by the elastic properties of the aorta in the circumferential direction. Recent work of Patel et al. (13) has shown that in the thoracic aorta the in vitro value of the circumferential elastic modulus is only slightly lower than the comparable in vivo value. (2) In some of our experiments in which it was possible to compare the in vitro and in vivo pressure-radius functions, differences were small and within experimental errors. (3) When we perturbed the pressure-radius function used in our computations by amounts comparable to the above differences, the computed flow fields did not differ significantly; thus the flow computation is relatively insensitive to small differences in the pressure-radius function.

**FLOW FIELD**

Many of our dogs, studied under general anesthesia with open chest, had low cardiac outputs. In some instances we increased the cardiac output by intravenous infusion of isoproterenol. Measurements on dog A, shown in Fig. 4, were obtained during infusion of isoproterenol (3 $\mu g/min$). The computed velocity profiles for this

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1A listing of the program used on the CDC 3200 hybrid computer is archived and may be obtained from National Technical Information Service, U. S. Department of Commerce, 5285 Port Royal Road, Springfield, Virginia 22151. Request report no. NIH-NHLI-ATHERO-73-1.
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dog are shown in Figure 4a. In this figure, \( T \) is the cardiac period, and \( r = t/T \) represents the normalized time. The radial ordinate is normalized by the instantaneous inside radius \( R_\tau(r) \), and the axial velocity \( w \) is normalized by the maximum centerline velocity \( w_n \). Note that for this case the velocity profiles are blunt with no retrograde flow. The associated centerline velocity wave is shown in Figure 4b. It has a relatively large peak value of \( w_n = 248 \text{ cm/sec} \). The radial velocity profiles are shown in Figure 4c. Note that these profiles tend to fold toward the left. This tendency is caused by the arterial taper. It is important to note that these radial profiles have a steep slope near the vascular wall, which implies that, in this case, blood in the region slightly away from the wall is not following the radial motion of the wall and that a centripetal flow of blood exists with respect to the wall. This motion could influence convective transport processes between the blood and the vascular wall. The radial velocity of the arterial wall as a function of the cardiac period is plotted in Figure 4d. The peak radial velocity for this case is relatively large and is responsible for the large convective accelerations which tend to maintain the blunt flow profiles. The pressure wave is shown in Figure 4e. It has a mean pressure of \( p = 104 \text{ mm Hg} \) and a large pulse pressure of 44 mm Hg (Fig. 6e). The mean pressure of \( p = 58 \text{ mm Hg} \). A complete set of data obtained under this condition is shown in Figure 5. Note that for this case the velocity profiles are slightly more blunt than they are in the previous case (Fig. 4). Also there is retrograde flow during diastole. Note also that the peak wall shear is still relatively high, although the mean blood flow has been reduced by a factor of six. This reduction is due to concentration of forward flow early in systole as a result of the large capacitance of the artery in this low pressure range.

Results from dog B during infusion of isoproterenol (3 \( \mu \)g/min) are shown in Figure 6. The velocity profiles for this dog are more blunt than those in Figure 4. Dog B has a mean pressure of 107 mm Hg and a pulse pressure of 44 mm Hg (Fig. 6c). The dicrotic notch on both the pressure and flow waves is more pronounced than it is in Figure 4, indicating a stiffer vascular wall. The mean pressure gradient computed for the wall-friction effect is approximately a fifth of the total mean pressure gradient shown in Figure 6f. Hence, a major part of the mean pressure gradient is used for balancing the convective accelerations. We also note that near the vascular wall there are significant positive and negative radial velocity gradients as seen in the radial velocity profiles in Figure 6c. They indicate near the wall the existence of radial flows moving...
FIGURE 4
Complete flow field in the descending thoracic aorta of dog A with normal mean pressure and large cardiac output (5.7 liters/ min). a: Axial velocity profiles. b: Center-line velocity wave. c: Radial velocity profiles. d: Radial velocity of the vessel wall. e: Pressure wave. f: Pressure gradient wave. g: Circumferential dis;ention wave. h: Velocity-gradient wave at the wall. i: Discharge wave. See text for abbreviations.
FIGURE 5

Complete flow field in the descending thoracic aorta of dog A with low mean pressure and low cardiac output (0.9 liters/min) produced by bleeding. See Figure 4 for explanation.
FIGURE 6

Complete flow field in the descending thoracic aorta of dog B with normal mean pressure and normal cardiac output (2.6 liters/min). See Figure 4 for explanation.
away from the wall during systole and toward the wall during diastole. For this experiment both the center-line velocity wave and the stroke volume are verified independently by a hot-film and an electromagnetic flow probe, respectively; the agreement in both cases is reasonable (Fig. 6b and i).

Results from dog C are shown in Figure 7. This case demonstrates the effect of hypertension and low cardiac output. The mean systemic pressure is 127 mm Hg, and the pulse pressure is 30 mm Hg (Fig. 7c). There is a large dicrotic notch on the pressure wave, indicating stiffness of the arterial wall. The dynamic circumferential distention of the arterial wall is 7.5% of the minimum radius (Fig. 7a). We also note that the mean pressure gradient due to wall friction is approximately half of the total mean pressure gradient shown in Figure 7f. It is evident that as the artery becomes more rigid the net effect of the convective accelerations is proportionally reduced. Here again, both the calculated center-line velocity wave and the flow rate are verified by two independent measurements (Fig. 7b and i).

As mentioned previously, it was not feasible to measure in vivo velocity profiles using a hot-film anemometer simultaneously with the determination of the pressure gradient. However, in model studies when this simultaneous measurement was done, the measured velocity profiles agreed well with those predicted by the nonlinear theory (4).

COMPARISON WITH LINEAR THEORY

The linear theory of pulsatile flow developed by Womersley and others (14-16) is satisfactory for describing certain aspects of flow in small arteries. However, in large arteries, as was shown by Fry et al. (17) by comparing the theoretical predictions with the experimental values of flow resistance and by Ling et al. (9) by comparing theoretical and measured values of the velocity distribution, the linear theory fails to give an adequate representation of the flow field. Since the linear theory is widely known and has been used extensively by physiologists, it becomes important to compare it with the present nonlinear theory. Such a comparison was done for a model artery by Ling and Atabek (4); however, it would be interesting to do a similar comparison using data from living dogs. We used the data from dogs A, B, and C to calculate the various flow parameters by the linear theory as shown in Figures 4-8. The results differed from those of the nonlinear theory in the following ways. (1) All velocity profiles were essentially parabolic in form, having values of peak center-line velocity \( u_m \) approximately two to four times the corresponding values predicted by the nonlinear theory. (2) The discharge waves \( q \), although having a general similarity in wave form, had a mean value which was six to eight times higher than the corresponding values from the nonlinear theory. (3) The wall shear waves predicted by the two theories were similar in dynamic form; however, the mean value from linear theory was larger. The failure of the linear theory to accurately predict the various flow parameters lies in the process of linearization. As can be seen from Eq. 2, the longitudinal pressure gradient \( \frac{dp}{dz} \) is balanced by the inertia forces due to both local and convective accelerations and by the frictional forces generated by velocity gradients. In the linearization process the convective acceleration terms are neglected. As a consequence, the linearized equations are forced to produce the required balance by artificially increasing the frictional forces with higher flow rates.

Thus, we conclude that the nonlinear theory is essential for predicting exact values of the flow parameters. However, it is important to note that the wave forms of the discharge waves predicted by both theories are similar in many instances except for a base-line shift (Fig. 7i). Thus, knowing the value of the discharge rate at any given time in the cardiac cycle, one could adjust the flow curve from the linear theory to obtain "true" flow. Greenfield and Fry (18), when comparing flow curves predicted by the theory of Womersley (14) and that of Fry et al. (19) with the measured flow in the descending thoracic aorta, adjusted the base line by assuming zero flow at the end of diastole; this assumption is valid in the ascending aorta but may cause errors in the descending aorta.

Finally, it is important to point out that as the arterial wall becomes more distensible and the arterial taper increases, the discrepancy between the nonlinear and the linear theories will become more pronounced; under these circumstances even the dynamic wave forms may no longer be similar (Fig. 8b and e).

In summary, we have (1) used the nonlinear theory for predicting the flow field in the descending thoracic aorta of dogs and (2) developed a precise pressure-gradient measurement technique. Using these techniques we have demonstrated the importance of the convective accelerations caused by both the arterial distention and the arterial taper.
FIGURE 7

Complete flow field in the descending thoracic aorta of dog C with high mean pressure and low cardiac output (1.3 liters/min). See Figure 4 for explanation.
Comparison of axial velocity profiles and discharge waves predicted by the linear and the nonlinear theory: a and b, c, d and e, and f represent results obtained by using the input data from dogs A, B, and C given in Figures 4, 5, 6, and 7, respectively. It is important to note that the plot of linear velocity profiles includes the contribution due to the mean flow in contrast to similar plots given in literature (12, 13), which only show the velocity profiles due to the pulsatile part of the flow. These latter profiles look more blunt and tend to resemble the nonlinear velocity profiles. See Figure 4 for explanation.
on the dynamics of blood flow. The method is capable of accurately predicting the flow field in symmetric arteries of sufficient length. In practice, the method is reasonably simple and straightforward when executed under proper computer programs. The new technique should provide quantitative data for the study of the transport of various blood components across the endothelial surface which may be of importance in understanding the pathogenesis of atherosclerosis. For instance, the radial flows near the vessel wall shown in Figures 4-7 must be taken into account when considering the convective transport of materials from the blood phase to the vessel wall (20). For work that does not require absolute precision, a double-lumen catheter may be used to obtain the pressure-gradient and pressure-wave information from intact animals and man.

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