Aortic Valve Leaflet Motion during Systole

NUMERICAL-GRAphICAL DETERMINATION

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ABSTRACT

An approximate numerical-graphical solution was made for the motion of a human aortic valve leaflet in a meridional plane through the center of the sinus of Valsalva. The observed buckling, whipping action of the leaflet was obtained. Of primary significance in the results was the location and the determination of the maximum curvature of the leaflet in this plane, since this point corresponds to the point of maximum fatigue bending stress. The approximate dimensionless curvature was on the order of 3.5 (referred to the aortic diameter) at a dimensionless radial location of 0.55-0.6. The most important feature determining leaflet curvature and motion was the initial contour shape. The initial shape of a normal human leaflet produces smaller curvatures than the curved shapes usually illustrated for aortic valves.

KEY WORDS

human trileaflet and aortic valve motion
curvature of valve leaflets during systole
leaflet motion dynamics
buckling of membrane valves
fatigue stress buckling of aortic valve

Some observations on the aortic valve during opening have indicated a wavy, buckling, whipping, flutter motion of the leaflets. If such a motion exists, bending fatigue stresses would be produced. It has been observed also that trileaflet prostheses fail at a location in the leaflet that could correspond to the point of maximum bending stress. An approximate numerical-graphical determination of valve motion is presented that indicates this motion in a meridional plane through the sinus of Valsalva.

Methods

NOMENCLATURE

A = Area.
d = Diameter of aorta.
f = Frequency (rate).
f = Te/6 (Fig. 1).
M = Time index at t = r/2, at end of pulse delivery.
m = Time index.
n = Velocity profile exponent.
Q = Instantaneous flow rate.
\( Q' \) = Time-averaged flow rate.
\( Q'' \) = Instantaneous flow rate.
\( \tau \) = Time period.
\( \omega \) = Radial frequency.

Subscripts and Superscripts

ao = Aorta.
e = Effective.
max = Maximum.
N = Normal.
0 = Value at center line (at \( t = 0 \)).
\( t \) = Tangential.
\( \bar{v} \) = Space-averaged value.

FLOW DELIVERY CONDITIONS

The cardiac output, the average flow rate as a function of time during systolic ejection, may be represented by \( Q(t) \), as indicated on Figure 1. The representative flow curve was obtained from observations of several wave shapes (1, 2). A sine wave is shown for comparison. This representative wave has its peak at 0.4 of the ejection period, and the sine wave peaks at 0.5. Systolic ejection occurs over less than half of the heart beat period. The effective period for blood flow delivery during systolic ejection may be expressed as

\[ T_e = \frac{f_c T}{6} \]

where \( r = \frac{80}{f_c} \) is the best period in seconds, \( f_c \) is the pulse rate in beats/min, and 0.75 < \( k_e < 0.8 \) (i.e., systole requires approximately 40% of the beat period). The radian frequency is then \( \omega_c = \frac{2\pi r}{60 k_e} \).

The radial velocity distribution at the base of the aorta may be represented by

\[ v(r) = v_0(1 - r^2) \]

where \( v_0(t) \) is the center-line velocity at \( r = 0 \). For fully developed Poiseuille (laminar Newtonian) tube flow, the exponent \( n = 2 \). For the accelerated flow during systole, \( n \) is determined by the flow acceleration; therefore, it is a function of \( f \) and may be approximated in the range 5 < \( n < 9 \). For the deceleration phase,
Flow pulse (cardiac delivery) wave shape. Note that the maximum flow occurs at 0.4 of the delivery period (at 0.4 r). A sine wave (broken line) with maximum flow at 0.5 r is shown for comparison. Total pulse period = r, equivalent delivery pulse cycle time = r.

3 < n < 5. As the flow rate approaches zero, the profile becomes complex with peripheral backflow (3).

The area-averaged velocity through an area A for a flow rate Q is \( \bar{v}(r,t) = \frac{Q(t)}{A} \). The velocity from Eq. 2 over the base area \( A_b \) is

\[
\bar{v}(r,t) = \frac{n+2}{n} \frac{Q_{max}}{\bar{v}} \left( 1 - \left( \frac{r}{r_b} \right)^n \right) \sin nt. \tag{7}
\]

where \( \omega = \frac{2\pi}{\beta} \). 

AORTIC GEOMETRY

A cross section of the geometry in the region of the aortic valve is sketched in Figure 2. Dimensions are shown as fractions of base aortic diameter (approximately the annulus diameter at the base of the aortic valve). The coaptation ratio of 0.2 at the apex of the leaflets is of interest. The maximum coaptation ratio of 0.4 is about midway along the contact or the free edge of the leaflets. The leaflet length ratio is 0.75-0.8. The maximum diameter ratio of the sinus cavities is about 1.5. The area-averaged diameter ratio of the sinus region calculated from the measured maximum cross-sectional area through the sinuses is 1.38. These measurements are the averages of measurements on silicone rubber casts of nine human valves (made by the authors) at pressures from 0 to 100 mm Hg corresponding to those during the pulse cycle.

No change in leaflet dimensions occurs following the time when left ventricular pressure reaches aortic pressure, since there is no longer any pressure gradient across the valve.

CALCULATION OF LEAFLET DISPLACEMENT

Several assumptions were made for calculating leaflet displacement. (1) The fluid is incompressible, and the geometry of the aortic-sinus region remains fixed. The diameter of the aortic-sinus region does increase about 20-25% during systole from zero to maximum systolic pressure. However, the length of the region only increases about 3-5% so that the length relative to the length of the leaflets does not change significantly. This expansion during systole does not significantly affect the flow field close to the heart, i.e., the entire source geometry of the aortic outflow tract is increasing. For the following calculations, for a first approximation, dimensional changes were neglected. This simplifying approximation is restrictive, but the results are significant as a first approximation. Neglecting dimensional dilation during the initial stages of systole affects a more rapid motion than that actually produced. (2) The cusp behaves like a neutrally buoyant membrane in the fluid and therefore offers no resistance to the fluid motion. (3) The original cusp shape during diastole is taken as the starting contour. (4) The leaflet contour is determined by the displacement of the original contour along streamlines with the additional constraints that it is hinged at the base and remains of constant length. (5) The displacement as a function of radius and time is approximated by integration of Eq. 6. (6) The streamline displacement from a constant radius is small enough to be neglected for first approximation calculations. (7) The cusp is freely hinged at its base.

The assumed axisymmetric velocity profile is an approximation that is difficult to evaluate. The actual flow conditions in the three-lobed sinus region are extremely difficult to formulate.
Results

DISPLACEMENT

The displacement along the contour from the initial equilibrium (stasis) position is obtained by integration of Eq. 6. To perform a finite difference analysis, the time is first expressed as a fraction of the period \( \tau_o \).

\[
i = \frac{m \tau_o}{2M}, \quad 0 \leq m \leq M \tag{8}\]

(or \( \omega t = 360^\circ M/M \) in degrees). If \( M = 100 \), the systolic phase \( 0 < t < \tau_o/2 \) is essentially a percent of the delivery period \( \tau_o/2 \). The index \( m \) may take on integer values for a finite difference analysis. The displacement \( S \) can be nondimensionalized with respect to the aortic radius \( r_o \); then with \( \Delta t = \frac{2\pi}{\omega r_o} \) and \( \tau_c = \frac{k\tau_o}{r_o} \), integration of Eq. 6 gives (with \( n \) assumed constant)

\[
S(r, m) = S'(r, m) = n + 2 \frac{Q_m}{A_o} \left[ 1 - \left( \frac{r}{r_o} \right)^n \right] \left[ 1 - \cos \frac{m\pi}{100} \right]. \tag{10a}
\]

For the sine wave, Eq. 9 gives

\[
S' = \frac{n + 2}{r_o} \left[ 1 - \left( \frac{r}{r_o} \right)^n \right] \left[ 1 - \cos \frac{m\pi}{100} \right]. \tag{10b}
\]

The total travel of a particle along the center line of a constant-diameter tube during systole would be

\[ S' = 9.1 \left[ 1 - \left( \frac{r}{r_o} \right)^n \right] \left[ 1 - \cos \frac{m\pi}{100} \right]. \tag{10c}\]

For the representative wave, \( Q_m/Q > 4.4 \) (by numerical integration), and, representing the integral numerically,

\[
S' = 32.1 \left[ 1 - \left( \frac{r}{r_o} \right)^n \right] \sum \frac{Q'dm}{100}. \tag{10c}
\]

LEAFLET MOTION

Since the leaflets are very compliant (thin and low-modulus material) and the fluid is incompressible, the initial fluid displacement as injection begins will displace the cusp membrane with essentially no relative motion between fluid and cusp. The cusp will move with the fluid with continuity of the velocity fields on both sides. There is little motion of the fluid until the left ventricular pressure equals the aortic pressure (i.e., until the pressure gradient across the valve equals zero). Following this condition, the pressure-loading stresses on the cusp itself are negligible so that its dimensions remain fixed. It then merely folds back. The only relative fluid motion is tangential to the cusp as it "slips" back through the advancing fluid. The fluid motion component normal to the cusp is the same on both sides (it is continuous through the cusp).

The calculated motion is shown in Figure 3 for a number of time increments during systole for the representative wave. A buckling, whipping motion is readily evident. This cusp motion in a sinus meridional plane was determined by starting with a representative cusp geometry determined from the silicone rubber casts. The cusp section contour was approximated by eight straight-line segments. The motion was constructed by applying the condition that each segment maintained its length as it swung along an arc from the previous segment starting with the commissure segment. The displacement of the ends of the segments was determined by the
Valve motion for normal initial contour. Contours are shown for even-\(m\) intervals from \(m = 0\) to \(m = 28\) (28\% of delivery pulse time or 11\% of beat period). See text for explanation of other abbreviations.

Motion along streamlines through the initial segment points. Seven streamlines were used. Approximate streamline shapes were assumed for this analysis. The displacement along these streamlines was calculated from Eq. 10c. The resulting meridional cusp shapes for \(m\) from 0 to 28 are shown in Figure 3. These contours are merely displacements of the initial contour line \((m = 0)\) produced by the velocity field of Eq. 6. The fact that points on the cusp contours correspond to flow displacement points indicates that the cusp moves along with the fluid with a continuity of the normal velocity components \(v_n\) across the cusp. The tangential (or slip) velocity \(v_t\) can be obtained by subtracting the normal vector velocity component of the cusp motion \(v_n\) from the flow velocity at any point on the cusp as shown at point \(A\) in Figure 3.

During the initial phases of the motion, the tangential velocity is very small (as shown at point \(B\), \(m = 4\)), because the velocity is small and the fluid and leaflet motions are nearly in the same direction. During the final stages, the fluid motion is primarily tangential but produces little effect on the valve motion, since the valve motion \((v_e)\) is now very small.

The point of maximum curvature of the leaflet is of interest since it is the point of maximum bending stress; it is at point \(K\). The dimensionless radius of curvature at this point is calculated as \(R' = R/d = 0.29\), or the dimensionless curvature is \(K' = Kd = 3.5\) \((d = 2r_0)\). Another large curvature point is near the mutual coaptation point of the three leaflets at the center line.

The stresses at \(K\) appear to be more critical as indicated from fatigue tests of trileaflet prosthetic valves (according to unpublished data obtained by R. E. Clark on composite prosthetic valves). For any point on the surface of the leaflet, the change of curvature from the initial to the maximum value is more significant for determining the bending stress than is the maximum curvature. For the case of Figure 3, the leaflet is “precurved” at the starting position in the direction of the maximum curvature so that the change in curvature is less than the maximum curvature. The change in curvature for point \(K\) is \(\Delta K' = 2.4\).
The trajectories for two other initial contours are shown in Figures 4 and 5. The initially convex shape of Figure 4 reverses its curvature, thereby developing a more severe stress condition than the precurved shape of Figure 3. The maximum curvature variation for this case is $\Delta K' = 5.6$. For the "bulged" valve initial contour of Figure 5, the stress is still larger as indicated by a maximum $\Delta K' = 6.7$. A "weak" valve of this contour shape is seen to be subjected to a bending stress nearly twice as large as that for the "normal" contour of Figure 3, since the bending stress is directly proportional to $\Delta K$. Initial approximate calculations of the maximum bending stresses indicate that they are only a small fraction of maximum membrane stresses developed at valve closure when the valve is subjected to a pressure loading on the order of 100 mm Hg. The full significance of the bending stress is therefore difficult to assess. It appears qualitatively that the maximum stress loading during diastole also occurs in the same region as the maximum bending.

Valve shape trajectories were also constructed for the sine wave flow delivery pulse shape. The results were only slightly different from those of Figures 3-5. The shapes for a uniform inlet velocity profile (rather than the exponential) were also constructed. As expected, curvatures near the wall were more severe. For example, a profile with the initial shape of that in Figure 3 gave a maximum $\Delta K' = 3.6$.

Larger curvatures are apparent at the base of the leaflet and at the line of coaptation. It is difficult to make an accurate assessment of the motion or of the stresses near the base because of the complications of the flow and the structural reinforcement here. Near the coaptation line, the leaflet is thinner, providing a smaller bending stress (inversely proportional to thickness). Bending stresses in the region of point $K$ (Fig. 3) are emphasized at this point because of the coincidence with observed valve failure in this region.

**AREA**

The valve does not physically open until about the eighth time increment, and it is 90% open by the twenty-second increment (at 95% radial position). In this case, however, there is no significant relation between blood flow and valve opening, since a significant quantity of blood has been displaced into the aorta before the leaflets part and since the valve presents no physical restriction to the flow during...
the opening phase except for very small boundary-layer effects of the slip flow near the surface. This boundary-layer displacement flow defect is quite small for this type of accelerated flow. Once the leaflet is in its extreme position, it presents a barrier between the mainstream and the fluid in the sinus region. Its effect is to prevent any significant shearing stress from being applied to the sinus fluid. Since there is very little coronary flow during systole and since the fluid is incompressible, there is essentially no flow into the region between the sinuses and the leaflets after the leaflets are in the fully open position and up until the time when flow deceleration begins. The only motion in this sinus region is a vortex eddy generated during the acceleration period.

A plot of the open area ratio $A/A_{\infty}$ vs. the time $m$ for the valve of Figure 3 is shown in Figure 6 with the flow rate ratio $Q/Q_{\text{max}}$. The flow rate is about 17% of its maximum value before the valve leaflets start to separate. The valve is fully open in approximately half the time required for the flow rate to reach its maximum value. These results are in close agreement with those presented in Figure 6 of reference 4. The open area vs. the time for the two valve contours of Figures 4 and 5 vary negligibly from that of Figure 6. The same observation holds for the sine wave delivery pulse and for the uniform inlet velocity profile.

![Figure 5](image)

Valve motion for initial convex contour bulged toward left ventricle. $m = 0 \text{ to } m = 20$.

![Figure 6](image)

Valve open area ratio $A/A_{\infty}$ as a function of the time index, $m$, as a percent of the systolic ejection period, for contour of Figure 3. Flow ratio, $Q/Q_{\text{max}}$, is shown by the broken line for reference.
References

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Circ Res. 1973;32:42-48
doi: 10.1161/01.RES.32.1.42

Circulation Research is published by the American Heart Association, 7272 Greenville Avenue, Dallas, TX 75231
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Print ISSN: 0009-7330. Online ISSN: 1524-4571

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