Letters to the Editor

Pulmonary Alveolar Blood Flow

In a recent paper on their intriguing sheet-flow model of the pulmonary alveolar circulation (Circ Res 30:470-490, 1972) Drs. Fung and Sobin conclude that flutter or oscillation is impossible because the Reynolds number of capillary blood flow is in the range of $10^{-1}$ to $10^{-4}$. The flow is dominated by viscous forces because, for a Reynolds number much less than 1, the inertial forces are small compared with the viscous forces. They contend that my observations on oscillations in collapsible tubes (IEEE Trans on Bio-med Eng, vol. BME-16:284-295, 1969) do not apply to their tubes because our Reynolds numbers differ by several orders of magnitude. In this letter, I will briefly recapitulate my findings and describe the model I proposed for a collapsible tube. I shall then give a condition for no oscillations which does not depend on the Reynolds number.

In my paper I showed that, at constant external pressure, the pressure drop across a collapsible tube held open at its ends was a nonlinear multiple-valued function of the steady flow. The pressure drop vs. flow or the characteristic curve had a region of negative slope between two regions of positive slope. The dynamic resistance, which is defined as the derivative of the characteristic at the operating point, went from positive in the open tube to negative as the tube partially collapsed to positive for the almost completely collapsed tube. Flow in the collapsed tube was through open side tunnels. Oscillations could be obtained only when the operating point was in the region of negative dynamic resistance. In both its steady and its nonsteady behavior, a collapsible tube is analogous to a tunnel diode or a transistor. The experimental demonstration of a negative resistance confirms the theoretical prediction of Rideout and Dick (IEEE Trans on Bio-med Eng, vol. BME-14:171-177, 1967), but not in the form expected.

On the basis of these results, I proposed a modification of the well-known linear lumped-L network for a collapsible tube (see for example Rideout and Dick). The proposed network contains an empirical pressure-flow characteristic, i.e., a non-linear resistance rather than a linear resistance, in series with the fluid inertance. The wall compliance is in shunt across the elements at the upstream end. The collapsible tube proper is connected to the rest of the system by resistors representing the rigid end tubes (leads) on which it is mounted. A collapsible tube held open at its ends is always embedded in a larger circuit even if that circuit only consists of its own leads.

Oscillations in the tube and an external circuit could be described by the second order nonlinear van der Pol equation (Phil Mag 2:978-992, 1926). The conditions for oscillation are given in terms of the van der Pol parameter $\epsilon$ where

$$\epsilon = \frac{(r-R)}{C/L},$$

Here $r$ is the dynamic resistance of the tube at the operating point, $R$ the viscous resistance of its leads plus the rest of the circuit, $L$ the inertance of tube and circuit, and $C$ the compliance of the tube. Assuming that the operating point is in the dynamic negative resistance region, oscillations will grow or decay depending on the sign of $\epsilon$, and the magnitude of $\epsilon$ will determine their rate of growth or decay. For $\epsilon > 0$, that is, when $r > R$, the oscillations grow. If $\epsilon << 1$, the oscillations are sinusoidal and grow slowly; for $\epsilon >> 1$, they are of the relaxation type and grow rapidly. As van der Pol has shown, the inertial forces are small compared with the viscous forces in relaxation oscillations, but some inertance, however small, must be present to have sustained oscillations. By rewriting $\epsilon$ as

$$\epsilon = \frac{(r-R)}{\omega L},$$

$$\omega = \frac{1}{(LC)^{1/4}},$$

where $\omega$ is the angular resonant frequency of the tube and circuit, it can be seen that $\epsilon$ does indeed represent the ratio of viscous to inertial forces. However, because $\epsilon$ depends on the difference between the dynamic and the viscous resistance as well as on the inertance, it characterizes flow in the circuit more completely than does the Reynolds number. The van der Pol $\epsilon$ is roughly the reciprocal of the Reynolds number.

It does not necessarily follow then that for small Reynolds numbers there will be no oscillations. A sufficient condition for no oscillations to appear is $r < R$, that is, $\epsilon < 0$. This condition is not contrary to the analysis of Drs. Fung and Sobin because in their Appendix A they postulated that a full nonlinear analysis might contain a limit cycle.
Motions tending to a limit cycle are characteristic of negative resistance oscillators.

William A. Conrad
30 West 71st Street
New York, New York 10023

REPLY TO THE ABOVE LETTER

Although Mr. Conrad did not dispute our theoretical analysis in any manner or offer any new experimental evidence in the range of Reynolds numbers pertinent to the pulmonary alveolar blood flow problem, he did propose a new criterion for flutter. He based his criterion on the belief that the flow in a Starling resistor can be exactly represented by a one-dimensional analogue with a van der Pol oscillator.

I agree that the relaxation-oscillation analysis by van der Pol is most suggestive to biological problems. The original paper by van der Pol (Arch Neurol Physiol 3:300 Annu 14:418-445, 1929) was written as a simulation of the beating of the heart and as a tool for analyzing the electrocardiogram: an oscillating Starling mechanism indeed looks like a van der Pol oscillator. Thinking of the lung as a collection of van der Pol oscillators suggests many facets for exploration and leads to strong heuristic reasoning. But to stop at a heuristic argument is to stop progress; one must proceed beyond it.

Although a certain analogue may be convincing, its reverse may not be true: a Starling resistor is not exactly a van der Pol oscillator, a heart is not exactly a combination of van der Pol oscillators. A relaxation oscillation may be identified as flutter, but all flutter is not relaxation oscillation. In the aeronautical, civil, and mechanical engineering fields, there are numerous examples of flutter which have nothing to do with relaxation oscillation. In other words, an analogue needs to be proven, not merely argued at.

An exact formulation of the flutter problem of a collapsible tube is quite simple: write down the Navier-Stokes equations for the fluids inside and outside of the tube, the equations of motion of the tube wall, the constitutive equation for the material of the tube wall, the equations of continuity for both the wall material and the fluids, the matching velocity boundary conditions at the tube wall to express consistent deformation and no-slip conditions for the viscous fluids, the conditions at the ends of the tube (pressure or flow fields outside the tube), and the initial conditions. These equations and conditions lead to a set of nonlinear partial differential equations. The problem at hand is to show that these all boil down to a single van der Pol equation and nothing else. Mr. Conrad did not do this and I cannot do it.

Mr. Conrad’s criterion may very well be right, but I am not convinced. The minimum he should show is that (1) at very low Reynolds numbers the pressure-flow curve remains the same as what he found at high Reynolds numbers, (2) the mode of deformation of the tube remains the same, (3) at very low frequencies the upstream oscillatory flow remains small compared with the downstream flow, and (4) his ε, which is variable throughout a cycle, works the same way as does the van der Pol ε, which is a constant.

To say that Mr. Conrad’s criterion may be right is not to say that our paper was wrong, because Mr. Conrad did not show what the value of the dynamic resistance must be in a collapsible tube at very low Reynolds numbers. His criterion and our results might even be coincident. On the other hand, as we said in our paper, the nonlinear differential equation was not fully investigated. Limit cycle (relaxation oscillation) was not investigated. The stability we spoke about is stability with respect to small perturbations. We did not find relaxation oscillation in our experiments at low Reynolds number. Whether such oscillations can be created by some device during an experiment is not yet known.

It is obvious that further research is needed on this problem. In mathematics, the proof of an existence theorem should not be confused with the demonstrations of special cases. Certainly a suggestion, no matter how reasonable, should not be confused with a criterion.

We do not agree with Mr. Conrad’s statement that the van der Pol ε is the reciprocal of the Reynolds number. If this statement were so, then, at very small Reynolds numbers, ε >> 1, and, following his argument, a Starling resistor would be infinitely prone to oscillation. If this phenomenon were so, then our experimental findings could already deny his criterion.

Y. C. Fung
Department of Applied Mechanics and Engineering Science
University of California San Diego
La Jolla, California 92037

Inferior Interatrial Pathways in the Dog

We have read with great interest the detailed studies concerning the inferior interatrial pathways...
Pulmonary Alveolar Blood Flow
WILLIAM A. CONRAD

Circ Res. 1973;32:117-118
doi: 10.1161/01.RES.32.1.117
Circulation Research is published by the American Heart Association, 7272 Greenville Avenue, Dallas, TX 75231
Copyright © 1973 American Heart Association, Inc. All rights reserved.
Print ISSN: 0009-7330. Online ISSN: 1524-4571

The online version of this article, along with updated information and services, is located on the World Wide Web at:
http://circres.ahajournals.org/content/32/1/117.citation

Permissions: Requests for permissions to reproduce figures, tables, or portions of articles originally published in Circulation Research can be obtained via RightsLink, a service of the Copyright Clearance Center, not the Editorial Office. Once the online version of the published article for which permission is being requested is located, click Request Permissions in the middle column of the Web page under Services. Further information about this process is available in the Permissions and Rights Question and Answer document.

Reprints: Information about reprints can be found online at:
http://www.lww.com/reprints

Subscriptions: Information about subscribing to Circulation Research is online at:
http://circres.ahajournals.org//subscriptions/