Current and Potential Fields Generated by Two Dipoles

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ABSTRACT

The distribution of currents and potentials in a circular conducting medium surrounding two eccentric dipoles was studied to establish how much information on the number, location, and orientation of the dipoles could be deduced from measurements of potential in the medium at various distances from the generators. When a single, eccentric dipole was active, the curve illustrating the distribution of potentials along the boundary exhibited different kinds of asymmetry, which revealed that the dipole was eccentric and gave some information about its orientation. When both dipoles were active, two maxima and two minima, revealing the presence of two generators, appeared along the boundary when the angle between dipole moments was 150° or more. Along internal circumferences two maxima and two minima appeared at smaller angles between dipole moments and the location of the maxima was closely related to that of the dipole anodes. When the dipoles lay on the same diameter and had opposite polarity, the presence of two generators was clearly detectable from boundary measurements, whereas vector representation was zero. These data improve our understanding of electrical signals recorded from the body surface, whether in the form of electrocardiograms, vectorcardiograms, or equipotential contour maps.

ADDITIONAL KEY WORDS

Mathematical-physical models, electrocardiography, vectorcardiography, electromaps

Recent investigations on the distribution of cardiac potentials on the body surface of normal subjects and cardiac patients have shown that multiple maxima and minima are simultaneously present on the chest walls during some phases of the cardiac cycle. During the other phases, only one maximum and one minimum were observed (1-5). The number, location and time-course of surface maxima and minima exhibited typical changes in various heart diseases (6-8).

Electrophysiological interpretation of chest maps requires the surface potential patterns to be correlated with the intracardiac location of active fibers at the various instants considered. Knowledge of this correlation may be gained in different ways: (1) The distribution of chest potentials and the location of intracardiac generators can be determined experimentally in the same animal (9), and (2) the distribution of potentials in a conducting medium can be established for any number and location of current generators by physical or mathematical models (10-19). This paper reports an investigation on the current and potential fields generated by two dipoles located eccentrically in a circular, homogeneous conducting lamina. This model has been chosen because it simulates electrical situations actually occurring in the heart when two excitation waves travel simultaneously in different directions through the ventricular or atrial walls.

The distribution of potentials generated by two dipoles can be determined with either a physical or a mathematical model. When this study was being planned, computer facilities were not available in the laboratory, and it was decided to use a physical model. Later on, a computer became available and some of the experimental data were compared with those obtained by using a mathematical model.
Methods

Two dipoles were placed in a circular perspex bath with a radius of 9.5 cm. Each dipole consisted of two 0.5-mm diameter silver wires, 10 mm apart, rising vertically from the bottom of the bath to a height of 8 mm. The wires were coated with silver chloride. A sufficient amount of 0.1\% NaCl solution was put into the bath to reach the top of the dipole wires, i.e., 8 mm. The centers of the dipoles lay on the same diameter (0°-180°, Fig. 1B), 15 mm from the center of the bath. The axis of the left dipole (fixed dipole) lay on the 0°-180° diameter. The orientation and polarity of this dipole were kept constant throughout the
experiments. The orientation of the right dipole was varied by 30° in each experiment. Square waves lasting 5 msec were generated by a battery-powered, solid-state multivibrator, at a rate of about 100 cps. The multivibrator was insulated from ground and its output could be connected to either dipole (Fig. 1C). At the beginning of each experiment, the dipole current was determined by measuring the potential difference across a resistor in series with the dipole circuit. Two tracings were then simultaneously recorded (Fig. 1D) by two Tektronix 122 preamplifiers and a two-channel Tektronix 502 oscilloscope. The first tracing displayed the potential difference between two silver-silver chloride electrodes located in a fixed position near the perimeter of the bath. This “control” tracing served to detect the potential variations which might occur in the bath during an experiment due to changes in dipole output. The second tracing recorded the potential difference between one of the fixed electrodes and a third electrode that was moved to explore 161 points in succession, these being distributed in the bath along a series of concentric circumferences (Fig. 1A). A synchronized calibration signal was fed into both amplifiers at every dipole cycle (Fig. 1C and D). The tracings were photographed from the screen of the oscilloscope, optically magnified and carefully measured. In the first experiment the multivibrator was connected to the left dipole in such a way that every square wave made the left pole positive to the right. The potentials were then recorded at the selected points in the conducting medium. The multivibrator was then switched to the right dipole which was oriented at 0°-180°. In this case too, the left pole was made positive to the right. The 161 points were explored again. Thereafter, the right dipole was rotated counterclockwise 30° at a time until it reached the 180°-0° orientation. For the sake of brevity, the expression “dipole at 0°, 30°... 180°” will be used to indicate the sequential orientations of the right dipole. A fresh exploration of the medium was carried out after each 30° rotation. No clockwise rotations were made since they would have produced potential distributions symmetrical to those obtained by counterclockwise rotation. The potential values measured at each point when the left dipole was energized were then added algebraically to those measured at the same points when the right dipole was energized in the 0° position. The same procedure was repeated for each orientation of the right dipole. In effect, it was considered that if the two dipoles had been simultaneously activated, the potential at each point in the medium would have been equal to the sum of those generated by each dipole acting separately. The procedure described gives the potentials produced by the two generators combined, measured against the reference electrode. Since the location of this electrode was arbitrary, the potential values were corrected so as to relate them to the average potential at the boundary. The final results were plotted on maps of the circular conducting medium. A separate map was thus obtained for each orientation of the right dipole. Equipotential lines were then drawn on all the maps. In some cases the potential distribution along several circumferences was also calculated by an Olivetti “Programma 101” desktop computer using the electrical image method described by Nelson (20). The computed data were in good agreement with the experimental data, discrepancies rarely exceeding 5%.

**Results**

It is known that when a dipole is located at the center of a circular conducting lamina, the potential distribution in the medium is characterized by two symmetry axes, one of which coincides with the axis of the dipole, while the other is perpendicular to it. This is shown in Figure 2, taken from a paper by Nelson (20). The potential varies along the boundary of the circular medium according to a sine law (21, 22). In our experiments, the dipoles were
FIGURE 3

A: Potential and current fields generated by an eccentric dipole (left dipole). Currents (broken lines) diverged from the anode and converged on the cathode. A potential maximum was located where currents arrived at the periphery of the bath; a potential minimum was located where currents flowed inward (arrows). B: Potential and current fields generated by two dipoles (left and right at 0°). Four current systems were present in the central portion of the medium. Only one system was present in peripheral areas. C: Potential distribution at the periphery of the lamina, related to the left (fixed) dipole. D and E: Potential distribution along the boundary and the 7 cm diameter circumference, generated by two dipoles located as in B. The superimposed dots represent a sine wave. F: First derivative of curve E, with two positive and negative peaks.

eccentric. When the left dipole was energized, the potential field was of the kind illustrated in Figure 3A, which also shows the current lines diverging from the positive pole and converging on the negative pole. The curve illustrating the potential distribution along the boundary exhibited one maximum and one minimum but was not perfectly sinusoidal in that it presented some degree of asymmetry (Fig. 3C). Obviously, a curve of the same kind was observed when the right dipole was at 180° (Fig. 4A). This curve consisted of one positive and one negative portion, which differed from each other. However, in the
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The current field produced by both dipoles being energized simultaneously (Figs. 3B, 5 and 6) was more complex in that it comprised several systems of current lines. The term "system of current lines" is here meant to indicate all the current lines traveling from one positive pole to one negative pole. In most cases there were four of these systems, i.e., (a) currents flowing from the anode of the left dipole to the cathode of the left dipole, or (b) to the cathode of the right dipole, (c) currents flowing from the anode of the right dipole to the cathode of the right dipole, or (d) to the cathode of the left dipole. When the right dipole was at 0° (Fig. 3B), the central portion of the medium was crossed by currents belonging to all four systems, whereas in peripheral areas there were only currents belonging to one system. These flowed from the anode of the left dipole to the cathode of the right dipole (Fig. 3B). Near the boundary the current pattern was therefore similar to that which would be observed in the case of a single centric dipole. The curve illustrating the potential distribution along the external circumference was similar to, but not identical with a sinusoid (Fig. 3D). The difference between the experimental curve and a sinusoid appeared more clearly when we plotted the potentials along a circumference of smaller radius (Fig. 3E), and was still more evident when we compared the first derivative of a sinusoid with that of the experimental curve.

The first derivative of a sinusoid is still a sinusoid, 90° out of phase in relation to its original function, and it therefore exhibits only one maximum and one minimum, whereas the first derivative of the experimental curve showed two positive and two negative peaks (Fig. 3F).

When the right dipole was turned to 30°, 60°, 90° and 120° (Fig. 5A, B, C, D), the area covered by several current systems expanded from the center toward the periphery. At the same time, the peripheral area occupied by a single system of currents shrank progressively. When a circumference was crossed by current lines belonging to different systems, this was associated with more than one potential maximum and minimum (Fig. 5A', B', C', D'). When the right dipole was at

\[ \frac{dV}{dL} = \frac{AV}{\Delta L} \]

where \( V \) = potential and \( L \) = distance along the circumference. In practice, we measured the potentials every 15 degrees of arc.
FIGURE 5

Potential and current fields generated by two simultaneously active dipoles. The left dipole maintained a fixed location and orientation. The right dipole was oriented at 30° (A), 60° (B), 90° (C) and 120° (D). The corresponding potential distributions along the boundary and a 7 cm diameter circumference are illustrated in A', B', C' and D'. The higher voltage curves correspond to the 7 cm diameter circumference.

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150° and 180°, currents coming from both anodes reached the periphery of the circular lamina, and currents heading toward both cathodes left the periphery (Fig. 6A and B). This was associated with two maxima and two minima appearing along the external circumference (Fig. 6A' and B').

Let us now consider the potential fields more closely. When the right dipole was at 0°, the peripheral maximum and minimum (Fig. 3B) faced the anode of the left dipole and the cathode of the right dipole, respectively. When the right dipole was rotated to the 30° position, both the peripheral maximum and minimum turned counterclockwise (Fig. 5A). However, they did not rotate as much as 30° but only about 15°. The straight line joining the peripheral maximum and minimum had an orientation which was close to that of the resultant vector obtained by summating the moments of the two dipoles. It will also be noted that the peripheral maximum no longer faced the anode of the fixed dipole, although this dipole had not moved. When the right dipole was rotated to 60°, 90° and 120°, the peripheral maximum and minimum continued to rotate counterclockwise (Fig. 5B, C and D). The minimum rotated more than the maximum. Consequently, the line joining the peripheral maximum and minimum did not pass through the center of the circle. Again, the orientation of this line approximated to

FIGURE 6
Same experiment as in Figure 5. The right dipole was oriented at 150° (A) and 180° (B). The corresponding potential distribution along the boundary is illustrated in A' and B'. In A' the potential distribution along the 7 cm diameter circumference is also represented (higher voltage curve).
that of the resultant vector. However, when the right dipole was rotated to 150° (Fig. 6A), the peripheral maximum reversed its direction of rotation, and turned back toward its original location, while the minimum continued rotating counterclockwise. Meanwhile, a new maximum and a new minimum appeared in the lower right quadrant. If we consider the potential distribution along an internal circumference having a diameter of 7 cm (Fig. 5A', B', C' and D' and Fig. 6A' and B'), we can see that the potential maximum permanently faced the anode of the left dipole, in that it was located at nearly 180° throughout the rotation of the right dipole. Meanwhile, the minimum turned 90° counterclockwise. A new maximum and a new minimum appeared earlier than on the external circumference, namely, when the second dipole had rotated only 90° (Fig. 5C'). The new maximum constantly followed the rotation of the right anode.

Discussion

In the course of this discussion, we will attempt to establish how much information about the number, location, and orientation of the generators can be deduced from knowledge of the potential distribution at the boundary of the conducting medium and inwards from the boundary. This "inverse" problem (23-27) can be solved mathematically, provided some constraints are introduced into the computation. However, we considered that a graphic representation of the potential and current fields generated by known sources would provide intuitive insight into the relationships between current sources, current distribution and location of potential maxima and minima at the boundary of the conducting medium.

When a single eccentric dipole was active in the bath, the curve illustrating the potential distribution along the boundary showed some degree of asymmetry, as previously stated. The features of the curve suggested that the dipole was eccentric and enabled us to establish whether it lay along a diameter, or perpendicular to it, or in a different direction (Fig. 4). This observation is in agreement with Gastonguay and Nelson's recent studies, which showed that the orientation, location and magnitude of an eccentric dipole in a circular conducting medium can be deduced from a series of potential measurements at the boundary (28). When both dipoles lay on the same diameter and had the same polarity (Fig. 3B), the curves illustrating the potential distribution along the boundary or along an internal circumference were not perfectly sinusoidal (Fig. 3D and E) and their first derivative showed two maxima and two minima (Fig. 3F). We were interested in finding out whether or not these features were related to the presence of two dipoles in the medium. In effect, a previous investigation (29) had shown that even a single, centric dipole may generate an imperfectly sinusoidal potential distribution along a circumference when the diameter of the circumference is less than 5 times the distance between poles. In our experimental conditions (Fig. 3B), the peripheral currents came from an anode located 4 cm away from the cathode toward which they were headed. Would a single, centric dipole, with poles 4 cm apart, generate a similar distribution of potentials along circumferences A and D having a diameter of 19 and 7 cm? To answer this question, we performed another experiment in which we energized a centric dipole with poles 4 cm apart. The curves illustrating the potential distribution along circumference A and D were not perfectly sinusoidal, and showed some similarity with those relating to the two aligned dipoles. Furthermore, the first derivative of the potential curve relating to circumference D exhibited two positive and two negative peaks (Fig. 7, left). It ensues from this finding that the presence of two dipoles in the circular medium cannot be inferred from the first derivative of the potential along a circumference exhibiting two maxima and two minima. Here, it will be noted that a distance of several centimeters between current source and sink can actually be observed in the heart during the recovery process (30).

We have seen that when the two dipoles
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First derivatives of the curves illustrating the potential distribution along the 7 cm diameter circumference, generated by a single centric dipole with its poles 4 cm apart (left graph) and by both dipoles, i.e., the fixed dipole plus the right dipole at 0° (right graph). Both curves show two positive and two negative peaks. These are more prominent on the right graph.

were aligned on the same diameter and had the same polarity, the potential distribution along the boundary did not indicate the presence of a double generator in the medium. When the right dipole was turned to 30°, 60° and 90°, the peripheral distribution was similar to that which would have been generated by a single eccentric dipole, whose orientation approximated to that of the resultant vector. No "local" effects were apparent and the potential distribution along the boundary only provided information on the overall electrical activity of the generators. However, when the right dipole was turned as far as 120° (Fig. 5D'), a small dip appeared on the potential curve, and became a clear-cut additional minimum at 150° (Fig. 6A'). When the right dipole was at 180° (Fig. 6B'), the potential curve once again resembled a sine wave which, however, had a period of 180° instead of 360°, and therefore exhibited two maxima and two minima. Thus, with some orientations of the right dipole, "local effects" were clearly visible even at the boundary of the circular lamina in the form of multiple maxima and minima which indicated the presence of multiple current sources and sinks in the medium. It is worth noting that when the right dipole started rotating counterclockwise (Fig. 5), the peripheral maximum, which had initially faced the anode of the left dipole, also moved counterclockwise, although the left dipole had not moved. This behavior should be kept in mind when interpreting body-surface electromaps. It shows that the migration of a surface maximum does not necessarily imply any motion of the underlying wavefront, but can be due to the appearance or motion of another wavefront elsewhere in the heart.

If we now consider the potential distribution along circumferences closer to the current generators, we can see that the smaller the radius of the circumference, the smaller the rotation of the right dipole needed to bring about two maxima and two minima. For instance, two potential maxima and two minima appeared along circumference D (radius = 3.5 cm) at a rotation of 90° (Fig. 5C'), whereas a rotation of 150° was necessary for them to appear at the boundary. Also the location of the potential maxima along circumference D was closely related to that of the dipole anodes in the medium. Thus, the presence of multiple dipoles was more evident, and could be detected at smaller angles between dipole moments, when areas close to the generators were explored in detail. In accordance with these results, one would expect that detailed exploration of the precordial region in man should provide more information about the number and location of intracardiac wavefronts than can be obtained from a small number of distant leads. Equipotential contour maps obtained from normal subjects and cardiac patients support this conclusion (1-8).

In a few cases, "vectorcardiograms" were obtained by recording potential differences between opposite ends of two orthogonal diameters. Even when local effects were present at the boundary, this procedure yielded vectors whose orientation was close to that of the true resultant vectors. These were obtained by vectorially adding the moments of the two dipoles. The vectorial representation obviously did not enable us to establish whether a single or a multiple generator was present in the medium. In addition, when the
right dipole was at 180°, the recorded vector was 0, and this could lead to the false conclusion that there were no active dipoles in the medium. In these experimental conditions, much more detailed information on the generators could be obtained by studying the potential distribution along peripheral or internal circumferences, as it is shown by the results discussed above.

Clearly, the conclusions of this investigation cannot be directly applied to human electrocardiography. In human beings and experimental animals, the heart is a much more complex generator than the group of two dipoles studied here. Also, the heart is surrounded by a conducting medium which is neither homogeneous nor of geometrical shape. Nevertheless, our simplified model illustrates a number of elementary potential distributions, the knowledge of which may be of use in understanding the more complex fields observed in vivo. These results could therefore be useful in interpreting the electrical signals recorded from the surface of the body, whether in the form of electrocardiograms, vectorcardiograms or equipotential contour maps.

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References


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