ABSTRACT

Segments of dog carotid artery were held at in-situ length and studied in vitro after excitation of the muscle with norepinephrine and after poisoning of the muscle with potassium cyanide. In-situ length corresponded to a longitudinal strain of .57 ± .02 relative to the unstretched length. The longitudinal elastic modulus was about $4.1 \times 10^6$ dyn/cm² at in-situ length and zero transmural pressure. This value was not altered by excitation of the vascular smooth muscle. The longitudinal stress due to traction decreased as the longitudinal stress due to transmural pressure increased, and it was suggested that this interaction underlies the relative constancy of vessel length in situ. The Poisson’s ratio between the circumferential and longitudinal directions was about 0.3 and was found to decrease slightly with activation of the vascular muscle. The data for the longitudinal modulus and for Poisson’s ratio were used to compute the circumferential elastic modulus. Activation of the muscle increased the circumferential elastic modulus when plotted as a function of circumferential strain. Comparison between the moduli in the two directions revealed that the arterial wall is not isotropic at physiological pressures because the circumferential elastic modulus is greater than the longitudinal modulus. Calculations indicated that assuming isotropy slightly underestimates the true circumferential modulus at small circumferential strains, and greatly overestimates the true circumferential modulus at large circumferential strains. Active smooth muscle has little direct effect on these estimations, but does alter the error by contracting the vessel to smaller strains.

ADDITIONAL KEY WORDS

- elastic modulus
- Poisson’s ratio
- changes in vessel length
- isotropic
- anisotropic
- materials
- elasticity
- incompressible
- isovolumetric
- cardiac cycle
- arteries

Materials which exhibit elastic moduli that are equal in all directions may be termed isotropic, while materials which exhibit elastic moduli that are not equal in all directions may be termed anisotropic. Poisson’s ratio, the ratio of strains in perpendicular directions, is inseparably related to elastic modulus and must be known to determine elastic moduli when forces are applied simultaneously in two or more directions. Most studies of vessel elasticity have assumed, for lack of more specific information, that the arterial wall at in-situ length is isotropic and exhibits a Poisson’s ratio of 0.5 (1-4). These assumptions are supported by the fact that the wall is isovolumetric (3, 5, 6) and that arteries in vivo manifest relatively constant length during the cardiac cycle (7, 8). This agrees with the prediction that an incompressible, isotropic tube should not change length upon pressurization (9). On the other hand, studies on excised, retracted arteries (9-12) indicate that the wall is not isotropic. However, the properties of excised arteries may not accurately reflect the properties of vessels at in-situ length because excised, retracted vessels exhibit a marked reduction in longitudinal elastic modulus (12). Two studies on vessels in situ (13, 14) also indicate that the arterial

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wall is anisotropic, but these data differ as to whether the elastic modulus is greater in the circumferential or longitudinal direction. The purpose of the present study was twofold: (1) to evaluate Poisson’s ratio and the circumferential and longitudinal elastic moduli of arteries held at in-situ length, and (2) to determine the influence of vascular smooth muscle upon these wall characteristics.

Methods

If a material is extended by application of a force in one direction, then the change in strain which occurs in that direction may be written

\[ \Delta \varepsilon = \frac{\Delta \sigma}{E} \]  

(1)

where \( \Delta \varepsilon \) is the change in strain, \( \Delta \sigma \) is the change in stress and \( E \) is the incremental elastic modulus. However, if a material is subjected to forces applied simultaneously in two directions, then the change in strain which occurs in any one direction will be a function of the ratio of the stress increment to the elastic modulus in that direction less a portion of the change in strain which occurs in the perpendicular direction. Poisson’s ratio \( \nu \) describes the magnitude of this portion. For a tube which is isotropic, the change in strain in the longitudinal and circumferential directions may be written

\[ \Delta \varepsilon_L = \frac{\Delta \sigma_L}{E} - \nu \frac{\Delta \sigma_C}{E} \]  

(2)

\[ \Delta \varepsilon_C = \frac{\Delta \sigma_C}{E} - \nu \frac{\Delta \sigma_L}{E} \]  

(3)

where \( \Delta \varepsilon \) is the change in strain, \( \Delta \sigma \) is the change in stress, \( E \) is the incremental elastic modulus, \( \nu \) is the Poisson’s ratio and the subscript indicate the longitudinal \( (L) \) and circumferential \( (C) \) directions. Radial stress, i.e., the stress which tends to compress the wall, amounts to only 5 to 10% of other stresses at physiological pressures and has been neglected to simplify analysis. Equations 2 and 3 may be added and rearranged to find the wall incremental elastic moduli for the condition where the length of the tube is held constant.

\[ E = \frac{\Delta \sigma_C}{\Delta \varepsilon_C} (1 - \nu^2). \]  

(4)

For a tube which may be anisotropic, the change in strain in each direction may be written

\[ \Delta \varepsilon_L = \frac{\Delta \sigma_L}{E_L} - \nu_{CL} \frac{\Delta \sigma_C}{E_L}. \]  

(5)

Radial stresses again have been neglected. In this case the direction of the moduli are specified. Presented in this way, Poisson’s ratio \( \nu_{CL} \) represents the ratio of strain in the circumferential direction to that in the longitudinal direction when the material is subjected to only longitudinal stress. On the other hand, a different Poisson’s ratio \( \nu_{LC} \) could be defined as representing the ratio of longitudinal strain to circumferential strain when the material is subjected to only circumferential stress. The relationship between the two Poisson’s ratios is

\[ \nu_{LC} = \frac{E_L}{\nu_{CL} E_C}. \]  

(6)

The second term in both equations 5 and 6 contains the coefficient \( \nu_{CL} \). This is necessary in order to preserve the symmetry of the stress-strain relationships (10, 15). If the material considered were isotropic then \( E_C \) would equal \( E_L \) and, necessarily, \( \nu_{LC} \) would equal \( \nu_{CL} \). Under these conditions equations 5 and 6 would become identical to equations 2 and 3. This analysis is based upon the assumption of orthotropy, i.e., that only normal strains occur in the three principal geometric directions when the vessel is loaded by inflation and longitudinal traction. Patel and Fry (16) have provided evidence to support this assumption for the arterial wall. In the present experiments these equations were solved simultaneously for Poisson’s ratio and for the incremental circumferential elastic modulus; the values for the remaining variables were obtained experimentally.

Carotid arterial segments were excised from dogs immediately after death and were restored to in-situ length. The following technique was employed to establish this length. Dogs were placed on their backs with the head back in a natural resting position resembling that seen in normal standing posture. A carotid artery was exposed in the neck and the unopened artery was held firmly with a clamp fabricated of a lucite plate. This clamp was mounted on the sensing arm of a Grass FT-10 force transducer, the output of which was recorded on a Grass model 7 polygraph. The location of the force transducer was adjusted along the longitudinal axis of the vessel until the output indicated no measured force. The artery then was severed between the transducer and the animal’s head to shift the load to the force transducer. The force necessary to maintain in-situ length was indicated by an immediate deflection of the polygraph pen. Table 1 presents the results of 20 measurements in 14
TABLE 1
Retractive Force Exerted by Carotid Arteries in Situ

<table>
<thead>
<tr>
<th>Dog</th>
<th>Artery</th>
<th>Retractive force (dyn X 10^-3)</th>
<th>Retractive force/body weight (dyn X 10^-9/kg)</th>
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<td>17</td>
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<tr>
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<td>23</td>
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</tbody>
</table>

**MEAN**

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**SE**

.05

animals. Although there is variability from animal to animal, the data show consistency when expressed as dynes force per kilogram body weight, probably because large dogs tend to have large vessels. A mean and standard error of 1.04 ± .05 X 10^-1 dynes force per kg was observed. These data were used to estimate the correct degree of elongation of the vessels studied in vitro. Five-centimeter segments of carotid artery were excised and cannulated with polyethylene PE 240 cannula at both ends. One cannula was mounted to a fixed upright block in a tissue bath and the other cannula was sealed at one end and mounted with the lucite clamp to the Grass FT-10 force transducer. The vessel, in turn, was suspended from a rigidly fixed micrometer. The retracted vessel segment was then elongated gradually by turning the micrometer until the force gauge indicated a force, in dyn X 10^2, equal to the animal's body weight in kilograms. The length-force relationship was sufficiently steep at these vessel lengths so that any error made in establishing the force resulted in a much smaller error in the vessel length. Thus, the method tended to restore accurately vessels to in-situ length. The length of several vessels before excision was compared with the length achieved after mounting and elongation in vitro. Before excision, fine surgical ligature was stitched and knotted through the vessel wall in two places. The distance between these knots was measured with calipers. The vessel was then excised, mounted in vitro and extended until the force gauge indicated the proper force. The length was again measured with calipers. The length of the mounted vessels in vitro agreed to within .04 cm (about 1.0%) with the lengths measured in situ before excision. Longitudinal force was measured in the in-vitro experiments, and the movement of the arm of the force transducer permitted slight changes in vessel length. However, the force transducer was sufficiently stiff (4.0 X 10^7 dyn/cm) so that the maximum changes in length occurring during experimental procedures were less than 0.2% of the length of the unpressurized vessel segment in-situ length. Therefore, the vessel length was treated as constant during the experimental procedures. The diameter of the arterial segments was measured in vitro with a linear displacement transducer consisting of a differential transformer with a lightweight, movable core (3). An adjustable foot was raised to limit the lower movements of the vessel and the movable core of the displacement transducer was used to measure wall movements. The core of this instrument weighed 613 mg. The area at contact between the disc and the vessel was about .10 cm^2 so that the weight of the core reduced transmural pressure by about 6 mm Hg at the site of contact. The inflation pressure was not corrected for this 6 mm Hg. A Statham P23Dc pressure transducer was connected by a T to the open cannula at the upright block in the tissue bath to measure intraluminal pressure.

The tissue bath was filled with Krebs-Ringer dextrose solution composed of 120 mM NaCl, 4.8 mM KCl, 2.5 mM CaCl_2, 1.2 mM MgSO_4, 7 H_2O, 15 mM phosphate buffer-pH 7.4, and 11.0 mM dextrose. A water jacket surrounded the tissue bath. Thermostatic control of water in the water jacket maintained the fluid in the tissue bath at 36 to 37° C.

**PROCEDURE**

Vessel segments were inflated with 100% O_2 under steady transmural pressures while the diameter and longitudinal retractive force were continuously monitored with the displacement transducer and the force gauge. The vessel segments were inflated with a gas in order for the present methods to be comparable with previous experiments which assumed isotropy (1, 3). This method of inflation also provided a means of immediately detecting leaks in the vessel wall. In a nonleaking vessel the luminal gas should become saturated with water vapor and therefore should not dry the vessel wall. Vessels which leaked when pressurized were discarded. The vessels were relaxed by subjecting them to several
cycles of stepwise pressure increments followed by pressure decrements. Then the pressure was brought to 100 mm Hg and sufficient norepinephrine was added to the bath to give a final bath concentration of 2 mg/liter. Previous studies indicated the effectiveness of the doses of the drugs used (3). When evidence of contraction was observed, the pressure was reduced to zero and the longitudinal elastic modulus was determined as follows: the length micrometer was turned, first to shorten and then to elongate the vessel segment in 0.5-mm steps. Each step was maintained until the force recording was steady for 2 minutes. Steady states were achieved almost immediately with little or no stress relaxation, unless the vessel segment was elongated to values greatly exceeding in-situ length. The vessel segment then was returned to in-situ length and the transmural pressure was increased in 15 mm Hg steps from 0 to 225 mm Hg. Each step was maintained until the diameter and force recordings were steady for at least 2 minutes. The bath then was drained, rinsed and refilled with Krebs-Ringer solution free of dextrose. The pressure was reduced to 75 or 100 mm Hg and sufficient potassium cyanide (KCN) was added to the bath to give a final bath concentration of 100 mg/liter. After 1 hour the pressure was decreased to 0 mm Hg and the length micrometer was again manipulated to first retract and then to elongate the vessel segment in 0.5-mm steps to determine the longitudinal elastic modulus. The arterial segment was then returned to in-situ length and the pressure was again increased in 15-mm Hg steps to 225 mm Hg. Once the vessel exhibited a stable radius at this high pressure, the displacement transducer which had been used to measure changes in diameter was used to measure the absolute diameter of the vessel segment. The differential transformer of the displacement transducer was lowered with a micrometer to advance the coil downward until the core was centered precisely within the coil. This was indicated by a movement of the polygraph pen to the center of the recording channel. The micrometer reading was noted. The displacement transducer was then removed from the tissue bath, and the core was permitted to fall onto the foot which had been in contact with the lower surface of the vessel. Then the micrometer was adjusted to advance the coil downward. When the polygraph pen was again at the center of the recording channel, the micrometer reading was again noted. The second micrometer reading was subtracted from the first to provide the absolute diameter of the vessel at the final level of inflation. To determine the thickness of the arterial wall the inflation pressure was reduced to zero and the displacement transducer was moved to a portion of the vessel which was between the ligatures and which covered some of the polyethylene cannula. Care was taken to avoid the regions of the vessel immediately adjacent to the ligatures. The diameter of the cannula (.203 cm) was subtracted from the external diameter of the vessel in this location to give the thickness of two vessel walls. Since it is known that the vascular wall is virtually incompressible (3, 5, 6), wall thickness data could be used to compute the wall cross-sectional area in these fixed-length preparations. Cross-sectional area, in turn, was used to determine the wall thickness and internal radius at each recorded external diameter. The ratio of wall thickness to external radius at 105 mm Hg was .15 ± .01 (mean and standard error) for the vessels after contraction with norepinephrine, and .09 ± .01 for the same vessels after dilation with KCN. The length of the vessel segment of in-situ length was measured with calipers for the computation of longitudinal strains.

The final procedure was the determination of the original radius and original length used for the computation of strains. The vessel segment was removed from the tissue bath and was permitted to hang freely in a vertical position with no applied forces. The diameter was measured with a micrometer and the length was measured with a fine rule. These values were taken as the original dimensions for the circumferential and longitudinal directions. Lee et al. (17) have used a similar measure and have emphasized the theoretical importance for determining zero strains under unloaded conditions.

Results

In-situ length corresponded to a longitudinal strain of .57 ± .05 (mean and standard error) relative to the unstretched length. This agrees closely with the value that would be predicted from the carotid artery retraction data of Bergel (1). Longitudinal stresses were determined from the force-extension curves obtained by the stepwise elongation of the unpressurized vessel segments. The slopes of these stress-strain curves at in-situ length were measured for the determination of longitudinal elastic moduli. Since the applied force was unidirectional, the longitudinal modulus could be calculated from equation 1. Longitudinal elastic modulus data are shown in Table 2. Some vessel segments exhibited a small change in elastic modulus when the vascular muscle was excited, although others did not. A mean value of 4.17 ± .43 x 10⁶ dyn/cm² was
observed after treatment with norepinephrine as compared with a mean value of $4.06 \pm 0.40 \times 10^6 \text{ dyn/cm}^2$ after poisoning with KCN. The difference between these values was tested with the Wilcoxon matched-pairs signed rank test (18) and was found to be not statistically significant ($P > .10$).

Figure 1 (bottom) presents a polygraph record illustrating the behavior of a KCN-poisoned vessel. Each step increase in pressure was associated with an increase in radius and a decrease in longitudinal retractive force. The latter indicates a decrease in the tendency for the vessel to retract. Similar responses were observed after treatment with norepinephrine and after poisoning with KCN, although the radii were smaller at each pressure level after the muscle had been activated with norepinephrine. The changes in radius were also more gradual after treatment with norepinephrine.

Figure 2 presents data obtained from the inflation of a norepinephrine-treated artery held continuously at in-situ length. The data are plotted as a function of circumferential strain; longitudinal strain was held constantly at in-situ length. Circumferential strain was calculated as

$$
\varepsilon_c = \frac{\Delta r}{r_0},
$$

where $\varepsilon_c$ is circumferential strain, $\Delta r$ is the change in external radius and $r_0$ is the original external radius. Original radius ($r_0$) was designated as the external radius observed in the excised, retracted, entirely unstressed vessel segment after poisoning with KCN. This designation of the basal or natural state considers the vessel as an unstressed, passive tube whose characteristics are influenced by the imposed strains and by the mechanical properties of the vascular muscle. The circumferential strains observed for the vessel segments of in-situ length were both smaller and larger than this original radius, depending upon the inflation pressure. In Figure 2 (top) both longitudinal and circumferential stresses are plotted as a function of circumferential strain. Circumferential stress was calculated as

$$
\sigma_c = P_T \times \frac{r_1}{L},
$$

where $\sigma_c$ is circumferential stress, $P_T$ is
transmural pressure, \( r_i \) is internal radius, and \( h \) is wall thickness. Longitudinal stress was calculated as the sum of two components,

\[
\sigma_L = \sigma_{Lp} + \sigma_{LT},
\]

(10)

where \( \sigma_L \) is the total longitudinal stress, \( \sigma_{Lp} \) is the longitudinal stress due to pressure and \( \sigma_{LT} \) is the stress due to longitudinally applied traction. Longitudinal stress due to pressure was calculated as

\[
\sigma_{Lp} = \frac{P_T \times r_i^2}{r_o^2 - r_i^2},
\]

(11)

where \( \sigma_{Lp} \) is longitudinal stress due to pressure, \( r_i \) is internal radius and \( r_o \) is external radius. Longitudinal stress developed in response to the externally applied traction was calculated as

\[
\sigma_{LT} = \frac{F_{LT}}{\pi(r_o^2 - r_i^2)},
\]

(12)

where \( \sigma_{LT} \) is the longitudinal stress due to the externally applied traction, \( F_{LT} \) is that force, in dynes, as measured by the force transducer and \( \pi(r_o^2 - r_i^2) \) describes the cross-sectional area of the vessel wall. It may be shown algebraically that the longitudinal stress due to pressure (\( \sigma_{Lp} \)) is about one half the circ-
Top: Circumferential and longitudinal stresses plotted as a function of circumferential strain. Longitudinal stress has two components, stress due to pressure and stress due to traction.

Bottom: Circumferential and longitudinal stresses plotted as a function of circumferential strain. Treatment with norepinephrine increases the slope of the circumferential stress curve, with little effect on the slope of longitudinal stress curve.

Circumferential stress ($\sigma_c$). However, in Figure 2 (top) it may be observed that, at small circumferential strains, the longitudinal stress due to the applied traction ($\sigma_{t,r}$) elevates the total longitudinal stress ($\sigma_l$) to a value which is more than one half the circumferential stress ($\sigma_c$). Indeed, up to moderate strains the longitudinal stress ($\sigma_l$) is actually greater than...
Poisson's ratio between circumferential and longitudinal axes. Closed circles indicate data after norepinephrine, open circles indicate data after poisoning with KCN. Closed arrows indicate statistically significant differences (P < .025) between norepinephrine and KCN data. Open arrows indicate no statistical significance. Top: Poisson's ratio plotted as a function of circumferential strain. Norepinephrine tends to lower Poisson's ratio. Bottom: Poisson's ratio plotted as a function of transmural pressure.

the circumferential stress ($\sigma_c$). With increasing circumferential strains, longitudinal stress due to traction ($\sigma_{l,p}$) gradually diminishes, so that at very large strains, the longitudinal stress ($\sigma_l$) does approach one half the value of the circumferential stress ($\sigma_c$). At these
large strains the longitudinal stress ($\sigma_L$) consists, almost entirely, of the longitudinal stress due to pressure ($\sigma_{lp}$). A striking feature of these data is the gradualness of the change in the longitudinal stress ($\sigma_L$).

In Figure 2 (bottom) circumferential stress and total longitudinal stress are plotted as a function of circumferential strain. Stresses in each direction are higher after treatment with norepinephrine than after poisoning with KCN. The slopes of the stress-strain curves are an index, although not precise, of elastic moduli. It may be observed that only the circumferential stress curve exhibits a distinct increase in slope after norepinephrine; the longitudinal stress curve exhibits a slight change in slope, and then only at very large strains. There is an elevation in longitudinal stress after norepinephrine, but this is not a direct effect of the muscle upon longitudinal elastic properties. It occurs as an indirect consequence of the increase in circumferential elastic modulus by the activated muscle. This increase in circumferential stiffness means that a higher pressure must be applied to distend the vessel to any given radius after norepinephrine than after poisoning with KCN. This increases the value of $P_T$ in equation 11 and thus elevates the longitudinal stress at each circumferential strain.

Solution of equations 5 and 6 provided values for Poisson's ratio and for the circumferential elastic modulus. Poisson's ratio was calculated by employing equation 6. Since the vessel segments were held at constant length, the change in longitudinal strain ($\Delta \varepsilon_L$) was effectively zero. Equation 6 was then rearranged for Poisson's ratio,

$$v_{CL} = \frac{\Delta \sigma_L}{\Delta \sigma_C},$$

where $v_{CL}$ is Poisson's ratio, $\Delta \sigma_L$ is the increment in longitudinal stress, and $\Delta \sigma_C$ is the increment in circumferential stress. The average circumferential strain exhibited by the vessels at each pressure was determined and Poisson's ratio was then plotted as a function of this strain. Figure 3 (bottom) shows these data. The closed circles represent the means for 16 carotid artery segments after treatment with norepinephrine; the open circles represent the means for the same 16 vessel segments after poisoning with KCN. The bars represent the standard errors of the means. Values for Poisson's ratio after norepinephrine and after KCN were compared statistically with the Wilcoxon test. The arrows at the top of the graph indicate the pairs of data points compared and the levels of statistical significance found. Solid arrows indicate differences that are significant at the $P < .025$ level; open arrows indicate differences that are not significant. It is clear that Poisson's ratio varies between .25 and .45, and that activation of the vascular muscle gives somewhat lower values than when the muscle had been poisoned. This is most evident at small and moderate strains; at large strains the values for the contracted and poisoned vessels tend to converge. The tendency for Poisson's ratio to be of lowest value at moderate strains may be explained by considering the ratio of the change in longitudinal stress to the change in circumferential stress, as used in equation 13.

This may be assessed graphically by considering the experiment shown in figure 2 (top). At both small and large strains, the slope of the longitudinal stress curve is somewhat less than the slope of the circumferential stress curve, while at moderate strains, the slope of the longitudinal stress curve is markedly less than the slope of the circumferential stress curve. Thus, Poisson's ratio tends to be maximal at both small and very large strains, and minimal at moderate strains. Since activation of the muscle differentially increases the slope of only the circumferential stress curve at moderate strains (Fig. 2, bottom), this effect tends to further lower the value of Poisson's ratio. Figure 3 (bottom) presents the values for Poisson's ratio, plotted as a function of transmural pressure. Each point is plotted at the mid-point between the pressure steps used in the experiment. The solid arrows indicate statistically significant differences between the norepinephrine and KCN data at the $P < .025$ level; open arrows indicate differences that were not statistically significant. Activation of
Incremental circumferential elastic modulus. Closed circles indicate data after norepinephrine, open circles indicate data after poisoning with KCN. Closed arrows indicate statistically significant differences (P < .025) between norepinephrine and KCN data; open arrows indicate no statistical difference. Broken horizontal line indicates value for longitudinal modulus. Top: Circumferential elastic modulus plotted as a function of strain. Modulus is higher after norepinephrine than after KCN. Bottom: Circumferential elastic modulus plotted as a function of transmural pressure. Circumferential modulus is greater than longitudinal at physiological pressures.

The muscle with norepinephrine tends to lower Poisson's ratio as compared with that after poisoning with KCN. This occurs most clearly at high pressures. However, at physiological pressures (75 to 150 mm Hg) the values for Poisson's ratio are between 0.25 and 0.35 and...
The values obtained for Poisson's ratio were substituted into equation 5 and this was arranged to solve for circumferential modulus \( E_c \).

\[
E_c = \frac{\Delta \sigma_c}{\Delta \varepsilon_c + \nu_{NL} \Delta \sigma_L - \Delta \sigma_L}, \tag{14}
\]

where \( E_c \) is the incremental elastic modulus, \( \Delta \sigma_c \) is the change in circumferential stress, \( \Delta \varepsilon_c \) is the change in circumferential strain, \( \nu_{NL} \) is Poisson's ratio, \( \Delta \sigma_L \) is the change in longitudinal stress, and \( E_L \) is the longitudinal elastic modulus. Figure 4 (top) presents the incremental circumferential modulus, plotted as a function of circumferential strain. The closed arrows indicate statistically significant differences \((P < .025)\) between the norepinephrine-treated vessels (closed circles) and the KCN-poisoned vessels (open circles). These data indicate that the elastic modulus tends to increase gradually with increasing strain, and that activation of the muscle with norepinephrine increases the elastic modulus of the wall above that seen after poisoning with KCN. Therefore, the longitudinal and circumferential moduli are equal only at, or between, these strains, depending upon the degree of muscular activity. Recent reports \((19)\) indicate that the static radial elastic modulus in prestrained vessels is about \(4.2 \times 10^8\) dyn/cm\(^2\), so it is possible that the wall might be isotropic at these strains. However, inspection of Figure 4 (bottom) reveals that the circumferential and longitudinal moduli are equal only at pressures between 60 and 75 mm Hg. At 120 mm Hg the circumferential modulus is almost twice the longitudinal modulus, indicating that the artery at in-situ length is not isotropic in the physiological pressure range.

In view of these findings regarding anisotropy and the value of Poisson's ratio, it was of interest to determine the magnitude of the error incurred by assuming that the vessel wall is isotropic and exhibits a Poisson's ratio of 0.5. To evaluate this error the present circumferential stress-strain data were used to calculate elastic moduli according to equation 4, which is based upon the assumption of isotropy. A ratio of the circumferential elastic modulus, as calculated by assuming isotropy with equation 4, to the true circumferential elastic modulus, as calculated by solution of equation 14, was then computed. This ratio is presented in Figure 5 (top), plotted as a function of strain. It is apparent that at small strains the calculation assuming isotropy tends to underestimate the true circumferential elastic modulus by about 20%. At moderate strains the ratio is approximately one, indicating that the isotropic calculation provides a good estimate of the true circumferential modulus. At large strains the calculation assuming isotropy
Ratio of circumferential modulus, as calculated by isotropic calculation ($E_{ISO\text{CALC.}}$), to circumferential modulus, as calculated by anisotropic calculation ($E_{ANISO\text{CALC.}}$). Closed circles indicate data after norepinephrine, open circles indicate data after KCN, closed arrows indicate statistical significance ($P < .025$) between norepinephrine and KCN data, open circles indicate no statistical significance. Broken horizontal line denotes equality between isotropic and anisotropic calculation. **Top:** Data plotted as a function of strain. Note the absence of muscle influence. Calculation assuming isotropy underestimates the true modulus at small strains, and overestimates the true modulus at large strains. **Bottom:** Data plotted as a function of transmural pressure.

greatly overestimates the true modulus. The consistent open arrows indicate that the differences in the data caused by treatment with norepinephrine and poisoning with KCN were not statistically significant, as tested by the Wilcoxon test. It thus appears that the error incurred by assuming isotropy is essentially a function of strain and that this error is minimally influenced by the presence or absence of active muscle. Figure 5 (bottom) presents the same ratio values plotted as a function of pressure. At low pressures the calculation assuming isotropy tends to underestimate the true modulus, while at high pressures the calculation assuming isotropy tends to overestimate the true modulus.
physiological pressures (75 to 120 mm Hg) the isotropic calculation provides a good estimate of the true modulus for both the contracted and poisoned vessels. At higher pressures, poisoning the muscle with KCN results in a statistically greater overestimation \((P < .025)\) of the true circumferential elastic modulus than when the vessel had been contracted with norepinephrine. This occurred because the KCN-poisoned vessels were dilated to larger radii than the norepinephrine-treated vessels and, as illustrated in Figure 5 (top), the error in estimation depends upon the strain. Thus, the differences between the data after norepinephrine and after KCN shown in Figure 5 (bottom) resulted largely from changes in geometry brought about by the active muscle rather than from the direct contribution of the muscle to the elastic characteristics of the wall.

Discussion

The present data indicate that the Poisson's ratio \((\nu_{CL})\) of the wall is about 0.3 in fixed-length arteries. This agrees with the values reported by Fenn (9) and by Lambossy (10) for various retracted arteries. Since the arterial wall is incompressible (3, 5, 6) the mean of all the Poisson's ratios must be 0.5 (15), but the individual Poisson's ratio may be more or less than 0.5. For a tube, where the ratio of radius to wall thickness is 10:1, the radial stress is only 5% of the circumferential stress and 10% of the longitudinal stress. Therefore it is possible to neglect the radial stress and, with it, the Poisson's ratios involving the radial direction. The significance of the Poisson's ratio that is reported here \((\nu_{CL})\) is that it is between the circumferential and longitudinal directions and therefore enters into the calculations of circumferential elastic modulus, as shown by equations 5 and 6.

Poisson's ratio is also related to changes in vessel length, as shown by equation 6. In an unstretched, pressurized artery the change in circumferential stress \((\Delta\sigma_c)\) is twice the change in longitudinal stress \((\Delta\sigma_L)\). Since Poisson's ratio is less than half, the product of Poisson's ratio times the change in circumferential stress \((\nu_{CL} \Delta\sigma_c)\) is less than the change in longitudinal stress \((\Delta\sigma_L)\), and therefore the change in longitudinal strain \((\Delta\varepsilon_L)\) must be positive, i.e., the vessel must lengthen. This agrees with reports that retracted vessels actually do elongate when pressurized (9, 10).

In a stretched artery the longitudinal stress results from pressure and also from longitudinal traction. Figure 2 (top) shows that with increasing circumferential strain, the change in longitudinal stress due to pressure \((\sigma_{Lp})\) is slightly greater than the decrease in the longitudinal stress due to traction \((\sigma_{Lt})\). The result is a gradual increase in the net longitudinal stress \((\sigma_L)\). Since in the present experiments the length was held constant, the change in longitudinal strain was zero, i.e., \((\Delta\varepsilon_L = 0)\). Equation 6 shows that under these conditions the increase in longitudinal stress \((\Delta\sigma_L)\) must be exactly equal to the product of Poisson's ratio times the change in circumferential stress \((\nu_{CL} \Delta\sigma_c)\). Poisson's ratio is about 0.3 in both fixed length (Fig. 3) and retracted arteries (9, 10), and this value probably applies to arteries in vivo as well. If it does, then the interaction between pressure and longitudinal traction observed in the present experiments may also underlie the relative constancy of vessel length in vivo. Equation 6 also shows that the change in longitudinal strain \((\Delta\varepsilon_L)\) is inversely related to the longitudinal elastic modulus \((E_L)\). This variable is very low in unstretched arteries, and should permit large changes in length to occur. In stretched arteries the modulus is elevated (12) so that any change in length that occurs should be relatively small.

The present results confirm previous reports that the arterial wall is anisotropic (9-14) and show that at in-situ length and at physiological pressures the elastic modulus of the carotid artery is greater in the circumferential direction than in the longitudinal direction. Anliker et al. (13) analyzed the propagation velocity of axial and radial waves in the carotid artery of living dogs, and they also concluded that the circumferential elastic modulus is higher than the longitudinal modulus. By contrast, Patel et al. (14) and Apter et al. (21) have shown that
the thoracic aorta exhibits a higher elastic modulus in the longitudinal direction than in the circumferential direction. The difference between the carotid artery and the aorta results from both differences in the properties of these vessels under longitudinal strain (14, 21) and also from the relative influence of the perivascular connective tissue (20). Although the arterial wall is strikingly anisotropic at most strains and pressures (Figs. 4, top and bottom), the error incurred by assuming isotropy is small at physiological pressures (Fig. 5, bottom). However, this does not mean that the wall is truly isotropic, or even that just the circumferential and longitudinal moduli are equal. The relatively small error made by assuming isotropy may be clarified by considering the equations used for the calculation of elastic moduli. Equation 4 presents the determination of circumferential elastic modulus, calculated assuming that the wall is isotropic and exhibits a Poisson's ratio of 0.5. Equation 14 presents the determinations of the true circumferential modulus assuming neither isotropy nor that the Poisson's ratio is 0.5. Inspection of these equations indicates that in both cases computation of circumferential elastic modulus depends largely upon the slope of the circumferential stress-strain curve. The calculation assuming isotropy reduces this slope by multiplication by the constant \((1 - \nu_s)^2\). If Poisson's ratio is 0.5, then \((1 - \nu_s^2)\) is equal to 0.75. By contrast the calculation of the anisotropic modulus reduces the value of the slope by the increasing of the denominator by \(\nu_s\). The value for the longitudinal modulus \((E_L)\) is, on the average, about \(4.10 \times 10^6\) dyn/cm² and has been assumed to be constant for each experiment. The value for Poisson's ratio \((\nu_{CL})\) is about 0.3 and is also relatively constant. Thus, \(\frac{\nu_{CL}\Delta \sigma_c}{E_L}\) is small and has little influence on the calculation of circumferential elastic modulus, except at large strains where the circumferential strain increment \((\Delta \varepsilon_c)\) becomes quite small. The slope of the longitudinal stress curve is variable but does not change greatly until, again, large strains are encountered (Fig. 2, bottom). Thus, at small and moderate strains, the magnitude of the expression \(\frac{\nu_{CL}\Delta \sigma_c}{E_L}\) is negligible and this results in a true circumferential modulus which is about equal to the slope of the stress-strain curve. This is larger than the value of \(75\%\) of the slope of the stress-strain curve, obtained by assuming isotropy. At large strains, the circumferential strain increments decrease and the slope of the longitudinal stress curve gradually becomes steeper (Fig. 2, bottom). This increases the influence of \(\frac{\nu_{CL}\Delta \sigma_c}{E_L}\) in the denominator of equation 14, and thus reduces the value computed for the anisotropic modulus to a level which is smaller than that computed assuming isotropy.

It is likely that the static circumferential elastic moduli previously calculated assuming isotropy for the carotid artery (1, 3) are in error to the extent predicted by Figure 5. It was observed previously (3), as in the present experiments, that active smooth muscle decreases the wall modulus when this variable is plotted as a function of pressure. However, it was observed in the earlier study that the modulus curves after norepinephrine and KCN were widely separated, especially at very high pressures. This was somewhat surprising because the maximum active muscle stress occurred at moderate pressures. However, at high pressures, the KCN-poisoned vessels had dilated to much larger strains than the norepinephrine-treated vessels. The present data indicate that the error incurred by assuming isotropy greatly overestimates the true modulus at large strains (Fig. 5, top). Since the KCN-poisoned vessels were at larger strains than the norepinephrine-treated vessels, it may be anticipated that computation of elastic modulus assuming isotropy would overestimate the modulus of the dilated, KCN-poisoned vessel more than it would the modulus of the contracted, norepinephrine-treated vessel. This prediction is supported by the present finding that when the true, anisotropic elastic moduli are plotted
as a function of pressure, the KCN- and norepinephrine-treated vessels do tend to converge at high pressures (Fig. 4, bottom).

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References

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