The Zero of Potential of the Electric Field Produced by the Heart Beat

The Problem with Reference to Homogeneous Volume Conductors

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The acceptable zeros of the potential of the electrical fields produced by certain dipole distributions in homogeneous volume conductors is discussed. A bridge circuit is described by which a solution of the three-arm and the four-arm central terminals of Wilson may be solved for a zero of potential of the field produced by an arbitrary distribution of dipoles in a homogeneous volume conductor. An acceptable zero of potential for evaluation of the potentials in a locus on the “body” surface distant from the heart is described.

Modern electrocardiographic theory is based upon the laws which define the flow of electric currents in volume conductors. The dipole theory is inherent in these concepts and was developed independently by Wilson and associates and Craib. Conformity to the theory places definite requirements on the zero of potential. It is the purpose of this report to present a brief discussion of these requirements and to offer what we believe to be a satisfactory solution of the problem.

If the extent of the conductor is infinite the potential function for the dipole vanishes at infinity with the inverse square of the distance from the dipole to the point of observation. The potential function for the accession double-layer vanishes at infinity with the solid angle. The potential function for the regression double-layers vanishes at infinity with a large number of solid angles. If the region of observation is near the dipole distribution in a large finite conductor a point on the surface of the conductor as far as possible from the dipole distribution is a satisfactory zero reference point. If the smallest sphere which surrounds the dipole distribution has a radius small in comparison with the distance from the center of the sphere to the point Po of observation, the superpositional effect at Po is the function of an unweighted resultant \( \mathbf{sE} \) (or vector sum) at the center of the sphere. The space curve of the terminus of \( \mathbf{sE} \) is the vectorcardiogram, a vector function of time. The assumption that \( \mathbf{sE} \) exists dates back to a report from Einthoven. Nevertheless, the theorem of superposition requires that the potential function at any point in the conductor exterior to the distribution be at least a weighted resultant at the centroid of the distribution, and in the present discussion we are not concerned with a multipole of this kind. The weighting may require a dipole of variable eccentricity or multipoles of higher order.

In 1934 Wilson and co-workers introduced an averaging network or central terminal for the “zero of potential.” The network assumes a symmetric orientation of \( \mathbf{sE} \) with respect to the electrodes on the branches of the node. In 1942 Bayley felt that the potential \( V_P \) of the central terminal should be carefully evaluated for the bearing this function might have on vectorcardiographic leads. The several attempts are regarded as unsatisfactory for the...
purpose indicated. As a first step it appeared desirable to develop the potential function for the arbitrary dipole. The spherical surface was chosen because the geometry of this surface greatly simplifies the boundary requirement of zero normal derivative at the surface of the conductor. L. J. Chu has informed us personally that the problem can be solved for the oblate and the prolate spheroids and has stated that the problem for the ellipsoid cannot be solved by technics at present available.

A Reference Point on the Generator Circuit

Initial investigations developed a bridge circuit similar to that depicted in figure 1, with two exceptions. The bridge side of $T_2$, connected to the ring $o$, is connected instead to the junction of two RC-impedances which form a parallel branch across the “pick-up” poles of a four-pole electrode. Each of the “pick-up” poles is mounted close to a generating pole ($o'$ and $o''$) shown in figure 1. In 1949 Bayley and Head (unpublished) were able to solve the central terminal for a zero of potential when the bridge was applied to a homogeneous volume conductor. In this circuit the zero potential reference point may be eliminated by utilizing an additional position of the dipole axis. However, when the bridge was applied to a nonhomogeneous conductor the reference point on the generator circuit varied with a change in direction of the field axis. The change in the reference potential was ascribed to a change in the “contact resistances” at the surfaces of the four-pole electrode.

The Integrating Electrode for the Zero of Potential

In order to avoid the difficulty encountered with a reference point on the generator circuit, the theorem of Gauss on the arithmetic mean may be applied to the potential function of the field due to an arbitrary dipole distribution in the homogeneous spherical conductor. Let $V_r$ denote the potential at any point upon the surface $s$ of the spherical conductor and let the function take the form

$$V_r = \frac{1}{4\pi R^2} \sum_{\alpha} (V_{r,\alpha} + V_{r,\beta} + V_{r,\gamma})$$

which, by Gauss’ theorem, gives the relation

$$V_r = \frac{1}{4\pi R^2} \sum_{\alpha} \oint (V_{r,\alpha} + V_{r,\beta} + V_{r,\gamma}) ds$$

Relation 2 states that the average value $\bar{V}_r$ of the potential on $s$ of the arbitrary dipole distribution within $s$ is independent of the position of the dipoles within $s$ and is zero. The two-dimensional equivalent of equation 2 is

$$\bar{V}_r = \frac{1}{2\pi R} \sum_{s'} (V_{r,\alpha} + V_{r,\beta}) ds' = 0$$

wherein $s'$ is now a circle or ring in the plane of which the axis of the dipole (or dipoles) lies. Let the volume conductor be in the form of tap-water which fills a large hemispherical plastic bowl. Let $s'$ be an integrating electrode in the form of a wire circle $o$ of radius $R'$ which encircles the “volume conductor” a short distance beneath the surface of the solution. If now the bridge circuit shown in figure 1 is applied to the conductor, the solution for the zero of potential of the field which is produced by an eccentric dipole is readily obtained. The following experiment will serve to illustrate the details of the method.

Experiment. Let the electrodes $(R)$, $(L)$, $(F)$ in figure 1 be mounted on the rim of the bowl in such a way that the electrode tips reach into...
Fig. 2. In front, from left to right, is (a) the generator component of the bridge, only the lower two panels shown of the relay rack are utilized, the resistance tuned beat frequency oscillator works into the metering panel through an audio power amplifier; (b) the small field axis switching box is on top of the slope-panel box, the latter contains the variable central terminal; (c) the right-hand slope-panel box contains transformers $T_1$ and $T_2$ along with two variable capacitors which may be switched into parallel with any two arms of the central terminal; (d) the frequency tunable detector amplifier rests on top of the Wagner ground device; and (e), a cathode ray oscilloscope serves as a visual detector. The unit located on top of the relay rack is a small cathode ray null detector used in the two-dimensional problem. The spherical integrating electrode $O$ is seen in the background suspended to the I-beam by a one-quarter ton electric hoist. The immersion tank is below and to the right of the sphere. Its catwalk may be seen just below the center of the bridge.

the solution at points inside of the ring at distances $r_1$, $r_2$, $r_3$ from the center. For ease of computation let the electrodes $(R)$, $(L)$ and $(F)$ be located at the apexes of an equilateral triangle the center of which coincides with the geometric center of the integrating electrode $o$ (fig. 1). Let the dipole electrode $(o'-o'')$ be inserted into the solution on the line from the center to the $(F)$ electrode at a distance $f$ from the center. Actually the positioning of the electrodes within $o$ is quite arbitrary and the positions chosen for this experiment make a shorter computation of theoretic values of the electrode potentials. If the generator is connected to the isolation transformer $T_1$, the detector is connected to the isolation transformer $T_2$, and if desired they may be interchanged. The generator circuit was activated by a sine-wave signal in the low audio frequency range ($= 400 - 20$ cycles per second), and 1 milliampere was made to flow through the generator side of $T_1$. A resistance tuned beat-frequency oscillator was used for initiating the signal. A frequency-tunable audio amplifier was connected to the high side of $T_2$ and its output voltage was fed into a high-gain sharply tuned cathode-ray null detector (see legend, fig. 2).

The axis of the dipole was first directed toward the $(R)$ electrode and the appropriate resistance ratios were measured by null detection. The dipole axis was then directed toward the $(F)$ electrode, and the final balance operations were then completed.

Let the resistance ratios $a = r/f$ and $b = l/f$ be defined as those required for $V_{xy} = 0$. The solution for $a$ and $b$ both by theoretic and experimental means follows. The theoretic computations involve a calculation of the potentials $(V_R)_1$, $(V_L)_1$, and $(V_F)_1$ for the first position $0 \rightarrow (x)$ of the dipole axis and a calculation of the potentials $(V_R)_2$, $(V_L)_2$, and $(V_F)_2$ for the second position $0 \rightarrow (y)$ of the dipole axis. In these calculations use is made of the formula for the potential due to the eccentric dipole.

The numerical values for these six potentials are then put into formulae similar to those de-
The results follow: desired ratios $a$ and $b$. The results follow:

$$R' = 9.5\cdot 9^2, \ r = 8.5\cdot 9^2, \text{ and } f = 4.27.$$ Also,

$$0 \rightarrow (x)$$

$$\begin{align*}
(V_{a})_{1} &= -1.44 \quad \lambda = .866 \\
(V_{a})_{1} &= +1.44 \quad \lambda_{1} = 0 \\
(V_{r})_{1} &= 0 \\
(V_{a})_{1} &= 0 \\
(V_{r})_{2} &= -1.41 \quad \mu = -0.5 \\
(V_{a})_{2} &= -1.41 \quad \mu_{1} = 1 \\
(V_{r})_{2} &= +9.17
\end{align*}$$

whence

$$a = r/f = (V_{a})(V_{a})_{2} - (V_{a})(V_{a})_{2} = 1 \\
b = l/f = (V_{a})(V_{a})_{2} - (V_{a})(V_{a})_{2} = 1$$

\begin{align*}
\text{If the resistances in the right arm branch and} \\
\text{and the left leg branch is required by 5 to be} \\
\text{162.5K ohms each, the resistance in} \\
\text{the left leg branch is required by 6 to be} \\
\text{162.5K ohms for } V_{r_{1}} = 0 \text{ for all positions of the field}.
\end{align*}$$

**Bridge Operation for the Two-Dimensional Conductor**

The bridge circuit shown in figure 1 is a double-bridge network and is operated by a double-balance, null detection method. By one method of operation the ratios $a = r/f$ and $b = l/f$ for $V_{r_{1}} = 0$ may be determined quickly without computation. This method will be described first. In figure 1, $T_{1}$ and $T_{2}$ are double-shielded isolation transformers. The generator $G$ and the detector $D$ have been described. In parallel with the bridge side of $T_{1}$ is the Wagner ground device which bring the potential $V_{b}$ of the integrating electrode to that of ground have no theoretic value in making $V_{b} = 0$. These adjustments serve to eliminate errors on the bridge arms due to the capacitance-to-ground of these arms. Also, the capacitance-to-ground of the generator and the detector circuits may introduce error in spite of the isolation transformers $T_{1}$ and $T_{2}$. This source of error is also eliminated by the Wagner ground device. Usually only two or three alternations of the switch are necessary. When equation 7 is satisfied, repeated alternations of the switch show no disturbance in the null balance. Let the first direction of the dipole axis be approximately perpendicular to a line from the center of the conductor to the $(P)$ electrode. Let the resistance $f$ in the $(P)$-arm of the terminal be adjusted to zero value. This procedure brings the potential $V_{r_{1}}$ of the central terminal to the potential $(V_{l})_{1}$ of the $(F)$ electrode. Any error in balance (switch on $T$) is eliminated by gently rotating the axis of the dipole. With the switch on $W$, $r_{p}$ and $C_{w}$ are now adjusted for balance and the switch is returned to $T$. If the balance is disturbed the dipole axis is again rotated gently for balance. Thus the double-balance $V_{r_{1}} = (V_{i})_{1} = V_{i} = V_{w}$ is obtained. The $f$ decade is now returned to its starting value $r_{s} = l_{s} = 50K$ ohms. If this last procedure does not disturb the double-balance the eccentric position of the dipole is symmetric with respect to the $(R)$ and $(L)$ electrodes. Generally, this will not be the case and the balance will go out. With the switch on $T$ the balance is regained by adjusting either the $r$ or the $l$ resistance decades of the terminal. The $f$ decade is again set to zero value. The balance may be disturbed slightly. If so, the direction of the dipole axis needs a very small additional rotation. If necessary, these operations are repeated. The procedure brings $V_{r_{1}} = V_{b} = V_{w}$ by an adjustment of the decade $r$ or $l$ when the potential $(V_{l})_{1}$ is zero, and thus the decade $f$ may be alternated between its resting value $f_{s}$ and zero without disturbing the balance. That adjusting the $f$ decade to zero brings $V_{r_{1}}$ to the potential $(V_{l})_{1}$ may be verified by the relations

$$V_{r_{1}} = \frac{1}{r_{l} + r_{f} + i f} \left[ f(V_{r_{1}}) + r_{f}(V_{l}) + r_{l}(V_{r_{1}}) \right]$$

and

$$\text{Lim } (V_{r_{1}}) = \frac{1}{r_{l} + i f} \left[ r_{l}(V_{r_{1}}) \right] = (V_{l})_{1}.$$
The initial double-balance is thus defined by the relation

$$[V_{T_1} = (V_R)_1 = V_0 = V_w]_1 = 0$$  \hspace{1cm} (8)

wherein the subscript indicates the axis position and the superscript indicates the balance number for the axis position. In this manner the ratio $a/b = r/l$ for $V_{T_1} = 0$ is solved, and may be verified by the relations

$$V_{T_1} = l(V_R)_1 + r(V_L)_1 = 0$$

whence

$$- \frac{(V_R)_1}{(V_L)_1} = \frac{a}{b}$$

The dipole axis is now rotated into the general direction of the $(F)$ electrode and the double-balance

$$[V_{T_1} = V_0 = V_w]_1 = 0$$  \hspace{1cm} (9)

is satisfied by adjusting the $f$ decade (switch on $T$) and the Wagner ground (switch on $W$). During this procedure care is taken not to disturb the setting of the $r$ and $l$ decades for they have been adjusted to the desired values in the first double-balance operation. The terminal $T$ is now solved for a zero of potential, and the dipole axis may be rotated into any desired direction in the plane of the integrating electrode $O$. The balance $V_{T_1} = 0$ will remain undisturbed as long as $V_0$ is kept at the potential $V_w$ of ground by adjusting the Wagner ground. This result is experimental proof, not only that $V_{T_1} = 0$, but that the integration by electrode $O$ is in accordance with relation 3 and is zero. In the present experiment, $a/b = r/l = 1$ and the value of the $f$ decade was 3.2 times that of the $r$ decades; that is,

$$\begin{align*}
r &= 50 \text{ K ohms} \\
l &= 50 \text{ K ohms} \\
f &= 160 \text{ K ohms}
\end{align*}$$  \hspace{1cm} (10)

The chief advantage of the foregoing method is the elimination of computation. A disadvantage lies in the requirement that the first position of the dipole axis be given a direction which makes $(V_R)_1 = 0$. An adjustment of this kind is done quickly in the idealized model but might prove very tedious for an electrode in the heart's substance.

We next suppose that the dipole axis cannot be controlled with sufficient accuracy to make one of the potentials $(V_R)_1$, $(V_L)_1$, or $(V_R)_1$ zero. The method described above must be abandoned. However, the problem can be solved by utilizing two arbitrary positions of the dipole axis, one each for two double-balance operations. Let the axis of the dipole be in the general direction of the $(R)$ electrode an operation indicated by $0 \rightarrow (R)$. The resistance $r$ may be adjusted to satisfy the balance

$$[V_{T_1} = V_0 = V_w]_1 = 0 \quad 0 \rightarrow (R)$$  \hspace{1cm} (11)

while the resistances $l$ and $f$ remain at their starting values $l_s = f_s$. At balance the setting $r_1$ of the $r$ decade is recorded and the ratio

$$R_1^r = r_1/r_s$$  \hspace{1cm} (12)

is computed. The $r$ decade is now returned to its starting value $r_s = r_s$. As a second step the two decades $l$ and $f$ are adjusted singly or in combination to satisfy the balance

$$[V_{T_1} = V_0 = V_w]_1 = 0 \quad 0 \rightarrow (R)$$  \hspace{1cm} (13)

the values $l_1$ and $f_1$ of the $l$ and $f$ decades at balance are recorded and the ratios

$$L_1^l = l_1/l_s, F_1^f = f_1/f_s$$  \hspace{1cm} (14)

are computed. The information obtained in relations 12 and 14 may be put in the form

$$\begin{align*}
(V_R)_1 + a(V_L)_1 + a c (V_R)_1 &= 0 \\
(V_R)_1 + R_1 (V_L)_1 + R_1^a (V_R)_1 &= 0 \quad 0 \rightarrow (R) \\
(V_R)_1 + \frac{1}{L_1} (V_L)_1 + \frac{1}{F_1^a} (V_R)_1 &= 0
\end{align*}$$  \hspace{1cm} (15)

wherein $a = r/l$ and $C = f/l$ are the ratios required to satisfy $V_{T_1} = 0$ for all positions of the dipole axis in the plane defined by the integrating electrode $O$. The subscript denotes the position of the field axis and the superscript denotes the balance number for that position of the field axis.

As a third step the dipole axis is rotated in the direction of the $(F)$ electrode (the electrode $(L)$ might have been chosen) and the $r$, $l$, and $f$ decades are returned to their starting values $r_s = l_s = f_s = 50$ K ohms. With the dipole axis in the $0 \rightarrow (F)$ position, the $f$ decade resistance is adjusted to satisfy the balance

$$[V_{T_1} = V_0 = V_w]_1 = 0 \quad 0 \rightarrow (F)$$  \hspace{1cm} (16)

The value $f_1^o$ of $f$ at balance is recorded and the ratio

$$F_1^o = f_1^o/f_s$$  \hspace{1cm} (17)

is computed. The $f$ decade is then returned to its starting value $f_s$ and the resistances $r$ and $l$ are adjusted for the balance

$$[V_{T_1} = V_0 = V_w]_1 = 0 \quad 0 \rightarrow (F)$$  \hspace{1cm} (18)

At balance the values of $r$ and $l$ are recorded and the ratios

$$R_1^r = r_1^o/r_s, L_1^l = l_1/l_s$$  \hspace{1cm} (19)
are computed. The information obtained in 17 and 19 may be put in the form

\[
C - I(V,\varphi) = 0
\]

Equations 15 and 20 are sufficient for a solution of \(a\) and \(c\) in terms of the ratios \(R, L, F, u, R, L, F, u^2\). The result is

\[
a = \frac{R_1 F_1 (A + 1)(B + 1) - AB}{A + F_2 (B + 1)}
\]

\[
c = \frac{R_1 F_1 (A + 1)(B + 1) - AB}{B + R_1 (A + 1)}
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\]

\[
c = \frac{R_1 F_1 (A + 1)(B + 1) - AB}{B + R_1 (A + 1)}
\]

The present experiment gave the values

\[
R_1 = 2.6 \quad R_2 = 3.2 \quad L_1 = 1 \quad L_2 = .32 \quad F_1 = 3.2 \quad F_2 = 3.2
\]

In computing the result we first note that \(B\) in relation 21 has by virtue of 22 a value \(-1\) so that 21 reduces to

\[
a = \frac{R_1 F_1 (A + 1)(B + 1) - AB}{A + F_2 (B + 1)}
\]

\[
c = \frac{R_1 F_1 (A + 1)(B + 1) - AB}{B + R_1 (A + 1)}
\]

\[
a = \frac{R_1 F_1 (A + 1)(B + 1) - AB}{A + F_2 (B + 1)}
\]

\[
c = \frac{R_1 F_1 (A + 1)(B + 1) - AB}{B + R_1 (A + 1)}
\]

Thus if we take 50K ohms for the final values of \(r\) and \(l\), we must have 100K ohms for the value of the \(f\) decade in order that \(V_{r_2} = 0\) for all positions of the dipole axis in the plane of the integrating electrode \(C\). It is to be noted that this result is identical to that obtained by the (first) direct method.

If we compare the bridge measured resistances with those determined theoretically we have

\[
\begin{align*}
\text{Bridge} & \quad \text{Theoretic} \\
r = 50K\text{ ohms} & \quad r = 50K\text{ ohms} \\
l = 50K\text{ ohms} & \quad l = 50K\text{ ohms} \\
f = 160K\text{ ohms} & \quad f = 162.5K\text{ ohms}
\end{align*}
\]

When it is noted that the decades have a tolerance of 0.1 per cent and that small error of electrode placement or position measurement must occur, the result is quite satisfactory.

The bridge circuit shown in figure 1 is readily adapted for a solution of the problem of bringing the potential of the central terminal to zero for all directions of the field axis. The three electrodes \((R), (L), (F)\) of the three-arm terminal define a plane (the \(RLF\)-plane). If the three-arm terminal is to be at the zero of potential of the field produced by a dipole distribution, the \(RLF\)-plane must bear a certain relationship to the "mid-point" of the resultant dipole. If the arbitrary dipole is in a homogeneous spherical conductor this relationship is defined by a cubic equation. As suggested by Wilson and associates it might be necessary to add a fourth arm to the central terminal which should be connected through a variable resistance, \(h\), to a fourth electrode \((B)\) located on the back. The four electrodes \((R), (L), (F)\) and \((B)\) then define the apexes of a tetrahedron. Adjusting the resistances in the four-arm terminal of potential \(V_{r_2}\) brings the potential to that of any point within the volume of the tetrahedron and possibly to the zero of potential at the "mid-point" of the resultant dipole provided, of course, that the dipole does not lie exterior to the tetrahedron. This last provision is based on the assumption that the electrode system at the apexes of the tetrahedron is immersed in an "infinite" (or very large) volume conductor. When the surface \(S_2\) which bounds the volume conductor is not at "infinity" and passes through the apexes of the tetrahedron, the configuration of \(S_2\) may be such as to permit a zero of potential at the terminal even though the resultant dipole be located in the nearby region exterior to the plane surfaces defined by the four-electrode system. In brief, the success of bringing \(V_{r_2}\) to the zero of potential of the field produced by the heart beat depends upon the location of the resultant dipole with respect to the volume of the tetrahedron formed by the four-electrode system at its apexes and upon the boundary of the conductor. Let us suppose that the bridge circuit of figure 1 contains a four-arm
Within the tetrahedron is suspended a generating electrode $g$. The latter consists of a plastic cube with six spherical copper poles, one mounted in each face of the cube. Each of the six poles is connected separately to a switching arrangement which permits the bridge generating circuit to apply an electromotive force to six combinations of the electrode poles. The first three switch positions involve the two poles on opposite faces of the cube, and thus each pair may generate a field, the axes of which are mutually perpendicular. If we define these axis directions as parallel to the $X'$, $Y'$, $Z'$ coordinates, then the last three switch combinations rotate the field axis to equal direction angles in the $X'Y'$- and the $X'Z'$-planes and finally into a position of equal direction angles with respect to all three coordinates.

The large spherical electrode $O$ (fig. 2) is immersed completely in a large tank of tap water. The immersion tank is iron-alloy painted with aluminum and mounted on a three-button insulator suspension to prevent forming a "ground loop." The tank is grounded by a cable at a connection common with that of the Wagner ground of the bridge circuit (fig. 1). The large capacitance and low resistance-to-ground of the tank permits the potential $V_o$ of the integrating sphere to be brought to the potential $V_w$ of the Wagner ground by adjustment of the potentiometer $r_w$ only. When this has been done, there is little need for further adjustment of the Wagner ground circuit during the nine balance operations.

**Bridge Balance Procedure for the Three-Dimensional Conductor**

Bridge balance procedure for the present problem is considerably more complicated than that encountered earlier in the two-dimensional problem. In general, a solution of $V_{T_i} = 0$ for all directions of the field axis requires three balance procedures for each of three noncoplanar positions of the field axis. Furthermore, the adjustment of the resistance decades $r$, $l$, $f$ and $h$ of the four-branch terminal which are made during any one of the three positions of the field axis must be such as to bring $V_{T_i}$ into the zero potential surface of the field at three nonlinear points.

At the onset of each balance operation the resistances of the terminal satisfy the relation

$$r_s = l_s = f_s = h_s = 50K$$

(24)
For the first direction of the field axis, the three balances conducted satisfied the relations
\[ \begin{align*}
(V_{T_1} = V_o = V_w)^1 &= 0 \quad r_1^1 = 90K \\
(V_{T_2} = V_o = V_w)^2 &= 0 \quad f_2^2 = 26K \\
(V_{T_3} = V_o = V_w)^3 &= 0 \quad h_3^3 = 15K
\end{align*} \]
wherein the subscript refers to the direction of the field axis and the superscript refers to the particular balance operation conducted on this direction of the field axis. The lower case letters on the right indicate the final values of the \( r, f, \) and \( h \) resistances at balance. When one letter appears for a given balance, only this resistance was adjusted away from its starting value indicated in \( 24. \)

For the second direction of the field axis the three balances conducted satisfied the relations
\[ \begin{align*}
(V_{T_1} = V_o = V_w)^1 &= 0 \quad l_1^1 = 80K \\
(V_{T_2} = V_o = V_w)^2 &= 0 \quad f_2^2 = 30K \\
(V_{T_3} = V_o = V_w)^3 &= 0 \quad h_3^3 = 17K
\end{align*} \]

For the third direction of the field axis the three balances conducted satisfied the relations
\[ \begin{align*}
(V_{T_1} = V_o = V_w)^1 &= h_1^1 = 150K \\
(V_{T_2} = V_o = V_w)^2 &= f_2^2 = 20K \\
(V_{T_3} = V_o = V_w)^3 &= r_3^3 = 10K, f_3^3 = 10K
\end{align*} \]

For the nine balances indicated by \( 25, 26 \) and \( 27 \) there are thirty-six ratios of the form \( r_i/r_s = R_i, \ldots, h_i/h_s = B_i \), wherein any such ratio has a unit value if the resistance was unchanged during a particular balance procedure.

Let the potential ratios of the electrodes \((R), (L), (F)\) and \((B)\) be defined by the relations
\[ \begin{align*}
(V_{L_1}) &= \alpha_1, \quad (V_o) = \beta_1, \\
(V_{L_1}) &= \gamma_1, \quad 0 \rightarrow (1) \\
(V_{L_2}) &= \alpha_2, \quad (V_o) = \beta_2, \\
(V_{L_2}) &= \gamma_2, \quad 0 \rightarrow (2) \\
(V_{L_3}) &= \alpha_3, \quad (V_o) = \beta_3, \\
(V_{L_3}) &= \gamma_3, \quad 0 \rightarrow (3)
\end{align*} \]
wherein the letter subscript indicates the electrode to which the potential refers and the numeral subscript indicates the dipole axis position. If now we define the ratios
\[ b = \frac{l}{r}, \quad c = \frac{f}{r}, \quad d = \frac{h}{r} \]
as those required to satisfy \( V_{T_1} = 0 \) for all directions of the field axis, the information obtained in the nine balance operations is sufficient to compute \( b, c, \) and \( d \) by the equations in \( 29 \)

\[ \begin{align*}
b &= \frac{\alpha_1 \beta_1 \gamma_1}{\alpha_2 \beta_2 \gamma_2} = .55, \\
c &= \frac{\alpha_1 \beta_1 \gamma_1}{\alpha_2 \beta_2 \gamma_2} = .49, \quad (29) \\
d &= \frac{\alpha_1 \beta_1 \gamma_1}{\alpha_2 \beta_2 \gamma_2} = 1.29
\end{align*} \]

and in \( 30 \)

\[ \begin{align*}
1/R_1 &= R_1 \\
1/F_1 &= 1 \\
1/B_1 &= 1
\end{align*} \]

\[ \alpha_1 = \frac{1}{1 + 1/B_1} = .118, \quad (30) \]

\[ \beta_1 = \frac{1}{1 + 1/B_1} = -.83, \quad (30) \]

\[ \gamma_1 = \frac{1}{1 + 1/B_1} = -.191, \quad (30) \]
Inasmuch as the potentials in 28 were not measured directly it is necessary to compute \( \alpha_i, \beta_i, \gamma_i \), \( \alpha_2, \beta_2, \gamma_2 \) etc. before solving for \( b, c, \) and \( d \) by equation 29. For example, the solutions for \( \alpha_i, \beta_i, \) and \( \gamma_i \) are given by the equations in 30 above, as determined from the relations

\[
\begin{align*}
R_0 \alpha_1 + R_1 \beta_1 + R_2 \gamma_1 &= -1 \\
\alpha_1 + \frac{1}{F_1} \beta_1 + \gamma_1 &= -1 \\
\alpha_i + \beta_i + \frac{1}{B_i} \gamma_i &= -1
\end{align*}
\]

and for which there are similar relations for axis positions 0 \( \rightarrow \) (2) and 0 \( \rightarrow \) (3).

The decade \( r \) of the terminal was arbitrarily chosen at a value of 100K ohms and the resistances for the 4-branch terminal of potential \( V_T \) = 0 for all directions of the field axis are, by 29.

\[
r = 100K, \quad l = 85K, \quad f = 49K, \quad h = 129K
\]

which is the result sought.

With the terminal decades set at the values indicated in 32, the dipole axis was rotated through all six directions (only three are required) available on the switch and the detector showed no disturbance of the balance.

\[
(V_{T_1} = V_0 = V_W)^* \quad (33)
\]

This result proves that for the dipole distribution used, the integrating electrode \( O \) of spherical surface \( S_1 \) and its nonhomogeneous content (see figs. 2, 3) serves as a satisfactory zero reference potential with which to solve the four-arm terminal for a zero of potential. The potentials \( V_{T_1} \) and \( V_0 \) will not remain equal for three or more nonco planar positions of the field axis unless both of the potentials \( V_{T_1} \) and \( V_0 \) are zero.

**Discussion and Conclusions**

A bridge circuit is described which solves the zero of potential of the three-arm terminal on a homogeneous conductor of two-dimensions. The basic principle of its operation is described by relation 3. Elaborations of this circuit are described by which the potential of the four-arm terminal when applied to a nearly homogeneous three-dimensional conductor can be brought to the zero of potential of the field produced by a slowly changing dipole distribution. The basic principle upon which the measuring system rests is defined by equation 2. Moreover, data are presented from two experiments which support the theory and its application to the problem in hand to an extent which is highly satisfactory.

**Implications.** Examinations of figures 2 and 3 at once suggest that the diameter of the spherical surface \( S_1 \) in comparison with the limits of the dipole distribution may justly raise the question of disproportional size when theory requires that \( S_1 \) be only large enough to surround the dipole distribution. The choice of a large surface \( S_1 \) has no direct bearing on the problem herein described but was influenced by the following considerations.

Let \( s_2 \) be the surface of a nonhomogeneous volume conductor of average conductivity \( K_2 \). Let the surface \( s_2 \) be of irregular contour and closed. Let the conductor of surface \( s_2 \) be located arbitrarily within the integrating spherical electrode \( s_1 \) and let the conductor between \( s_1 \) and \( s_2 \) be homogeneous. Let \( s_2 \) contain an arbitrary dipole distribution which produces an electrical field throughout the volume contained within \( s_1 \). A Helmholtz double-layer on \( s_2 \) acting alone will produce a field between \( s_1 \) and \( s_2 \) identical to that which is produced by the dipole distribution within \( s_2 \) acting alone. A single layer on \( s_2 \) takes care of the change in conductivity across \( s_2 \). Moreover, if the volume within \( s_1 \) is large in comparison with that contained by \( s_2 \) and if the conductivity \( K_2 \) is much greater than that between \( s_1 \) and \( s_2 \) the electrical effects at \( s_1 \) will be that of contracting the double-layer on \( s_2 \) toward some eccentric point within \( s_1 \). Consequently, \( s_1 \) acting as an integrating surface, may be considered a satisfactory zero of potential with which to evaluate the field within \( s_1 \).

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The Zero of Potential of the Electric Field Produced by the Heart Beat: The Problem with Reference to Homogeneous Volume Conductors

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