Magnetic Meters: Effects of Electrical Resistance in Tissues on Flow Measurements, and an Improved Calibration for Square-Wave Circuits

By Donald J. Ferguson, M.D., and Herbert D. Landahl, Ph.D.

ABSTRACT

Apparent sensitivity of magnetic meters applied to intact vessels can be altered by changes in conductivity of vessel walls or of the contained fluid, and by variable contact of the vessel with surrounding tissue or fluid. An experimental study of these problems was begun by designing a built-in magnetometer for complete electrical calibration of the square-wave meter. It was then found that flow of mercury, saline or blood of various hematocrits through nonconductive tubes gave signals equal to those calculated on the basis of Faraday's principle, provided that the magnetic field was practically uniform across the electrodes. Adequate uniformity was obtained with parallel pole faces at least 1.3 times as wide as the gap. Variable loss of signal (up to 50%) owing to contact of conductive vessels with surrounding tissue or fluid, could be practically eliminated by an electrical shield extending 2.6 times the diameter along the vessel at the electrodes. Specific conductances of canine aortas were from 0.52 to 0.73 times the conductances of blood from the same animals. Alterations in this ratio, readily produced by changes in composition of vessel or fluid, can result in significant changes of flow signal. Indirect flow calibrations require matching of the conductance ratio to that occurring at the time of measurement.

ADDITIONAL KEY WORDS electronics blood flow velocity
electrophysiology Faraday's principle magnetic field
calibration electrical shield

Magnetic meters function reliably when electrodes touch the fluid directly as it flows through a nonconductive tube. When the meter is applied to the outside of a blood vessel, however, several new variables require control. Some of these can be eliminated by direct calibration in situ after the measurement (that is by timed flow through the meter and into a graduate from the opened vessel). Direct calibration is moderately difficult and can be invalidated by changes in blood pressure, hematocrit, or various electrical and mechanical conditions from the time of the original measurement. Direct calibration cannot be used in some experiments or in human beings. Indirect calibrations by perfusion of excised vessels or tubes are generally preferred. But the apparent sensitivity of the meter can be altered by changes in conductivity of vessel walls or of the contained fluid, and by variable contact of the vessel with surrounding tissue or fluid. Atheromatous vessels present additional problems due to possible asymmetrical flow or altered tissue resistance. We have attempted to increase the reliability of the meter for use on blood vessels by an experimental study of these problems, supplementing more theoretical discussions already available (1-4).

The main problem with which this paper is concerned is the effect on flowmeter signals of variations in electrical conductivity of the flowing liquid, the vessel wall, and the surrounding medium. The first step was to devise an accurate calibration procedure for the magnet, amplifier, and recorder, and to
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show that flow signals measured with this apparatus in nonconductive conduits agreed with those calculated using Faraday's law. The second step was to determine the minimum width of the magnet along the axis of the conduit, relative to the gap, to obtain uniformity of field across the gap at the electrodes, and to obtain the full calculated signal. With these variables controlled, the third step was to perform experiments using cellophane (conductive) conduits, to establish the length of electrical shielding along the conduit on either side of the electrodes necessary to maintain the signal unaltered by changes in conductivity of the surrounding medium. Finally, using these results in setting up the experiments, the effects of alterations in the relative conductivity of flowing liquid and vessel wall were demonstrated.

Methods

The meter used has been described (5). It operates with a 240-cycle/sec square-wave magnet current. A new type of electrical calibration for this meter was devised, the circuit for which is given in Figure 1. Recording was done with a Grass direct writer.

The equation for magnetic meters, based on Faraday's law, is \( E = HLV \times 10^{-8} \), where \( E \) is voltage, \( H \) is magnetic field strength in gauss, \( L \) is internal diameter of the conduit in centimeters, \( V \) is average velocity of the moving conductor in centimeters per second, and \( 10^{-8} \) is the factor converting abvolts to volts. Magnetic permeability in air, blood, saline, or tissues need not be considered because it is very close to unity. When the diameter of the lumen is constant, the volume flow (\( F \) in cubic centimeters per second) should be a linear function of \( E \), which is amplified and recorded from electrodes in or on the wall of the conduit.

Magnets of varying width were made with stacks of U-shaped transformer laminations, forming a uniform air gap of either 0.95 or 1.9 cm, with arm lengths 4.5 or 5 cm. All were wound with 80 turns of doubled no. 26 wire and energized with 1 amp of current. Other magnet-electrode probes of various sizes were used, similar to those previously described (5). All magnets were insulated with a layer of epoxy resin.

Field strength was measured by the voltage induced in a coil of wire held perpendicular to the field. Current to the magnet was monitored and kept constant. With each reversal of the square-wave magnet current, a voltage spike was generated in the coil and registered on a Tektronix type 503 oscilloscope with type 122 amplifier; both were calibrated just before each measurement by use of a voltage reference battery. Integration of the area under the spike gave the volt-seconds (2Edt) induced as the magnetic field changed from one direction to the other. One-half this quantity is proportional to the steady part of the field in either direction, i.e., the part from which the flow signal is generated. By Faraday's rule, \( H = \frac{10^{-8}}{NA} E dt \), where \( N \) is the number of turns in the coil and \( A \) is the area within one loop of coil, in square centimeters. A single loop,
2 cm in diameter, was used to determine the field in the center of a magnet with pole faces 4.3 x 5 cm and gap 1.9 cm. A coil with many turns of fine wire about 2 mm in diameter was then in turn calibrated for its NA in the same magnet and used to measure field in smaller magnets. Because the square wave was not perfectly uniform, it was necessary to measure 4 cycles (8 consecutive areas) to get a representative mean. An electronic integration was devised which greatly facilitated these measurements. A 100-kilohm resistor (R) and a 0.1-microfarad capacitor (C), giving a time constant (RC) 0.01 sec, were connected in series with the coil and voltage was measured across the capacitor with the oscilloscope. The nearly square wave displayed was extrapolated to give height in millivolts. Field in gauss equals 500 x mv ÷ NA. Calculations explaining this formula are given in Appendix I. Results with the two methods of integration were practically identical.

The same device was used to provide an electrical calibration signal needed for assuring uniform amplification with each measurement. A single loop of wire of diameter equal to the lumen of the magnet-electrode probe was built into the probe with the loop flat against the center of one pole piece. Leads were independently shielded. Dimensions were such that the loop was within a uniform field extending across the gap. Connections were made as shown in Figure 1, so that the induced square wave could be read on the oscilloscope, while the amplified signal was also recorded, as the magnet current was turned on or off. Comparison of the oscilloscope and recorder tracings gave the total amplification with each measurement. A 100-kilohm resistor was read on an isolation transformer. (In some experiments frequency was varied between 20 and 1000 cycle/sec without affecting the result.) Balance obtained with a variable resistance was read on an oscilloscope. The addition of capacitance had no apparent effect in improving the balance. Resistances of solutions in nonconductive plastic tubes and of blood vessel walls were taken at two lengths and the differences were used in order to minimize the effect of potentials generated at the electrodes. Contacts with vessels were made by noncrushing vascular clamps made of stainless steel. Contacts with solutions were made with stainless steel plugs inserted in the ends of the tube.

Measurements of specific conductance of standard solutions of NaCl were repeated day after day with only minor variation. Blood presented no special problem as long as tubes were kept horizontal to prevent red cells from settling to one end. Blood vessels were gently compressed between parallel blades of calipers extending across the vessel, until the lumen was obliterated. The width and thickness were then measured simultaneously, at both ends and in the middle of the segment. For measurements on human aortas, strips 1-cm wide were cut longitudinally and circumferentially. It was impossible to measure vessel walls accurately because of their compressibility, varying diameter and thickness, branches, and, in human specimens, atheromata. It was also difficult to maintain the same degree of wetness. Specific conductances of vessels could therefore be estimated only approximately.

The importance of grounding to earth in the vicinity of the magnet, which is often essential to reduce noise, has been noted before (1, 5, 7). If grounding is symmetrical and metal contacts are more than a few millimeters from the electrodes, the signal is not appreciably affected. The use of ground connections within the electrode
Results

FLOW IN NONCONDUCTIVE TUBES

The recorded signals from flow of mercury, saline or blood through a nonconductive tube with electrodes touching the fluid match the expected or calculated signals, shown in Table 1, within the errors of measurement. The pole face of the magnet was 2.3 times as wide as the gap, in the directions parallel to the conduit. The blood used was canine, hematocrit 27, or human, diluted with saline to hematocrits of 30, 22, and 16 without appreciable change in response of the meter.

These results are as expected from Faraday's principle, and they confirm the validity of the procedure used to measure amplification by the meter. This type of flow system therefore probably serves as an adequate means of making such measurements.

WIDTH OF MAGNETIC FIELD

Uniformity of magnetic fields, measured with a 2-mm coil in a gap of 0.95 cm, is shown in Table 2 and Figure 2, in relation to width of pole faces. Examples of loss of calculated signal are shown for nonuniform fields in Table 2. The minimum width of pole face along the conduit that gives practical uniformity across the gap at the electrodes is 1.3 times the gap (Fig. 2). The other dimension of the pole face needs to be equally large and symmetrically placed with respect to the axis of the conduit.

Data in Table 3 indicate that additional width of pole faces along the conduit does not influence the loss of signal from flow of 0.9% NaCl. In these experiments the gap between pole faces was 1.9 cm and the conduit was 0.65 cm in outside diameter. These dimensions were chosen to avoid the electrical shielding provided by a pole face which is close to the conduit. The need for a uniform tube of adequate length made cellophane highly preferable to blood vessels for these experiments, aside from the problem of wall conductivity which will be considered subsequently.

ELECTRICAL INSULATION AROUND CONDUCTIVE CONDUIT

Tables 3 and 4 show that the maximum reduction of signal due to immersion in 0.9% NaCl, when there is no insulation around the conduit, is shown in Table 3 and Figure 2, in relation to width of pole faces. The blood used was canine, hematocrit 27, or human, diluted with saline to hematocrits of 30, 22, and 16 without appreciable change in response of the meter.

These results are as expected from Faraday's principle, and they confirm the validity of the procedure used to measure amplification by the meter. This type of flow system therefore probably serves as an adequate means of making such measurements.

<table>
<thead>
<tr>
<th>Width/gap</th>
<th>Fields in gauss</th>
<th>% of calculated signal recorded</th>
</tr>
</thead>
<tbody>
<tr>
<td>.33</td>
<td>.33</td>
<td>82</td>
</tr>
<tr>
<td>.67</td>
<td>.67</td>
<td>94</td>
</tr>
<tr>
<td>1.0</td>
<td>1.0</td>
<td>96</td>
</tr>
<tr>
<td>1.3</td>
<td>1.3</td>
<td>97</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>98</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>98</td>
</tr>
</tbody>
</table>

*Calculations based on means of the two field measurements.

<table>
<thead>
<tr>
<th>Width/gap</th>
<th>% reduction of signal by immersion in 0.9% NaCl</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.53</td>
<td>49</td>
<td>±2.4</td>
</tr>
<tr>
<td>1.0</td>
<td>52</td>
<td>±2.4</td>
</tr>
<tr>
<td>1.0*</td>
<td>54</td>
<td>±1.9</td>
</tr>
<tr>
<td>1.3</td>
<td>51</td>
<td>±1.9</td>
</tr>
<tr>
<td>1.3*</td>
<td>49</td>
<td>±1.9</td>
</tr>
<tr>
<td>2.3</td>
<td>53</td>
<td>±3.3</td>
</tr>
</tbody>
</table>

Flow of saline through cellophane tubing 20 cm long, 0.65 cm in diameter, 0.00875 cm thick. Magnet gaps all 1.9 cm, width of poles varied as listed. Electrodes were unsupported insulated wire.

*Plexiglass block added to each side of pole face to increase the width of the face along the conduit to 2.3 times the gap, in order to detect any electrical shielding effects given by the widest magnet.

SE is standard error of difference between means of six measurements with conduit in air and in saline.

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ELECTRICAL RESISTANCE AND MAGNETIC METER

POLE FACE

<table>
<thead>
<tr>
<th>Length insulated</th>
<th>% expected signal recorded</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>54</td>
<td>±6</td>
</tr>
<tr>
<td>1.1</td>
<td>73</td>
<td>±8</td>
</tr>
<tr>
<td>2.0</td>
<td>89</td>
<td>±7</td>
</tr>
<tr>
<td>2.6</td>
<td>100</td>
<td>±4</td>
</tr>
<tr>
<td>3.0</td>
<td>99</td>
<td>±3</td>
</tr>
<tr>
<td>7.4</td>
<td>99</td>
<td>±5</td>
</tr>
</tbody>
</table>

Insulation symmetrical to electrodes. Pole faces 2.82 diameters apart, 1.3 times gap in width along the conduit.

*External diameter of conductive conduit.
†Expected signal was that recorded with conduit in air.

vessel, is about 50%, compared to the signal obtained with the vessel in air. The experiments represented in Table 4 were performed with a cellophane tube surrounded by a non-conductive plastic tube through which the electrodes were inserted. Lengths were removed from each end of the nonconductive sheath until the flow signal began to be reduced during immersion. This occurred at a length less than approximately 2.6 times the outer diameter of the conductive tube. Variability in results of this experiment was increased by the difficulty of keeping the same concentration of saline at the electrodes in spite of repeated drying and reimmersion. Observations with magnet-electrode probes embedded in plastic confirmed the fact that complete circumferential shielding extending 1.3 diameters on each side of the electrodes will prevent a change in signal during immersion, and that appreciably less insulation will not prevent the change.

**Conductivity of the Wall versus that of the Fluid Within**

In the experiments with cellophane tubes...
perfused with saline, it has been assumed that
the conductivity of the wall is the same as
that of saline. This assumption is in agree-
ment with results given in the first two classes
in Table 7. The surface was not allowed to
dry because this increased the salt concen-
tration and caused a reduction of signal for a
given flow rate. Numerous attempts to obtain
consistent indirect calibrations with the use
of excised canine or human blood vessels
failed until it was recognized that conduc-
tivity of the vessel wall varied appreciably de-
pending on how the vessel was handled.

The flow data given in Table 5 were ob-
tained from living animals. The results clear-
ly indicate that higher signals were obtained
from saline than from blood. It has already
been shown (Table 1) that this difference
does not occur in a nonconductive tube.
Therefore it must be related to differences in
the relative conductivity of the fluid and the
vessel wall. If it is assumed that conductivity
of a living artery is not appreciably affected
by a brief perfusion with saline, then only the
conductivity of the perfusing fluid has varied.

The change in signal is due to the fact that
less of the induced current is shorted through
the vessel wall when the wall has a higher
resistance relative to that of the fluid. Con-
versely, a wall with lower resistance or a fluid
with higher resistance reduces the signal. The
value of L in the flowmeter equation is equal
to the inner diameter of a nonconductive con-
duit. L is equal to the outer diameter of a
conductive conduit only when the conduit
has the same conductivity as the fluid. (See
Appendix II).

Variations in mean intraluminal pressure be-
tween 20 and 200 mm Hg did not alter sig-
nals from perfused vessels, as long as they
were distended sufficiently to make contact
with the electrodes.

Specific conductance of aortas taken from
3 normal dogs (killed in the course of other
experiments) and measured immediately af-
ter excision ranged from 0.00319 to 0.00483
(ohm-cm)$^{-1}$ at 37°C. These values are
from 0.52 to 0.73 times the specific conduc-
tances of blood samples from the same ani-
mals (Table 6). Although the measurements

**TABLE 5**

<table>
<thead>
<tr>
<th>Dog no., Vessel</th>
<th>Perfusate</th>
<th>Recorded at 1 ml/sec</th>
<th>% diff</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, Aorta</td>
<td>Blood</td>
<td>1.36 ± 0.13</td>
<td>+ 22</td>
<td>.05</td>
</tr>
<tr>
<td></td>
<td>Saline</td>
<td>1.66 ± 0.13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2, Aorta</td>
<td>Blood</td>
<td>0.966 ± 0.37</td>
<td>+ 41</td>
<td>.001</td>
</tr>
<tr>
<td></td>
<td>Saline</td>
<td>1.36 ± 0.089</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3, Femoral</td>
<td>Blood</td>
<td>0.666 ± 0.018</td>
<td>+ 13</td>
<td>.01</td>
</tr>
<tr>
<td>artery</td>
<td>Saline</td>
<td>0.751 ± 0.039</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Different magnet-electrode assemblies were used for the three experiments because of
difference in size of vessels.

**TABLE 6**

<table>
<thead>
<tr>
<th>Fluid or tissue</th>
<th>Temp (°C)</th>
<th>Condition, treatment</th>
<th>Specific conductance (ohm-cm)$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plasma</td>
<td>26 ± 1</td>
<td></td>
<td>0.0128</td>
</tr>
<tr>
<td></td>
<td>37 ± 1</td>
<td></td>
<td>0.0156</td>
</tr>
<tr>
<td>Blood</td>
<td>27 ± 1</td>
<td>Hematocrits</td>
<td>0.0080</td>
</tr>
<tr>
<td></td>
<td>37 ± 1</td>
<td>36, 43, 42</td>
<td>0.0070</td>
</tr>
<tr>
<td>Aorta</td>
<td>28 ± 1</td>
<td>Fresh</td>
<td>0.0040</td>
</tr>
<tr>
<td></td>
<td>37 ± 1</td>
<td>Fresh</td>
<td>0.0047</td>
</tr>
<tr>
<td>Aorta</td>
<td>25 ± 1</td>
<td>In saline 5 days</td>
<td>0.0074</td>
</tr>
<tr>
<td>Vena cava</td>
<td>27 ± 1</td>
<td>Fresh</td>
<td>0.0074</td>
</tr>
<tr>
<td></td>
<td>37 ± 1</td>
<td>Fresh</td>
<td>0.0072</td>
</tr>
<tr>
<td>Vena cava</td>
<td>25 ± 1</td>
<td>In saline 5 days</td>
<td>0.015</td>
</tr>
</tbody>
</table>

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on vessels involve a number of sources of error, it is interesting to note that the $Q_{10}$ values are moderately stable. For saline, plasma, blood, aorta, and vena cava the average $Q_{10}$ is respectively 1.24, 1.22, 1.21, 1.19, and 1.26. No appreciable difference could be detected by taking measurements of aortas transversely (along the circumference) as compared to longitudinally. Errors in measuring the exact cross section were of the order of $\pm 10\%$ in fresh vessels and became larger in those stored for a day or longer due to swelling. Storage of vessels (with an antibiotic) in 0.9% NaCl (specific conductance 0.0152 at 25°, 0.0196 at 37° C) was followed by an increase in conductance, which approached 0.5 to 1.0 times that of saline after several days and then remained stable for several months. Ratios of specific conductances of aortas that were kept several days in saline to that of saline are similar in magnitude to the ratios of conductances of fresh aortas to that of blood. This fact explains the fair agreement of in vivo and in vitro calibrations using these systems. The difference in conductivity after soaking a fresh aorta overnight in saline, however, was sufficient to cause an apparent decrease of about 15% in the flow signal developed from saline flow through the vessel. A further decrease, up to 50%, followed immersion in saline for 64 hours. Still further reduction of about 50% of the remaining signal was found after soaking vessels overnight in a 20% salt solution. Conductance of the thin-walled vena cava was close to that of the surrounding fluid.

Specific conductances of 6 freshly excised human aortas obtained at autopsy ranged between 0.00267 and 0.00513 (ohm-cm)$^{-1}$ at 25 ± 1° C. There was no apparent correlation with the degree of atherosclerosis, which varied between negligible and severe in the specimens examined. Measurements in longitudinal and circumferential directions were similar.

By controlling conductivity of the vessel wall with the method of concentric tubes illustrated in Table 7, a close agreement between predicted and measured results was obtained. The first two situations diagrammed in the table, with the conduit in air, gave the signals expected from Faraday’s principle. In the third case, results are analogous to those previously found by immersing the cellophane tube in saline (Table 3). The fourth, fifth and sixth cases are those in which specific conductance of the perfusing fluid is the same as, less than, or more than that of the wall, respectively. Results are as predicted, within errors of measurement. There was some difficulty in keeping the tubes exactly concentric, and in maintaining the inner position of the electrodes, which were introduced from the end of the cellophane tube. Calculations necessary for the prediction are given in Appendix II.

### Table 7

<table>
<thead>
<tr>
<th>DIAGRAMS</th>
<th>Expected mv at 1 ml/sec</th>
<th>Recorded mv at 1 ml/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.785</td>
<td>0.767 ± 0.032</td>
</tr>
<tr>
<td>9</td>
<td>0.269</td>
<td>0.265 ± 0.019</td>
</tr>
<tr>
<td>5 9</td>
<td>0.430</td>
<td>0.362 ± 0.075</td>
</tr>
<tr>
<td>5 9</td>
<td>0.269</td>
<td>0.225 ± 0.021</td>
</tr>
<tr>
<td>5 9</td>
<td>0.187</td>
<td>0.177 ± 0.013</td>
</tr>
<tr>
<td>5 9</td>
<td>0.344</td>
<td>0.313 ± 0.016</td>
</tr>
</tbody>
</table>

Flow measurements in concentric cellophane tubes, 1.86 cm and 0.65 cm in diameter. Thickness of cellophane: larger tube 0.003 cm, smaller tube 0.005 cm. Magnet pole face 4.3 x 5 cm, gap 1.9 cm. Diagrams show cross section of tubes with position of electrodes and concentration of NaCl in %. Tubes not immersed:

IRREGULAR LUMEN

Atheromatous vessels may show little change in specific conductance, as noted above. Such vessels are, however, likely to cause flow patterns which are asymmetrical
to the axis of the vessel. Experiments illustrated in Figure 3 indicate the magnitude of change in signal that could occur in extreme cases. The differences would be larger if the field were not uniform, and if fluid and wall conductivities were not the same. In the group illustrated on the lower level, the blood vessel had been soaked in 0.9% NaCl for several weeks. Although its specific conductance was not measured, it was probably not far from that of saline. Results in these experiments are close to what would be predicted from the calculations given by Shercliff (ref. 3, p. 29, Fig. 13).

Discussion
A reasonable uniformity of magnetic field can be achieved with pole faces only 1.3 times as wide as the gap. Wyatt (7) had earlier reached a similar conclusion, and Shercliff (3) stated that a pole face 1.5 times the diameter adequately reduced the uncertainty caused by “end shorting.” The data given here would not necessarily apply to magnets with other types of core or to those without a core. A uniform field allows for use of the electrical calibration system we have described and permits calculation of signal amplitude. Uniformity of field also should reduce some types of error in indirect calibration, in which velocity profiles may be assymetrical in one of the systems or thick walls would confine flow to a weaker part of the field in the center of the gap.

Electrical shielding around vessels could be dispensed with if all measurements were made with vessels suspended in air. This is impractical under most conditions. Shielding might also be superfluous if all measurements could be made with the same degree of submersion in fluid of the same conductivity. This is also impractical. Therefore adequate electrical shielding, extending 2.6 times the diameter along the vessel and symmetrical to the electrodes, is desirable.

The conductivity of vessel walls relative to that of the moving fluid determines the amplitude of signal that will be recorded from a given flow (1, 8, 9). Our measurements suggest that blood vessels exceed normal blood in specific resistance (the reciprocal of specific conductance) by a factor less than 2, contrary to some other observations (9). If an average ratio of conductances is used and wall thickness is measured, a correction factor can be calculated as indicated in Appendix II, to be applied to the calibration obtained with any conductive fluid flowing through a nonconductive tube attached to the same probe. The considerable confusion in the literature as to

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**FIGURE 3**

Diagrams of cross section of the same electrode block with various conduits, all immersed in 0.9% NaCl. Flow is perpendicular to page. Numbers are relative voltage recorded at 1 ml/sec flow rate of saline, ± standard deviation.

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whether flowmeters respond differently to saline and blood can be understood, if it is assumed that variation in specific conductance of vessels or perfusing fluid under the conditions of measurement were present but not taken into account. It is evidently a mistake to calibrate with freshly excised vessels perfused with saline or with vessels stored in saline and perfused with blood, or to allow a stored vessel to be in contact with anything but the same concentration of salt to be used for perfusion. Drying of the vessel results in higher concentration of salt and a lower signal. Hence the vessel should be kept immersed throughout the experiment. It is also apparent that calibrations in vivo are best done with autogenous blood. Since the resistance of blood varies with the hematocrit, changes in the latter may alter calibrations.

The potentially large errors encountered in measuring flow through atheromatous vessels can best be overcome by selecting a relatively undamaged area of vessel on which to apply the meter. Measurements before and after endarterectomy would probably be valid only if the vessel wall at the point of measurement was unchanged by the surgical operation.

Appendix I
Measurement of Field Strength for Square-Wave Magnetic Meter
Calculation of absolute flow rate with a magnetic flowmeter requires measurement of both the induced voltage and the magnetic field strength. In the apparatus used to obtain the flow measurements given above, the alternating magnetic field is nearly constant between 0.16 and 2.08 msec and again between 2.24 and 4.16, at the end of the cycle. The flow-induced voltage has the same form and the apparatus gives a recorded output which is proportional to the absolute value of the voltage during the latter part of each half cycle, i.e., from 1.6 to 2.0 and 3.6 to 4.0 msec. If a coil is placed in the field, the output will be very nearly zero except between 0 and 0.16 and between 2.08 and 2.24. For the method to be given for measuring the magnetic field, we may have an arbitrary period, T, of alternation and we will only require that the voltage be fairly constant over the second half of each half cycle.

Let the output voltage, \( E(t) \), from a coil having \( N \) turns of area, \( A \), be applied through a resistor, \( R \), to an oscilloscope of high impedance (\( \approx 100R \)) across which is a capacitor, \( C \) (see Fig. 1). The values of \( A \), \( R \), and \( C \) are presumed to be accurately measured and the coil placed at right angles to the field where the field is uniform. The instantaneous voltage, \( V(t) \), at the oscilloscope input will be determined from

\[
E = Ri + V = R \frac{dq}{dt} + V, \tag{1}
\]

where \( i \) is the instantaneous current in the coil and \( q \) is the charge in the capacitor.

Since the voltage, \( V \), equals the charge divided by the capacity, \( V = q/C \), the above equation can be written

\[
\frac{dV}{dt} + \frac{V}{RC} = \frac{E}{RC}. \tag{2}
\]

If we multiply both sides of equation 2 by \( e^{t/RC} \) and note that the left hand side equals the derivative of \( Ve^{t/RC} \), we may integrate both sides of the resulting equation and obtain the following result after dividing through by \( e^{t/RC} \):

\[
V = V_o e^{t/RC} \cdot \frac{1}{RC} \int_0^t e^{t'/RC} E(t')dt'. \tag{3}
\]

The quantity \( V_o \) is an integration constant which can be seen to be equal to the value of \( V \) at \( t = 0 \).

Since the induced voltage is

\[
E(t) = 10^{-8} NA \frac{dH}{dt}, \tag{4}
\]
then

\[ V = V_0 e^{-t/RC} + \frac{10^{-8} NA}{RC} e^{-t/RC} \int_0^t e^{-t'/RC} \frac{dH(t')}{dt'} dt'. \] (5)

After a sufficient number of cycles the transient term containing \( V_0 \) will disappear. If also \( RC \) is large enough so that \( e^{-t/RC} \approx 1 \), the above can be written

\[ V(t) = 10^{-8} \frac{NA}{RC} \int_0^t e^{(t'-t)/RC} dH(t') = 10^{-8} \frac{NA}{RC} H(t), \] (6)

and therefore \( H(t) \) can be calculated from \( V(t) \). Since it is assumed that \( H(t) \) has been experimentally shown to be constant over more than half of each half cycle, then the magnitude of the magnetic field strength, \( H_m \), during the constant part of each half cycle can be calculated from the measured change in voltage, \( \Delta V \), from high to low using equation 6 or from

\[ H_m = \frac{\% 10^6}{\%} \frac{RC}{NA} \Delta V. \] (7)

But if \( RC \) is too large, the changes in voltage may become too small to measure. In practice, if \( RC \) has a value such that \( T/RC \leq \% \) and if the magnetic field is constant over most of the cycle, then \( V(t) \) will rise rather abruptly from a negative value to a maximum

\[ V = 2 \times 10^{-8} \frac{NAH_m}{RC - \tau} \left[ \frac{1 + e^{-T/\tau}}{1 + e^{-t/\tau}} e^{-t/RC} - e^{-t/\tau} \right], 0 \leq t \leq \% T, \] (8)

at some time well before \( \% T \), then it will fall slowly, nearly linearly, until \( t = \% T \), at which time it will abruptly drop toward a negative value. Under these conditions a linear interpolation of \( V \) to \( t = 0 \) can be easily made by using a straight edge along the linear portion of the curve to mark the intersection with the vertical axis at \( t = 0 \) and measuring \( \Delta V \) as the interval along the vertical axis from the point at which the voltage abruptly rises. If the trace is sufficiently sharp and there is a minimum of noise, the estimation of \( \Delta V \) under the above restrictions can be made to a few percent. Then from equation 7, \( H_m \) can be calculated.

As an example we may give the solution of equation 5 for the case in which the magnetic field begins abruptly to rise at \( t = 0 \) from a value, \(-H_m\), toward \(+H_m\) with a time constant \( \tau < 0.1 T \), e.g., \( H = H_m - 2H_m e^{-t/\tau} \). The corresponding voltage \( E(t) \) induced in the coil would rise abruptly from zero to some value, \( E_o \), and then decay exponentially with a time constant, \( \tau \). At \( t = \% T \), \( E \) would abruptly go from zero to \(-E_o \) and again decay towards zero. The steady state solution for this situation can be obtained as follows. Substitute the above expression for \( H \) into equation 5 and integrate. Eliminate \( V \) by using the fact that \( V \) at \( t = T \) must be equal to minus the voltage at \( t = 0 \), i.e., \( V(\% T) = -V_m \). In this way we find

\[ V = V_0 e^{-t/RC} + \frac{10^{-8} NA}{RC} e^{-t/RC} \int_0^t e^{-t'/RC} \frac{dH(t')}{dt'} dt'. \]

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[V(T/4) - V(T/2)] - V(0) = 2V(T/4), since V(0) = -V(T/2). This value is just V(T/4) - V(3T/4), the amplitude of the voltage measured from the midpoints of the positive and negative phases. But from equation 8

\[
\Delta V^*/\Delta V = 2V(T/4) = 1 + \frac{T}{RC} - \frac{T}{2R^2C^2} - 2e^{-T/RC} + \ldots
\]  

so that from chosen values of T, RC, and τ, we can estimate the error to obtain a corrected value of ΔV. Using the restriction on T, RC, and τ imposed above, the correction terms in equation 9 are respectively less than 0.031, 0.008, and 0.037. In the numerical example with RC = 2T and τ = T/16 this estimate of ΔV would be about 1% too low, even though there is a 10% decrease between T/4 and T/2. For the same value of RC and τ decreased by 2/3, the estimate would be about 1% too high. For larger values of RC and smaller values of τ the corrections are entirely negligible. Although an estimate of ΔV can be made from V(T/4) quite satisfactorily and the correction terms can generally be ignored, the graphical linear extrapolation was found to be more convenient, especially since τ was only about 0.04 m sec and the maximum occurred quite early in the cycle.

One of the problems in the measurement of flow by electromagnetic induction is the estimation of the amplification in the recording equipment in order to calculate absolute values of the flow-induced voltage. It was found that the output from a loop and circuit, as used above, can provide a known output voltage with the same frequency and essentially the same waveform as is produced during the flow measurements. The voltage during the part of the cycle which is used in the flow measurement is then easily measured. Furthermore, the voltages can be arranged to be of the same order of magnitude as that produced by flow. This enables one to calibrate the output in millivolts or microvolts with the equipment being operated in the same way as when used for flow measurements.

**Appendix II**

**Correction Factor for Effect of Conductivity of Conduit on Flow-Induced Voltage**

Consider a fluid of conductivity, \( \sigma \), to be flowing at a velocity, \( v(r) \), where \( r \) is the distance from the center of a long conduit, in a tube of inner diameter, \( 2a \), and outer diameter, \( 2A \), with \( \rho \) being the conductivity of the material of the tube, the conductivities being assumed to be isotropic. Let there be a magnetic field, \( H \), sufficiently large and uniform, at right angles to the direction of flow. Since an electric field, \( H \times v \) is induced only inside the tube, the intensity being \( H v(r) \), then the potential, \( V \), must satisfy the continuity equation \( \text{div grad } V = \text{div} (H \times v) \) for \( r < a \), while \( \text{div grad } V = 0 \), \( r > a \). If \( \theta \) is the angle from the \( H, v \) plane to a point, \( r, \theta \), the induced field, \( H \times v \), in polar coordinates with unit vectors, \( \hat{r} \) and \( \hat{\theta} \), is \( rH v \sin \theta + \hat{\theta}H v \cos \theta \), so that \( \text{div} (H \times v) = H \sin \theta (dv/dr) \). Assume that the potential, \( V = u \sin \theta \) within the fluid where \( u \) is a function of \( r \) only. Then introducing the expression for \( \text{div } \text{grad } V \) in polar coordinates, we have

\[
\frac{1}{r^2} \frac{\partial}{\partial \theta} (r \sin \theta \frac{\partial u}{\partial r}) + \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{r \partial u}{\partial r} \right) = \frac{d}{dr} \left[ \frac{1}{r} \frac{d(ru)}{dr} \right] \sin \theta = H \sin \theta \frac{dv}{dr}
\]

since \( u \) is assumed to be a function of \( r \) only. Factoring out the factor \( \sin \theta \), the resulting equation can be integrated twice to yield

\[
u = (1/r)H \int_0^r r v(r) dr + c_0/r + c_2/r,
\]

where the integration constant, \( c_2 = 0 \), since \( u(0) \) must not be infinite, having been chosen to be zero. If \( V = U \sin \theta \) within the vessel wall where \( v = 0 \), we obtain

\[
U = \frac{c_0}{r} + c_2/r = \frac{1}{2} U(A) \times (r/A + A/r),
\]

if \( U_0 = U(A) \), since \( U'(A) = 0 \), i.e., no current flows out from the outer surface of the vessel. Since the potential is continuous at \( r \), while the
electric fields are in proportion to the conductivities, \( u(a) = U(a) \) and \( u'(a) = \rho U'(a) \), the integration constants, \( c_1 \) and \( U(A) \), can be found by solving the two resulting equations and thus \( u(r) \) and \( U(r) \) can be obtained. These functions multiplied by \( \sin \theta \) give the potentials throughout the region. We need only \( u(a) = U(a) = u_0 \) and \( U(A) = U_0 \), the potentials at \( \theta = 90^\circ \), measured at the inner and outer surfaces respectively, the potentials being measured from the center. Since the integral from zero to \( a \) of \( 2\pi r v \, dr \) equals \( \pi a^3 v \), \( v \) being the average flow velocity, the potentials across the inner and outer diameters are thus found to be:

\[
2u = 2aHv \left( \frac{a^2 + A^2}{A^2 + a^2 + \rho(A^2 - a^2)} \right), \quad (4)
\]

\[
2U_A = -\frac{2aA}{A^2 + a^2} \left( 2u_0 \right), \quad (5)
\]

In making flow measurements, the outer wall can be made to fit snugly into a collar so that the diameter, \( 2A \), can be accurately measured, while the inner diameter is estimated by subtracting the wall thickness which may be more difficult to estimate. The potential difference, \( 2u_0 \), can only be measured by electrodes which penetrate the wall. If we use the measured potential, \( 2U_A \), then the flow rate, \( F \), can be calculated from

\[
F = \left( \frac{\pi a}{2H} \right) 2U_A f, \quad f
\]

\[
= \frac{A^2 + a^2}{2aA} \left( 1 + \rho \frac{A^2 - a^2}{A^2 + a^2} \right), \quad (6)
\]

Note that error in measurement of \( a \) enters directly through the first factor above, as well as into the correction factor, \( f \), even when \( \rho \) is small compared with 1.

The conductivities of aortas at body temperature were found to be 0.52 to 0.73 times that of blood (see Table 6). If the conductivities are equal, then \( 2U_A = 2HF/\pi A \) or \( F = (\pi A/H) U_A \), the quantity in parentheses being a constant for a particular collar electrode. If the conductivities are not equal, then the flow is also given by

\[
F = \left( \frac{\pi A}{2H} \right) 2H_A f^*, \quad f^*
\]

\[
= \frac{1}{2} (1 + \rho) + \frac{1}{2} (1 - \rho) - \frac{a^2}{A^2} . \quad (7)
\]

Note that error in measuring \( a \) has a decreasing effect as \( \rho \), the ratio of the conductivity of the vessel wall to that of blood, approaches 1. Since the outer diameter can be measured more accurately than the inner diameter, there is some advantage in using equation 7 in preference to equation 6, the correction factor, \( f^* \), being estimated from the conductivity ratio and inner diameter.

If one has an unshielded vessel in a magnet with insulated pole pieces and there is a conducting solution surrounding the vessel, there is an electrical shielding effect from the presence of the pole pieces which changes the flow signal. If the vessel wall is very thin, the fluid inside and outside the vessel have the same conductivities, the poles of the magnet are sufficiently wide, and the vessel is centered between the poles, then from the method of images one can calculate the potential difference \( 2u' \). The ratio of the potentials with the conduit immersed to that with the conduit in air \( (2u_0) \) is given by

\[
\frac{2u'}{2u_0} = \frac{1}{2} + \frac{1}{M} \left( \frac{1}{4 + M^2} + \frac{1}{2^2 + M^2} + \frac{1}{3^2 + M^2} + \ldots \right), \quad (8)
\]

where \( M \) is the ratio of the vessel diameter to the gap width, \( M \), being less than or equal to 1. For \( M = 1, 2/3, 1/2, 1/3, \) and 0, one obtains from the above the respective values 0.88, 0.67,
0.60, 0.54 and 0.50. Thus in Table 4, for example, where the ratio is $M = 1/2.82$, one would expect from the above values that the relative signal would be about 0.55, a value in agreement with that given in Table 4. Note that the vessel or conduit must not be too close to the bottom of the tray containing the external fluid.

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