Laminar-Turbulent Transition Process in Pulsatile Flow

By Edward L. Yellin, Ph.D.

ABSTRACT
A controlled ex-vivo study of a simple, sinusoidally oscillating flow in a rigid, constant-area, smooth tube, has produced significant insight into the laminar-turbulent transition phenomenon. The development of turbulence was studied by analyzing the dynamic characteristics of the transition process; i.e., the velocity, growth rate, and intermittency which describe the generation and propagation of turbulent slugs. A new concept, the relaxation time, has been introduced to interpret the effect of a periodic flow component superposed on a mean flow. Classical stability concepts, such as the point of inflection criterion and the Reynolds number, which have been derived from steady-flow analysis, are shown to require modification when applied to an oscillatory flow. Neither the mean nor the instantaneous Reynolds number is a sufficient criterion for determining the transition of laminar to turbulent flow in a pulsatile system. Other necessary criteria are: (1) a source of disturbances, (2) the relaxation time, and (3) the distance from the fluid under observation to the source of disturbance. The concept of relaxation time indicates that slowly oscillating flows of large amplitude tend to suppress or destroy turbulence downstream from sources of disturbance.

Qualitative observations are presented which indicate that systolic acceleration may be laminar regardless of the large value of the instantaneous Reynolds number, while diastolic deceleration probably produces disturbed, but not turbulent or highly dissipative, flow.

ADDITIONAL KEY WORDS
- turbulent flow patterns
- similarity parameters
- hydrodynamic instability
- hydraulic analog
- critical Reynolds number
- dissipation of turbulence

The conditions under which a laminar or turbulent flow can exist in the cardiovascular system have long been of concern to physiologists (1-5). Turbulence influences pressure-flow relations, generation and propagation of disturbances (e.g., audible sounds), local mixing of blood and of infused material with blood, wall drag and vibrations, hemolysis, and thrombosis.

The complexity of the physiological system defies a theoretical analysis of the transitional stages of laminar and turbulent flow, and renders even an experimental approach very difficult. A natural result has been a tendency to interpret physiological observations and to justify conclusions on the basis of commonly accepted theories and observations of engineering fluid dynamics. All too frequently, the engineering results are extrapolated to the physiological system without due regard to the underlying assumptions, either stated or unstated, or to the unique constraints of the engineering system. The problem is difficult also because until recently (6, 7) no experimental or theoretical investigations of transi-
tion in oscillatory tube flow have been done under controlled ex-vivo conditions.

The physiologist is concerned with those flow characteristics which depend not only on the existence of truly laminar or truly turbulent flow, but also on the existence of disturbed flow. We shall adopt a practical approach of discussing the cardiovascular system in terms of three possible flow regimes: laminar flow, disturbed flow, and turbulent flow. The following operational definitions are presented to facilitate description and discussion of the transition from laminar to disturbed to turbulent conditions in pulsatile flow through tubes.

**Definitions**

Laminar flow is, essentially, the sliding of fluid layers over one another with no mixing other than molecular; it is characterized by particle paths which do not cross.

Stable flow, in the classical hydrodynamic sense, is one in which small disturbances are spontaneously attenuated. In steady flow through tubes at sufficiently high flow rates, any source of instability will lead to turbulence. Sources of instability are the production of vortices at surfaces of discontinuity arising from an obstruction or from the mixing of two layers of fluid with differing velocities. Since most experiments, including this one, have been done with constant-area, smooth tubes, in which instabilities arise only in the inlet region, we will define the stable flow as one which when disturbed will not undergo transition to a self-preserving turbulent flow.

Disturbed flow is a transient response of a laminar flow to a source of instability which causes the flow to deviate from its streamlined motion. The energy relations in a disturbed flow regime are such that the disturbance decays as it propagates downstream. Frequently, the portion of the flow which is disturbed is referred to as turbulent, but unless the disturbances are self-preserving we shall not consider a disturbed flow turbulent. In flow through a tube, disturbances are usually either carried into the tube from the entrance region, or arise from a change in shape of the boundary; e.g., a branch, an obstruction, or a diverging container.

Transition is the process by which, in flow through tubes, a state of laminar flow is modified to a state of turbulent flow such that the turbulence is self-preserving.

Turbulent flow is "... an irregular condition of flow in which the various quantities [pressure and velocity] show a random variation with time and space coordinates so that statistically distinct average values can be discerned" (8).

The purpose of this paper is to describe experiments designed to provide some insight into the laminar-turbulent transition phenomena of a simple, sinusoidally oscillating flow in a rigid, constant-area, smooth tube. This approach may appear at first to have only slight relevance to the problems of in-vivo blood flow, but because the simplest system was studied, the problems have been rendered more amenable to analysis and understanding; and the results should provide a meaningful first approximation to the physiological system. Hence, the physiological implications of these results are discussed to reevaluate critically the application of classical concepts from engineering fluid dynamics to the cardiovascular system.

**Methods**

**Flow System**

The flow system is shown in Figure 1. The steady component of flow was supplied to the entrance chamber by a constant-head tank, from which it entered the test section via a well-rounded nozzle. The test section of 1-inch i.d. acrylic tubing was fabricated from four 6-foot lengths, carefully joined to provide a continuously smooth boundary. The periodic component of flow was supplied by a piston actuated by a variable-speed, variable-stroke drive providing a large range of frequencies and flow volumes. The piston was mounted in a Bellofram rolling diaphragm which insured isolation of the fluid, frictionless motion, and a linear volume displacement.

The periodic component of flow was calculated from the pulsator rate and stroke length. The steady flow component was measured by an orifice flowmeter. The combined flow was determined by weighing and timing. (In sinusoidal flow, the temporal average flow rate is equal to the steady flow; therefore, weighing the efflux provided a check on the flowmeter.) Distortion of the sinusoidal output was kept to a minimum (approximately 2%) by maintaining a large pressure.
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Diagram of the flow system. The components are discussed in the text.

Fluid
A dilute aqueous dispersion of bentonite was used as the working fluid. The birefringent properties of the suspension made it particularly valuable in this transition study; no probes were necessary and no extraneous disturbances were introduced into the system. Consequently, not only the generation but also the propagation of turbulence was observed visually at several locations and monitored at two places by photosensitive cells (at A and C, Fig. 1). The output of the photocells was amplified and recorded by a Grass polygraph. The preparation of the fluid has been described elsewhere (9). The composition and physical properties of the dispersion were as follows: concentration, 0.235 by weight; particle size, 0.5 μ or less; specific gravity, 1.0; viscosity relative to water, 1.12. At this low concentration the fluid was Newtonian in character, yet optically active. When viewed between crossed polaroids, the existence of laminar, disturbed, or turbulent flow regimes was readily observed (Fig. 2, top).

Transition characteristics
Transition to turbulence in steady flow is not a sudden phenomenon; there is a wide range of Reynolds numbers in which the flow is intermittently turbulent. Turbulence is generated at a site of instability and propagates downstream. At random intervals of time this production of turbulence ceases, so that a discrete slug or patch of turbulent fluid flows through the tube. A photograph of a typical turbulent slug and a schematic depiction of intermittently turbulent tube flow at three moments of time are presented in Figure 2. By recording the passage of slugs at two tube locations, one is able to compute the velocities of the front and rear of a slug between the two positions, and the time spent in turbulent flow at the two sites. Two pertinent transition characteristics, intermittency and growth rate, may then be defined (Fig. 2, below) and studied.

The existence of turbulence at any location is characterized by an intermittency factor (10), \( \lambda \), which is the percent of time a fluid is turbulent at a particular location, i.e., the relative velocities of the front and rear of a slug. A zero growth suggests stability, but does not exclude the existence of turbulence or disturbances. Stability only implies there...

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\(2\)Bentonite was generously supplied by the National Lead Co., Houston, Texas.

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Transition characteristics of a typical slug. Above: Composite photograph of a typical turbulent slug of a Bentonite dispersion in tube flow. a) Front of the slug of turbulence; this is an elongated cusp. b) Body; note its fine structure. c) Rear; the right half is undisturbed laminar flow. Below: Schematic view of the tube at three successive times, t₁, t₂, and t₃. The broken lines represent the trajectories of the rear of a slug, and the solid lines that of the front. At t₁, a turbulent spot has formed (bottom of tube) which erupts rapidly into a slug. Frequently, as shown at t₃, slugs merge with one another. This occurs because the front of a slug has a greater velocity than the rear. An observer sufficiently far downstream will not see any slugs, only fully turbulent flow. χ = distance along the tube.

will be no growth of disturbances into self-maintaining turbulence.

THEORY

The solution of the equations of motion has been presented by Womersley (12). In order to generalize results from one physical system to another, a nondimensional form of the equation of motion is developed as follows:¹

Under the conditions of this experiment the Navier-Stokes equations reduce to:

\[
\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right),
\]

which is made dimensionless by substitution of the following defined quantities:

\[ u' = \frac{u}{u}, \quad \tau' = \omega t, \quad \nu' = \frac{\nu}{\rho u^2/2}, \quad r' = \frac{r}{a}, \quad x' = \frac{x}{a} \]

which, when substituted into equation 1 yields,

\[
\frac{\partial u'}{\partial \tau'} = -\frac{R}{4\alpha^2} \frac{\partial^2 p'}{\partial x'^2} + \frac{1}{\alpha^2} \left( \frac{\partial^2 u'}{\partial r'^2} + \frac{1}{r'} \frac{\partial u'}{\partial r'} \right). \quad (2)
\]

If we now define \( u = U + \bar{U} \) in order to reflect the effects of the steady and oscillating flow, and since:

\[
\frac{D}{D t^*} = \frac{2\alpha u}{\nu}, \quad \bar{R} = \frac{2\bar{U}}{u},
\]

then,

\[ R = \bar{R} (1 + \bar{Q}/\bar{Q}) \]

and finally,

\[
\frac{\partial u'}{\partial \tau'} = -\frac{\bar{R} (1 + \bar{Q}/\bar{Q})}{4\alpha^2} \frac{\partial^2 p'}{\partial x'^2} + \frac{1}{\alpha^2} \left( \frac{\partial^2 u'}{\partial r'^2} + \frac{1}{r'} \frac{\partial u'}{\partial r'} \right). \quad (3)
\]
TURBULENCE IN PULSATILE FLOW

TABLE 1

Symbols and Definitions

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>radius of the circular tube</td>
</tr>
<tr>
<td>G</td>
<td>growth rate of a turbulent slug</td>
</tr>
<tr>
<td>R</td>
<td>instantaneous Reynolds number; ( = 2au/\nu )</td>
</tr>
<tr>
<td>RI</td>
<td>mean flow Reynolds number; ( = 2aU/\nu )</td>
</tr>
<tr>
<td>RC</td>
<td>critical Reynolds number</td>
</tr>
<tr>
<td>R</td>
<td>peak Reynolds number; ( = RI(1 + Q/Q) )</td>
</tr>
<tr>
<td>Rl</td>
<td>lowest Reynolds number during a cycle ( = RI - 0/0 )</td>
</tr>
<tr>
<td>Q</td>
<td>instantaneous volume flow</td>
</tr>
<tr>
<td>Q</td>
<td>steady component of volume flow</td>
</tr>
<tr>
<td>Q</td>
<td>amplitude of the periodic component of volume flow</td>
</tr>
<tr>
<td>t</td>
<td>time coordinate</td>
</tr>
<tr>
<td>tu</td>
<td>time required for a turbulent slug to pass a given location in the tube</td>
</tr>
<tr>
<td>T</td>
<td>total time during which slugs are observed at a given location</td>
</tr>
<tr>
<td>u</td>
<td>instantaneous spatial average fluid velocity</td>
</tr>
<tr>
<td>U</td>
<td>spatial and temporal average fluid velocity</td>
</tr>
<tr>
<td>v</td>
<td>instantaneous velocity</td>
</tr>
<tr>
<td>Vf</td>
<td>velocity of the front of a slug</td>
</tr>
<tr>
<td>VR</td>
<td>velocity of the rear of a slug</td>
</tr>
<tr>
<td>x</td>
<td>space coordinate in the axial direction</td>
</tr>
<tr>
<td>a</td>
<td>frequency parameter ( = a \sqrt{\omega/\nu} )</td>
</tr>
<tr>
<td>L</td>
<td>fraction of the total time that slugs pass a given location, i</td>
</tr>
<tr>
<td>nu</td>
<td>kinematic viscosity of the fluid</td>
</tr>
<tr>
<td>tau</td>
<td>real time spent by the fluid below the critical Reynolds number</td>
</tr>
<tr>
<td>om</td>
<td>circular frequency in radians/sec ( = 2\pi f ), where f is the number of cycle/sec.</td>
</tr>
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</table>

Equation 3 is the dimensionless form of the fundamental equation which most accurately reflects the existence of the steady and periodic flow components. Three parameters are necessary for dynamic similarity between flow systems.

1. \( \tilde{R} = \frac{2au}{\nu} \), the mean Reynolds number, which is the usual criterion for stability in steady flow.

2. \( a = \sqrt{\frac{\omega}{\nu}} \), the frequency parameter, which is a ratio of inertial to viscous forces and hence a Reynolds number for a periodic flow.

3. \( \tilde{Q}/\tilde{Q} \), the flow amplitude ratio, which insures similarity of waveforms.

The gross time variations of flows studied herein are described by the following relations for volume flow and instantaneous Reynolds number (Fig. 3):

\[ Q = \tilde{Q} \left(1 + \frac{\tilde{Q}}{\tilde{Q}} \sin \omega t\right) \]

and

\[ R = \tilde{R} \left(1 + \frac{\tilde{Q}}{\tilde{Q}} \sin \omega t\right) \]

It has also been shown\(^1\) that an additional valuable criterion for the existence of turbulence in a pulsating flow is the nondimensional relaxation time, \( \tau \), which is defined and illustrated in Figure 3. The concept of relaxation time has been formulated to provide an indication of the influence of dissipative forces in flows with large periodic components. It is a close approximation to the crosshatched area of Figure 3, and takes into consideration both the time spent in a stable flow, and the depth of penetration into the stable regime.

The use of relaxation time, \( \tau \), may be further justified as follows: flow disturbances and transient turbulence are attenuated by the dissipative action of viscous forces. The effectiveness of these forces is directly proportional to the fluid viscosity, \( \nu \), the ratio of inertial to viscous forces, \( R \), and the time available for dissipation, \( \theta \). The viscous forces dominate the inertial forces only when the flow is below the critical Reynolds number (Fig. 3), therefore, \( \tau \propto \theta \nu (R^c - R)/2 \). The effectiveness of the dissipative forces is also inversely proportional to the scale of turbulence, (small vortices dissipate more rapidly than larger) and small radius tubes contain more small vortices. Hence, \( \tau \alpha \frac{1}{a} \). The nondimensional form of the relaxation time may now be presented as:

\[ \tau = \frac{\theta \nu}{2a^2} (R^c - R) \]

The value of this approach will be demonstrated and discussed below.

Results

Because each flow system has unique transition characteristics, it was necessary first to undertake a thorough investigation of steady flow and then to observe the effects of the periodic flow component on the steady flow transition characteristics. The mean flow Reynolds number, \( \tilde{R} \), which is common to both steady and periodic flows, was used as the primary independent variable. The frequency parameter and the flow-amplitude ratio then became the parameters representing the influence of the periodic flow component on the generation and propagation of turbulent slugs.
STEADY FLOW

Intermittency (the percentage of time spent by the fluid in a turbulent regime) at measuring stations A and C (280 and 563 diameters from the inlet), and growth rate between the stations are presented as functions of the Reynolds number by the solid curves of Figure 4. These transition characteristics for steady flow indicate that below a Reynolds number of 2270 no turbulent slugs were formed. Between 2270 and 2450 few slugs were formed, and they propagated without change in size. Above 2450 the slugs grew with a characteristic velocity, with the percentage of intermittency depending on the distance from the entrance.

Changing the inlet disturbance level would change the values of the Reynolds number at which the various phenomena occur. Most studies indicate, however, that at a Reynolds number of approximately 2000, all disturbances will be damped (13). It should be noted that damping is a function of time, or, since the fluid is in motion, a function of distance down the tube. Consequently, disturbances originating at the entrance may persist for several diameters and give the illusion that the flow is turbulent.

PULSATILE FLOW

Selected results of intermittency and growth rate, as functions of the mean flow Reynolds number for pulsatile flow, are presented as broken curves in Figure 4. In this plot, the frequency parameter and flow-amplitude ratio are independent parameters. Since the inlet disturbance level remained unchanged, the differences between the steady flow and pulsatile flow transition characteristics are due only to the presence of a periodic component. Note that at higher Reynolds number the intermittency for pulsatile flow is greater than that for steady flow. Either more slugs are generated per unit of time, or the

\[ R = \bar{R} \left(1 + \frac{\dot{\alpha}}{\alpha} \sin \omega t\right) \]

\[ \tau = \frac{\theta \nu}{2 \alpha} \left(R^c - R\right) \]

\[ \frac{\theta \nu}{\alpha^2} = \frac{3\pi}{\alpha^2} - \frac{2}{\alpha^2} \sin^{-1} \left(\frac{R^c - R}{\alpha} \right) \]

FIGURE 3
Reynolds number (R) as a function of time (t) in simple sinusoidal flow. \( \bar{R} = \) mean flow Reynolds number. The relaxation time is very nearly equal to the crosshatched area.
growth rate increases. A study of the lower graph of Figure 4 shows that the growth remains unchanged or decreases. Consequently, the increased intermittency of periodic flow is due only to an increase in the frequency of slug generation. These results indicate further that a combination of high frequency and high flow-amplitude ratio (Δ) or low frequency and low flow-amplitude ratio (□) yield an unchanged growth rate. On the other hand, a marked decrease in growth rate is observed for flow with a low frequency and large flow-amplitude ratio (□). These observations can be explained by the relaxation-time concept and will be discussed.

The independent effects of the flow-amplitude ratio and frequency parameter on the growth rate are shown in Figure 5. Increasing
the flow-amplitude ratio depresses the growth; decreasing the frequency also depresses the growth. Both trends are accentuated at low Reynolds numbers. A striking result shown in the lower half of Figure 5 is the existence of suitable combinations of low frequency and high flow-amplitude ratio such that only laminar flow is observed at the two downstream locations, although simultaneous visual observation of the entrance region shows that this region is highly disturbed. For a given low frequency (Fig. 6) the intermittency is also observed to decrease at large values of flow-amplitude ratio. This decrease becomes most apparent if it is compared with a predicted intermittency (broken lines, Fig.

**Figure 5**

Upper graph: Growth rate (G) as function of the flow-amplitude ratio (Q/Q) at three different Reynolds numbers and one frequency. Q/Q = 0 represents results from steady flow.

Lower graph: Growth rate (G) as a function of the frequency parameter (a). Note particularly the absence of turbulence (outside of the entrance region, see text) at low values of frequency.
Intermittency at location A as a function of the flow-amplitude ratio at constant frequency, with Reynolds number as parameter. The broken curves are explained in the text.

6) obtained by a time integration of the steady-flow intermittency curve at A (Fig. 4).

The ability to destroy turbulence completely in the downstream portions of the tube is further indicated in Figure 7, a semiquantitative graph of the approximate distance traveled by a slug versus the relaxation time. At a relaxation time above 35, slugs will not propagate much beyond 280 diameters from the entrance.

The results of Figures 4 through 7 indicate that in pulsatile flow there exist combinations of Reynolds number, flow-amplitude ratio, and frequency parameter, such that the growth rate of slugs becomes substantially less than the steady-flow value, at times leading to the complete destruction of turbulence. We shall discuss this result further.

An important observation crucial to the development of a comprehensive theory of transition in periodic flow must be noted: in straight smooth tubes, no turbulent slugs were observed in the downstream region if they had not first appeared in the entrance region, a distance of approximately the first hundred diameters. This observation has been previously reported for steady flow (11) and was noted in both the steady flow and periodic flow portions of this study. It was observed, however, that under conditions of rapid deceleration, laminar flow frequently broke down...
spontaneously (Fig. 8). This breakdown occurred during the backflow phase of flows with large oscillatory components. This type of disturbance cannot be considered true turbulence because it does not have the fine structure of a turbulent slug, and because it is destroyed as soon as the fluid accelerates into forward flow.

Discussion

The results of this investigation are most meaningful when discussed within the framework of a physical model (Fig. 9), formulated by synthesizing certain well-known concepts of boundary layer theory and stability theory. The fluid in a large reservoir can be characterized by the absence of shear and the presence of random disturbances. As the fluid enters the tube, a shear flow develops as the viscous effects diffuse inward from the wall to form a boundary layer whose thick-

![Figure 8](image)

**FIGURE 8**

Transient breakdown of laminar flow due to rapid (right) and less rapid (left) deceleration.

![Figure 9](image)

**FIGURE 9**

The physical model. Upper drawing: the entrance region of a tube; $\delta$ is the boundary layer thickness, $l_e$ the entrance length, $l_c$ the critical length, and $d$, the tube diameter (not to scale, $le/d$ is too small). Lower drawing: the abscissa is the number of diameters from the inlet; the ordinate on the left is the Reynolds number $R_e$ based on boundary layer thickness (this Reynolds number is proportional to the Reynolds number based on tube diameter), and the ordinate on the right is the number of disturbances passing a cross section each second. If transition to turbulence does not occur within the critical entrance length, all disturbances will be damped as they are carried downstream into the developing boundary layer, and the flow will be laminar.
ness increases with distance from the entrance. Small random disturbances are constantly transported into the tube from the reservoir. Below the critical Reynolds number of about 2000, all disturbances will be dissipated by viscous effects in the boundary layer. Above this Reynolds number an instability may develop in the boundary layer leading to a transfer of energy from the mean flow to the disturbance, and transition to turbulence occurs.

In a smooth tube, that is, in the absence of wall disturbances, transition may occur only in the entrance region. Disturbances originating in the reservoir which fail to trigger turbulence in the entrance region will be dissipated once they leave this region regardless of the value of the Reynolds number. Above the critical Reynolds number production of turbulence is intermittent for two reasons. First, only certain types of disturbances will be amplified, and these types may not be carried continuously into the tube. Second, in a system under the action of a fixed pressure gradient between the inlet and outlet, the flow rate will depend on the pressure losses within the system. The increased pressure drop, due to the formation of a slug, results in a reduction of flow and a decrease of Reynolds number, thus enhancing stability and terminating the growth of a disturbance into a slug. The pressure loss then decreases, the Reynolds number increases, and new slugs may form.

This model is consistent with the results of numerous experiments on transition in steady flow. It is also consistent with mathematical theories of stability which predict the instability of the entrance region and the stability of the fully developed parabolic velocity profile (14). In the absence of a mathematical theory of stability for unsteady flows, one cannot assert rigorously the validity of this model for oscillating flow. However, the experimental result that turbulent slugs were never observed downstream without first having been observed in the entrance region, presents a strong argument that the velocity profile in oscillating flow is also stable to small disturbances. The increase in intermittency at low values of relaxation time is the result of an increased instability in the entrance velocity profile. The fully developed oscillating profile appears to be more stable than the steady parabolic profile, as indicated by the decreased growth rate. This apparent contradiction, whereby a flow is turbulent more of the time and yet is more stable, is a further indication of the need to separate the stability problem from the transition problem (15).

The effects of a periodic component on the mean flow transition characteristics can best be interpreted by the relaxation-time concept. In a given physical system, the viscosity, radius, and critical Reynolds number remain constant. The relaxation time (Fig. 3) will, therefore, vary inversely with the frequency parameter and mean flow Reynolds number, and directly with the flow-amplitude ratio. Since these quantities are also the similarity parameters which have been systematically varied in this experiment, the presentation of the results (Figs. 4 through 7) indicates directly the influence of relaxation time. Flows with a low frequency and large amplitude of periodic component will show a growth rate significantly lower than steady flows with the equivalent d-c level. Furthermore, at sufficiently large values of the relaxation time, entrance-generated turbulence can be destroyed within a finite distance from the point of origin. This conclusion follows logically from the result that the oscillating velocity profile is stable. If turbulence can only be generated in the unstable entrance region, and if the flow goes through a phase in which it spends a suitable period of time in a dissipative regime, then it necessarily follows that a turbulent slug will propagate no further from its origin than the distance it will travel during one cycle.

Let us imagine a Reynolds "demon" who sits about 100 diameters downstream from the entrance to a tube and becomes activated when the Reynolds number falls below critical. When active, our demon attempts to destroy all nonlaminar flow variations and permits the passage of laminar flow only. His effectiveness depends on the severity of the ap-
approaching disturbances and the time in which he is allowed to remain active. If all the disturbances in the system are upstream from the Reynolds demon, and if we provide him with sufficient activation time (from the point of view of the fluid, relaxation time), then it follows that the downstream flow will always be laminar.

The same reasoning may be used to extend these results to a disturbed flow. At low Reynolds number the flow downstream of an orifice, for example, may be quite disturbed although transition to turbulence cannot occur. In a pulsating flow these disturbances will propagate no further from their source than the distance they can travel in one cycle.

**PHYSIOLOGICAL IMPLICATIONS**

The sinusoidal flow studied here differs markedly from arterial blood flow. In a typical in-vivo flow (Fig. 10) the systolic phase occupies only one-third of the cycle and is characterized by extremely large accelerations to a peak flow which is very much greater than the mean flow. The flow-amplitude ratio in Figure 10 is 6.5. The diastolic phase occupies the remaining two-thirds of the cycle and is accompanied by a small amount of net back-flow, and by an extremely large relaxation time. Furthermore, the arterial system differs markedly from the system we used: during the rapid ejection phase, jet mixing occurs in the aorta; the direction of the flow is reversed in the aortic arch; the descending aorta is branched, tapered, flexible, and perhaps hydraulically rough. All of these differences preclude any direct application of our results to arterial flow; but in the hope that some further insight into the physiological problem could be achieved, an experimental program was undertaken to simulate qualitatively the systolic phase of flow. Visual observations were made under the extreme flow conditions listed in Table 2. Several general observations concerning the flow downstream from the entrance can be made.

1. Whenever reversal of flow occurred in the core region, disturbances appeared at the time of flow reversal (Fig. 8): as a rule these disturbances were restricted to the core and were destroyed as the flow accelerated. One would suspect that the breakdown of streamlined flow was due to the exaggeration of small flow asymmetries by the large decelerative forces. Apparently the lift and drag forces on the vortices brought them into the axial region, where they were destroyed by partial conversion to kinetic energy, rather than by complete viscous dissipation. We should expect, therefore, a large portion of the arterial system to be in a state of disturbed flow at the onset of diastole; but both the amount of energy loss due to viscous...
dissipation and the amount of mixing would be less than energy loss and mixing caused by turbulence.

2. At maximum flow, whether forward or backward, the flow was laminar and undisturbed. Therefore, large instantaneous Reynolds numbers during systole do not result in transition to turbulence everywhere, but only at sources of disturbances. The distal system need not be turbulent. This observation is consistent with the theory and experimental results presented above, and is significant because it departs from current ideas. We do not, however, conclude that *arterial* blood flow is laminar during systole, but rather that flow in tubes will be turbulent during systolic acceleration only if two conditions are met: a large shear rate and a source of instability.

3. At large values of the periodic component, fine structured slugs (Fig. 8, photograph on right) occasionally formed at the moment of flow reversal and were destroyed as the flow accelerated in the forward direction. Note that this disturbance did not appear at the peak forward flow, but rather at the minimum reverse flow.

4. The flow was much more stable to a disturbance created by removing fluid with a syringe in the wall (as a branch would do in an artery) than to disturbances created by injecting small amounts of fluid. Further work is necessary to evaluate the physiological implications of points 3 and 4.

On the basis of these observations it is reasonable to speculate that arterial blood flow is probably disturbed but not truly turbulent, with the disturbances being greatest in the ascending aorta. A recent in-vivo study by Freis and Heath (4), who used both infusion-detection and cineangiography, arrived at essentially the same conclusion.

The results of this investigation lead us to question seriously theories of transition which rely on instantaneous peak Reynolds numbers or on increased wall drag (16) without also considering possible sources of disturbance and the influence of relaxation during diastole. Furthermore, many of the classical aspects of the stability-transition problem, some of which have been applied indiscriminately to the physiological system, may be evaluated critically on the basis of this study.

The existence of a point of inflection in the velocity profile is a well-known cause of instability in boundary layer flow (17). Since a pulsatile flow may, under conditions of large flow-amplitude ratio, possess points of inflection, this has been suggested as a criterion of instability in arterial blood flow (18, 19). A careful reading of the engineering literature (17, 20) shows, however, that the point of inflection criterion has been studied only in flat-plate flow and not in tube flow; and in flat-plate flow, the point of inflection is always associated with backflow and separation due to an adverse pressure gradient (17). Points of inflection in oscillatory flow arise from inertial effects in the axial region and not in the boundary layer, and they need not produce separation even though they are accompanied by backflow. In effect, an adverse pressure gradient does not exist in constant-area tubes, as was observed in this experiment. According to Streeter et al. (21), the tapered arterial system may produce separation during backflow; more work is required to verify this, but it is certainly incorrect to apply stability theory arbitrarily from flat-plate flow to tube flow.

Studies in fluid mechanics have apparently also prompted both engineers and physiologists to consider the curvature of the aortic arch

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**Table 2**

<table>
<thead>
<tr>
<th>$\omega$ (sec$^{-1}$)</th>
<th>$a$</th>
<th>$R$ (cm/sec)</th>
<th>$Q/Q_0$</th>
<th>$R$ (cm/sec$^2$)</th>
<th>$t_{50}$ (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>4.2</td>
<td>0-3500</td>
<td>0-0.46</td>
<td>1000-5100</td>
<td>0.76</td>
</tr>
<tr>
<td>1.9</td>
<td>9.3</td>
<td>0-3500</td>
<td>0-2.2</td>
<td>7600-11,200</td>
<td>17.1</td>
</tr>
</tbody>
</table>

*See Table 1 for definition of symbols.
†Average over the first 50% cycle.
and the acceleration of blood during systole to be stabilizing factors. The large curvature of the arch, however, may result in flow separation and thus act as a source of disturbances (20); any stabilizing influence of centrifugal forces will only be of secondary importance. The influence of acceleration is also somewhat doubtful. It is well known that accelerating flows are more stable because they retard separation (17). The investigation discussed in this paper also indicates that in smooth tubes the accelerating fully developed flow is stable. However, present work by the author indicates that an accelerating flow is far more unstable at a source of disturbances, such as an orifice or a body placed in the stream, than is a steady flow. That is, a flow accelerated from zero velocity to a given Reynolds number may generate more disturbances than a steady flow at the same Re-ynolds number. Again, although further experimental work is required to ascertain the effects of acceleration and curvature, it is incorrect to extrapolate results and concepts indiscriminately from engineering systems to physiological systems.

The physiological system is decidedly more complex than the system studied herein; nevertheless, it is hoped that this study will provide some further insight into the in-vivo problem, and stimulate further research into more complex physiological analogues.

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References

Laminar-Turbulent Transition Process in Pulsatile Flow

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