Theoretic Analysis of the Influence of Heart-Dipole Eccentricity on Limb Leads, Wilson Central-Terminal Voltage and the Frontal-Plane Vectorcardiogram

By ERNEST FRANK, PH.D.

This is a theoretic investigation of the influence of the position of a current dipole on the electric potential produced at the boundary of a homogeneous conducting sphere. Thirty different eccentric dipole positions are selected within the sphere to bear analogy to locations of the equivalent dipole of the human heart within the chest, and boundary-potential calculations are made from points selected to bear analogy with body-surface points utilized in limb leads, Wilson central-terminal and frontal-plane electrocardiography. The point of view taken differs completely from the scalene-triangle geometric representation of eccentricity; instead, an equilateral triangle is used in conjunction with assigned strength and orientation time-variations for the eccentric dipole.

It is concluded that if electrocardiography were subject to no other limitation of the application of field analysis, eccentricity of dipole position alone can produce very significant deviations between the manifest heart vector and the true heart vector in both direction and magnitude. The dependence of the Wilson central-terminal voltage on dipole eccentricity is likewise appreciable and of the same order of magnitude as reported by several experimenters; however, the effects of eccentricity on the Wilson central-terminal voltage are generally less pronounced than on the manifest heart vector.

The use of a current dipole to represent the electrical activity of the human heart has been widespread in the theory and practice of electrocardiography. In particular, the unification by Einthoven of standard limb-lead data into a single entity, the manifest heart vector, is made possible by the current-dipole concept. Also, the mean electric axis, ventricular gradient and other quantities related to the manifest heart vector have provided useful criteria for the analysis of electrocardiographic waveforms, and have assisted in their interpretation. Moreover, the new field of spatial vectorcardiography has as its cornerstone the current dipole representation. Such extensive use of this concept is due in large measure to its simplicity. Despite its wide application, however, it is recognized that dipole heart theory involves many approximations whose limits of error have not been firmly established quantitatively. Many controversies on this matter have been indulged in over the past 20 or more years.

Among the approximations used in heart-dipole theory, attention is here focused on the position of the current dipole. It has long been recognized that the use of a dipole at the center of a spherical conducting medium is not in accord with the anatomic situation so far as the heart is concerned; nevertheless, a centric dipole has been and still is usually used as an approximate representation. However, the position of the dipole has received some attention in recent years with the outstanding work of Burger and van Milaan whose theoretic and experimental work is based upon an eccentrically located dipole; the work of Wilson and his colleagues has also included nongentric dipole considerations. They and others have found that the position of the current dipole in the medium is a factor which may not be ignored.

Burger and van Milaan's analysis of body-
electrode voltages produced by an eccentric dipole has included, in part, a geometric representation of these electrode voltages as projections of the manifest heart vector onto the sides of a scalene triangle, which becomes the equilateral triangle of Einthoven as a special case. This geometric representation has also been supported and elaborated upon by Wilson. In fact, the scalene-triangle device has come to be emphasized as one of the major implications of dipole eccentricity. This is indeed unfortunate because the geometric representation, although a formulation of great nicety, is a mathematic abstraction which demands considerable mathematical insight in order to appreciate the practical significance of dipole eccentricity. This was recognized by Burger and van Milaan in their original presentation of this remarkable extension of vector-projection ideas to the eccentric case, which also includes the effects of inhomogeneities and irregular boundary shape.

The principal aim of this analysis is to deemphasize the nonphysical geometric representation by displaying in graphic, quantitative terms the effects of dipole eccentricity on such practically useful quantities as limb-lead voltages, Wilson central-terminal voltage and the frontal-plane vectorcardiogram. Because it deals with some directly observable entities, this is a clearly understandable approach. The theoretic analysis presented here shows directly and distinctly the extreme importance of dipole eccentricity, while the interpretation of the electrocardiographic significance of various degrees of triangle distortion is rather obscure.

**Method**

It is common practice in the analysis of physical problems to introduce simplifying assumptions in order to acquire an understanding of the gross behavior of a physical system. It has also proved to be a generally fruitful practice to single out for particular study one characteristic of the simplified system by allowing this characteristic to undergo variations not included initially. In this manner, considerable insight can be gained regarding the influence of this selected characteristic, and the essence of the conclusions reached are frequently applicable to the actual physical system. The development of physical science is replete with examples of such procedures; the field of electrocardiography is no exception with its outstanding example of Einthoven's hypothesis.

This general type of approach is here applied to one characteristic of the physical system consisting of the heart and body; namely, the position of the heart dipole. In accordance with this general approach, the Einthoven hypothesis is adopted with one exception; the position and orientation of the heart dipole is left arbitrary in an equatorial plane (the frontal plane) of the homogeneous conducting sphere. The assumption of limb electrodes located at the apices of an equilateral triangle is retained since the major objective is to avoid the artifice of distorted triangles which tend to obscure the very pronounced effects of dipole eccentricity.

The simplified system to be studied is portrayed in figure 1. In this diagram the eccentric position of the dipole is determined by two parameters: \( b \), the distance from the dipole to the sphere center \( O \), and \( \beta \) the angular location of the dipole measured in the counter-clockwise direction from the fixed line joining \( O \) and \( F \). The angle \( \psi \), measured counter-clockwise from the \( z \) axis which passes through the dipole and the sphere center \( O \), serves to define the orientation of the dipole. Finally, the angle \( \theta \), measured clockwise from the \( z \) axis, locates a point on the spherical surface of radius \( R \) at which the electric potential is \( V \). The inclusion of eccentricity necessitates this complication of notation as compared with the more usual centric analysis.

It can be seen from figure 1 that considerable flexibility exists in the heart-dipole position and orientation despite its restriction to the RLF plane. An increase in \( b \) increases the amount of eccentricity, while \( \beta \) can be used to position the dipole around the sphere center. At any given position of the dipole, determined by fixing \( b \) and \( \beta \), the dipole orientation can be controlled by \( \psi \). Meanwhile, the angle \( \theta \) can be varied.

*The position and orientation considered in this analysis are not completely general inasmuch as it is assumed that the dipole lies in the frontal plane. Inclusion of a dipole component perpendicular to the frontal plane accentuates the adverse effects of eccentricity.*
independently so as to determine the boundary potential \( V \) at any point on the sphere for any prescribed dipole position and orientation. Moreover, the dipole moment, \( p \), is also independently controllable.

The method of analysis consists of:

1. Assigning to the heart dipole reasonable variations in strength and orientation as suggested by typical limb-lead voltages; that is, \( p \) and \( \psi \) are prescribed functions of time.
2. Assigning different fixed positions of the dipole which in the sphere are analogous to the space occupied by the heart within the chest; that is, various combinations of \( b \) and \( \beta \) are selected.
3. Allowing the dipole to undergo identically the same variations in strength and orientation, relative to the triangle RLF which is considered to be fixed, at each of the various assigned positions in 2.
4. Computing the limb-lead voltages, Wilson central-terminal voltage and the frontal-plane vectorcardiogram for each of the cases in 3.

Some imagination is required to capture the spirit of this analysis. The true heart vector, assigned in 1, has never been observed; it is only the manifest heart vector which has been measured. However, so far as the purposes of this analysis are concerned, an intelligent supposition of a plausible true heart vector can be made which cannot be seriously in error since similar conclusions regarding dipole eccentricity would be reached for a wide variety of true heart vectors which seem reasonable.

When the dipole is centric, the manifest heart vector is directly proportional to the true heart vector. But for an eccentric dipole the manifest heart vector can depart appreciably from the true heart vector as determined by the position of the dipole, as will be shown in detail. Consequently, the form of the limb-lead voltages depends upon dipole eccentricity as well as the true heart vector. In addition, the Wilson central-terminal voltage, which is theoretically zero only in the centric case, is derivable once the true heart vector is assigned, and the manner in which this voltage depends upon dipole eccentricity can be investigated theoretically.

**Analysis and Results**

This analysis is confined to the QRS complex; however, the method can also be applied to the P or T waves, if desired. The true heart-vector amplitude, \( p \), which is the dipole moment, and the true heart vector orientation \( \psi \) were synthesized as a compromise between QRS complexes as seen in normal subjects and a desire for simplicity. However, any variations of \( p \) and \( \psi \) could be used in an analysis of this type. The \( p \) and \( \psi \) shown in figure 2 represent about the simplest variations which are not unreasonably different from estimates derived from the normal QRS complex.

It can be seen in figure 2a that \( \psi \) is a simple linear function of time. The equation for this \( \psi \) is

\[
\psi = 220° - 40t 
\]

where the time \( t \) is in hundredths of a second; for example, at \( t = 3 \) (0.03 second), \( \psi = 220° - 40(3) = 100° \). The equation for \( p \) during the time interval \( 2 \leq t \leq 8 \) is

\[
p = \frac{p_{max}}{1 + (t - 5)^2}, \quad 2 \leq t \leq 8
\]

where \( p_{max} \) is a constant designating the maxi-
the maximum value of \( p \), and \( t \) is in hundredths of a second; thus, at \( t = 4 \) (0.04 second), \( p = \frac{p_{\text{max}}}{2} \). Equation 2 describes \( p \) only in the indicated time interval; the actual \( p \) used in the analysis, and given in figure 2, trails off more rapidly than given by equation 2 in the intervals \( 0 \leq t < 2 \) and \( 8 < t \leq 10 \). Moreover, \( p \) is zero for \( t \) less than zero or greater than 10 (0.01 second), and this establishes the duration of the QRS complex. Since \( p \) is so small for the first and last 0.01-second intervals, the QRS duration is substantially 0.08 second. A function of \( p \) that is symmetric about the maximum value \( p_{\text{max}} \) (which occurs at \( t = 0.05 \) second) was selected for simplicity and also to obtain a symmetric frontal-plane vectorcardiogram in the centric case.

The polar diagram in figure 2b, derived from the \( p \) and \( \psi \) presented in figure 2a, contains essentially the same information as the two curves in figure 2a and is the more commonly used form of presenting heart-vector data in electrocardiography. The angle at which the maximum value of \( p \) occurs is \( \psi_{\text{max}} = 20^\circ \) as can be seen from either figure 2a, figure 2b or equation 1 with \( t = 5 \).

In order to determine the limb-lead voltages produced by the prescribed \( p \) and \( \psi \) for the case of a centric dipole, the potential can be computed from the well known equation
\[
V = \frac{3p \cos (\theta + \psi)}{4\pi \gamma R^2}
\]
where \( (\theta + \psi) \) is the total angle between the dipole axis and a point on the boundary where

\[
\begin{align*}
V_I &= V_L - V_R \\
V_{II} &= V_V - V_R \\
V_{III} &= V_R - V_L
\end{align*}
\]
This procedure can be repeated for various instants of time during the QRS interval so that the limb-lead waveforms can be developed as a function of time. The frontal-plane vectorcardiogram can then be constructed in the usual manner from any two of these three limb-lead voltages.

The frontal-plane vectorcardiogram has, in the centric case, exactly the same shape as the true heart-vector loop given in figure 2b and the maximum value $E_{\text{max}}$ of the manifest heart-vector amplitude $E$ also occurs at $\psi_m = 20^\circ$. In terms of Einthoven’s angle $\alpha$ (measured clockwise from the line joining $R$ and $L$ of figure 1 to the manifest heart-vector direction) $E_{\text{max}}$ is seen to occur at $\alpha_m = 70^\circ$. However, the vectorcardiogram is diminished in amplitude owing to the short-circuiting effect of the medium. Thus, for the case of a centric dipole the manifest heart vector is proportional to the true heart vector and, consequently, gives a faithful reproduction of the relative variations of the heart dipole. The Wilson central-terminal voltage is not shown in figure 3 because, in the centric case, it is identically zero for all instants of time during the QRS complex for any values of $p$ and $\psi$.

The calculation of the limb-lead voltages produced by an eccentric dipole is no different, which can be assigned a specific numeric value, if desired. As a typical example, take $E_{\text{max}} = 1.5$ millivolts (mv). Then the maximum values of the limb-lead voltages are $V_L = 0.77$ mv, $V_N = 1.48$ mv, $V_M = 1.04$ mv.

Using this method of calculation, the limb-lead voltages and frontal-plane vectorcardiogram given in figure 3 for the case of a centric dipole, $b = 0$, result from the time variations of $p$ and $\psi$ prescribed in figure 2. It can be seen that these are not unreasonable waveforms for a normal subject, at least in general shape.*

* The relationship between $E_{\text{max}}$ and $p_{\text{max}}$ is, in the centric case,

$$E_{\text{max}} = \frac{3\sqrt{3}p_{\text{max}}}{4\pi R^2}$$

The limb leads are expressed as a fraction of $E_{\text{max}}$. 

Fig. 3. Limb-lead voltages and frontal-plane vectorcardiogram produced during the QRS complex by the true heart-vector variations given in figure 2 for the special case of a centric dipole. The time $t$ is expressed in hundredths of a second.

Fig. 4. Thirty eccentric dipole positions in the frontal plane at which calculations were made are indicated by the points on the circles $b = 0.1R, 0.2R, 0.3R, 0.4R$. The calculation of the limb-lead voltages produced by an eccentric dipole is no different, which can be assigned a specific numeric value, if desired. As a typical example, take $E_{\text{max}} = 1.5$ millivolts (mv). Then the maximum values of the limb-lead voltages are $V_L = 0.77$ mv, $V_N = 1.48$ mv, $V_M = 1.04$ mv.
in principle, from that described for the centric case. However, a more complicated equation than equation 3 must be used to compute $V_a$, $V_e$, and $V_f$; namely,

\[ V = \frac{p}{4\pi\gamma R b} \left\{ \cos \psi \left[ \frac{1 - f^2}{(1 + f^2 - 2f\mu)^{3/2}} - 1 \right] \right. \\
\left. - \sin \psi \frac{3f - 3f^3\mu + f^4 - \mu}{\sin \theta (1 + f^2 - 2f\mu)^{3/2} + \mu} \right\} \]  

(5)

Central-terminal voltage computation is based on the well-known expression

\[ V_{cr} = \frac{1}{3} (V_a + V_e + V_f) \]  

(6)

Limb-lead voltages, the Wilson central-terminal voltage and the frontal-plane vector-cardiogram were computed for 30 different dipole positions, indicated by the points in figure 4. These positions were selected to bear analogy to locations of the equivalent dipole of the human heart within the chest, based upon a study of frontal-plane x-ray photographs. A summary of the results is portrayed graphically in figures 5, 6, and 7. A more detailed and quantitative analysis of the results is given elsewhere.

\[ V = \frac{p \cos \psi (3 - f)}{4\pi\gamma R^2(1 - f)^2} \]

and

\[ V = \frac{-p \cos \psi (3 + f)}{4\pi\gamma R^2(1 + f)^2} \]

respectively.

* The limiting values of equation 5 for $\theta = 0^\circ$ and $\theta = 180^\circ$ become

\[ V = \frac{p \cos \psi (3 - f)}{4\pi\gamma R^2(1 - f)^2} \]

and

\[ V = \frac{-p \cos \psi (3 + f)}{4\pi\gamma R^2(1 + f)^2} \]

respectively.

* For a detailed description of the method of computation and a numeric table of $V$ computed from equation 5 for a range of values of $\theta$, $\psi$, and $b$ see Frank, E., "Current Fields in Homogeneous Volume Conductors with Special Reference to Electrocardiography," Paper #3, Microfilm available on Interlibrary Loan, University of Pennsylvania Library, Philadelphia, Pa.
FIG. 6. Superposition of frontal-plane vectorcardiograms calculated for the various dipole positions of figure 4. The time, indicated by points, is in hundredths of a second. In each case the frontal-plane vectorcardiogram for a centric dipole is shown for reference. The arrow marked Lead I coincides with the direction R to L of figures 1 and 4. Three of the $b = 0.1R$ cases are not shown.
DISCUSSION

The dependence of the form of the QRS complex on dipole position can be seen in figure 5. The calculated limb-lead voltages $V_l$ and $V_{III}$ all lie within the shaded regions of figure 5 for the indicated eccentricities. The solid curves are the limb-lead voltages obtained in the case of a centric dipole. The effects of eccentricity are quite pronounced. For example, with $b = 0.3R$, the $S$ deflection of $V_l$ can become more than twice that of the centric case (this occurs at $\beta = 100^\circ$). Other deviations can be estimated from the shaded areas of figure 5 which are all drawn to the same scale.

The frontal-plane vectorcardiogram can be seen in figure 6 to be extremely sensitive to dipole position. The value of $E_{max}$ and the angle $\alpha_m$ at which it occurs undergo such great variation with changing dipole position that it is obviously a serious error to ignore eccentricity. A detailed analysis of $E_{max}$ and $\alpha_m$ reveals that the deviation in $\alpha_m$ from the centric case, where $\alpha_m = 70^\circ$, can become as great as 50°. This worst case, of those considered, occurs with $b = 0.4R$ and $\beta = 100^\circ$ which may not be too likely a position for the human-heart dipole. However, even for eccentricities as small as 10 per cent of the radius ($b = 0.1R$) $\alpha_m$ can deviate from the centric case by as much as 10°. The maximum amplitude of the manifest heart vector $E_{max}$ also changes markedly as a function of eccentricity, becoming as much as 80 per cent greater (with $\beta = -10^\circ$ and $b = 0.4R$) than that obtained in the centric case with exactly the same heart-dipole variations.

Although the departures of the Wilson central-terminal voltage, $V_{CT}$, from the centric value of zero are appreciable, they are not as great as the departures of the manifest heart vector from the true heart vector. It can be seen in figure 7 that $V_{CT}$ can reach about 15 per cent of $E_{max}$ (centric) with $b = 0.3R$ which is not an unreasonable value of eccentricity for the human heart. The cases of maximum departures of $V_{CT}$ from zero occur in the vicinity of $\beta = 0^\circ$. However $V_{CT}$ is much smaller when the dipole is approximately equidistant from two apices of the triangle RLF (such as is the case with $\beta = \pm 60^\circ$) as can be seen in figure 7. These maximum departures of $V_{CT}$ from zero are quite close to those reported in experimental investigations and this suggests that dipole eccentricity may be a dominating factor which influences the central-terminal voltage. For example, the results in the following first two columns are summarized in reference 8 for immersion experiments:

<table>
<thead>
<tr>
<th>Reduction of limb leads owing to immersion</th>
<th>Maximum measured deviation of $V_{CT}$</th>
<th>Maximum deviation expected for immersion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eckey &amp; Fröhlich... slight</td>
<td>0.3 mv</td>
<td>0.3 mv</td>
</tr>
<tr>
<td>Burger .................................. 25%</td>
<td>0.26 mv</td>
<td>0.33 mv</td>
</tr>
<tr>
<td>Wilson .................................. 50%</td>
<td>0.15 mv</td>
<td>0.3 mv</td>
</tr>
</tbody>
</table>

If the unimmersed value of $E_{max}$ is taken as 1.5 mv then the per cent $V_{CT}$ departures in these cases are approximately 0.3/1.5 = 20 per cent.

There is both anatomic and electro-experimental evidence that the heart-dipole position...
Influence of Heart-Dipole Eccentricity

The method of analysis presented here can be readily applied to the case of variable eccentricity as a function of time, but there is insufficient information concerning the path taken by the dipole through the heart to assign a realistic variation. The scalene-triangle geometric representation of Burger and van Milaan rests upon the assumption of fixed eccentricity. However, vector-projection ideas can be extended to cases of variable eccentricity by using a sequence of triangles whose shape shrinks and grows from instant to instant during the heart beat. The pursuit of this nonphysical geometric representation thus becomes very awkward with variable eccentricity. This is because the coefficients in the limb-lead voltage equations of Burger and van Milaan are no longer constant if the assumption of fixed dipole position is violated.

The essential conclusion reached on the basis of a homogeneous spherical medium is that the dipole position can produce pronounced deviations of the manifest heart vector from the true heart vector in both direction and magnitude; these variations also show up markedly in the limb-lead voltages. It has also been shown that the effect of dipole eccentricity on the Wilson central-terminal voltage is appreciable but not as marked as on the manifest heart vector. It is, of course, pertinent to speculate on the extension of these results to the human torso which is neither spherical nor homogeneous. It can be safely stated that dipole position is also an important factor in affecting the potentials measured at the surface of the body of a human subject, and should be taken into account if possible. Although the quantitative conclusions based on the homogeneous conducting sphere obviously cannot be applied directly to the human body, they do indicate the order of magnitude of eccentricity effects that might be expected. It is likely that the effects of eccentricity are even greater than given here in the presence of a torso-shaped boundary and human-body inhomogeneities.

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References

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